کتابچه نمونه سوالات چیست:

سایت ما **افتخار** دارد برای اولین بار در ایران توانسته است کتابچه نمونه سوالات تمام دروس پیام نور که هر یک حاوی تمامی آزمون های برگزار شده پیام نور (تمامی نیمسالهای موجود حتی الامکان با جواب) را در یک فایل به نام کتابچه جمع آوری کند و هر ترم نیز آن را آپدیت نماید.

مراحل ساخت یک کتابچه نمونه سوال

(برای آشنایی با زحمت بسیار زیاد تولید آن در هر ترم) :

دسته بندی فایلها – سرچ بر اساس کد درس – چسباندن سوال و جواب – پیدا کردن یک درس در نیمسالهای مختلف و چسباندن به کتابچه همان درس – چسباندن نیمسالهای مختلف یک درس به یکدیگر – وارد کردن اطلاعات تک تک نیمسالها در سایت – آپلود کتابچه و خیلی موارد دیگر..

همچنین با توجه به تغییرات کدهای درسی دانشگاه استثنائات زیادی در ساخت کتابچه بوجود می آید که کار ساخت کتابچه را بسیار پیچیده می کند .

فهرست مطالب

فصل اول

مبانی دینامیک

۱-۱. جرم یک سیب به طور متوسط برابر است با:

$$m = \frac{2\ \text{kg}}{12\ \text{سیب}} = 0.167\,\text{kg}$$

$$W = mg = 0.167\,(9.81) = 1.635\,\text{N}$$

وزن این سیب‌ها تقریباً دو برابر قاعدهٔ سرانگشتی (2 N برای هر سیب) می‌باشد.

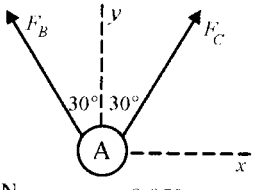
۱-۲. برای شخص ۸۰ کیلوگرمی:

$$W = mg = 80(9.81) = 784.8\,\text{N}$$

۱-۳.

$$F_B = \frac{G\,m_A\,m_B}{{d_{AB}}^2} = \frac{G\left(\rho_A \frac{4}{3}\pi r^3\right)\left(\rho_B \frac{4}{3}\pi r^3\right)}{{d_{AB}}^2}$$

$$= \frac{6.673\,(10^{-11})\left[\frac{4}{3}\pi\,(0.050)^3\right]^2 (8910)\,(2690)}{1^2} = 4.38(10^{-10})\,\text{N}$$



$r = 0.050$ m

برای تمام گوی‌ها

$$F_C = \frac{G m_A m_C}{d_{AC}^2} = \frac{G\left(\frac{4}{3}\pi r^3\right)^2 \rho_A \rho_C}{d_{AC}^2}$$

$$= \frac{6.673\,(10^{-11})\left[\frac{4}{3}\pi\,(0.050)^3\right]^2 (8910)\,(7210)}{1^2} = 1.175(10^{-9})\,\text{N}$$

$$\mathbf{R} = \mathbf{F}_B + \mathbf{F}_C = 4.38\,(10^{-10})\left[-\sin 30°\,\mathbf{i} + \cos 30°\,\mathbf{j}\right] + 1.75\,(10^{-9})\left[\sin 30°\,\mathbf{i} + \cos 30°\,\mathbf{j}\right]$$

$$\mathbf{R} = (3.68\,\mathbf{i} + 13.98\,\mathbf{j})\,10^{-10}\,\text{N}$$

۱-۴.

$$\mathbf{V}_1 = 15\left(\frac{4}{3}\mathbf{i} + \frac{3}{5}\mathbf{j}\right) = 12\,\mathbf{i} + 9\,\mathbf{j}$$

$$\mathbf{V}_2 = 12(-\cos 60°\,\mathbf{i} + \sin 60°\,\mathbf{j}) = -6\,\mathbf{i} + 10.39\,\mathbf{j}$$

$$V_1 + V_2 = 15 + 12 = 27$$

$$\mathbf{V}_1 + \mathbf{V}_2 = (12 - 6)\,\mathbf{i} + (9 + 10.39)\,\mathbf{j} = 6\,\mathbf{i} + 19.39\,\mathbf{j}$$

$$\mathbf{V}_1 - \mathbf{V}_2 = (12 - (-6))\,\mathbf{i} + (9 - 10.39)\,\mathbf{j} = 18\,\mathbf{i} - 1.39\,\mathbf{j}$$

$$\mathbf{V}_1 \times \mathbf{V}_2 = (12\,\mathbf{i} + 9\,\mathbf{j}) \times (-6\,\mathbf{i} + 10.39\,\mathbf{j}) = (124.7 + 54)\mathbf{k} = 178.7\,\mathbf{k}$$

$$\mathbf{V}_2 \times \mathbf{V}_1 = -(\mathbf{V}_1 \times \mathbf{V}_2) = -178.7\mathbf{k}$$

$$\mathbf{V}_1 \cdot \mathbf{V}_2 = (12\,\mathbf{i} + 9\,\mathbf{j}) \cdot (-6\,\mathbf{i} + 10.39\,\mathbf{j}) = 12(-6) + 9(10.39) = 21.5$$

۱-۵.

$$g_h = \frac{Gm_e}{(R + h)^2} = \frac{6.673\,(10^{-11}) \cdot 5.976\,(10^{24})}{[6.371\,(10^6) + 0.25\,(10^6)]^2} = 9.10\,\text{m/s}^2$$

جرم شخص $m = \frac{W}{g} = \frac{880}{9.81} = 89.7\,\text{kg}$

وزن شخص در ارتفاع 250 کیلومتری $W_h = m\,g_h = 89.7(9.10) = 816\,\text{N}$

عبارات «g صفر» و «بی‌وزنی» در این مورد بی‌معنا هستند.

۱-۶.

از معادله (۱-۲): $\frac{G m m_e}{(R_e + h)^3} = \frac{G m m_m}{R_m^2}$

m = جرم ماهواره

m_e = جرم زمین

جرم مریخ = $m_m = 0.107\, m_e$

شعاع زمین = $R_e = 6371$ km

شعاع مریخ = $R_m = 3394$ km

بنابراین: $R_e + h = R_m\sqrt{\frac{m_e}{m_m}} = 3394\sqrt{\frac{1}{0.107}} \Rightarrow h = 4000$ km

$g_{rel} = 9.780327\,(1 + 0.005279\sin^2\gamma + 0.000023\sin^4\gamma + \ldots)$

در : $\gamma = 40°$ $g_{rel} = 9.801698\ \text{m/s}^2$

$$g_{abs} = g_{rel} + 0.03382\cos^2\gamma$$
$$= 9.801698 + 0.03382\cos^2 40°$$
$$= 9.821544\ \text{m/s}^2$$

$$W_{abs} = m\,g_{abs} = 90(9.821544) = 883.9\,\text{N}$$

$$W_{rel} = m\,g_{rel} = 90(9.801698) = 882.2\,\text{N}$$

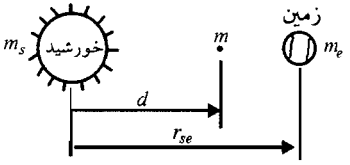
۸-۱. قانون جاذبه عمومی نیوتن: $\frac{Gmm_s}{d^2} = \frac{G\,m\,m_e}{(r_{se} - d)^2}$

$$d^2[m_s - m_e] - d[2\,m_s\,r_{se}] + m_s r_{se}^2 = 0$$

$m_e = 5.976\,(10^{24})$ kg

$m_s = 333000\,[5.976\,(10^{24})]$ kg

$r_{se} = 149.6\,(10^9)$ m



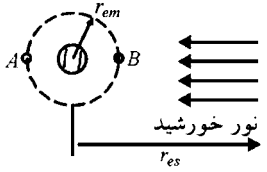
با جایگزینی مقادیر فوق و حل معادله خواهیم داشت:

$d = 149.3\,(10^9)$ m

$d = 149.9\,(10^9)$ m

۹-۱. نیرویی که زمین بر ماه وارد می‌کند:

$$F_e = \frac{G\,m_e\,m_m}{r_{em}^2} = \frac{(6.673\times10^{-11})\,(5.976\times10^{24})^2\,(1)\,(0.0123)}{(3.84398\times10^8)^2}$$
$$= 1.984\times10^{20}\,\text{N}$$



نیرویی که خورشید بر ماه وارد می‌کند:

$$F_{SA} = \frac{G\,m_s\,m_m}{(r_{es} + r_{em})^2} = \frac{(6.673\times10^{-11})\,(5.976\times10^{24})^2\,(333000)\,(0.0123)}{(1.496\times10^{11} + 3.84398\times10^8)^2}$$
$$= 4.34\times10^{20}\,\text{N}$$

$$F_{SB} = \frac{G\, m_s\, m_m}{(r_{es} - r_{em})^2} = 4.38\times10^{20}\,\text{N}$$

$$F_{SB} = \frac{F_{SA}}{F_e} = 2.19 \quad , \quad R_B = \frac{F_{SB}}{F_e} = 2.21$$

۱-۱۰.

$$C_D = \frac{D}{\frac{1}{2}\rho v^2 S}$$

$$|C_D| = \frac{M\,L\,T^{-2}}{\left(\frac{M}{L^3}\right)\left(\frac{L}{T}\right)^2 L^2} = 1 \quad \Rightarrow \quad C_D \text{ بدون بُعد است}$$

۱-۱۱.

$$W = mg \quad , \quad g = g_0\left(\frac{R}{R+h}\right)^2$$

طبق شکل ۱-۱ در عرض جغرافیایی 28° N و سطح دریا:

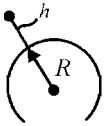
$g_0 = 9.818\,\text{m/s}^2$

در $h = 2440\,\text{m}$:

$$g = 9.818\left[\frac{6371\,(10^3)}{6371\,(10^3) + 2440}\right] = 9.810\,\text{m/s}^2$$

در $h = 8848\,\text{m}$:

$$g = 9.818\left[\frac{6371\,(10^3)}{6371\,(10^3) + 8848}\right] = 9.791\,\text{m/s}^2$$



$$\Delta W = m\,\Delta g = 80\,(9.810 - 9.791) = 1.576\,\text{N}$$

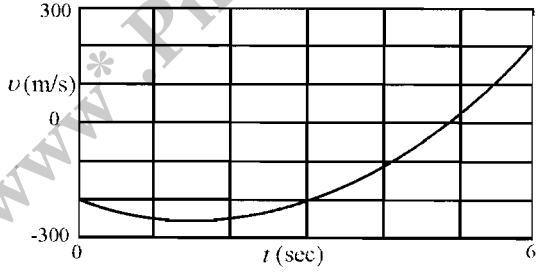
فصل دوم

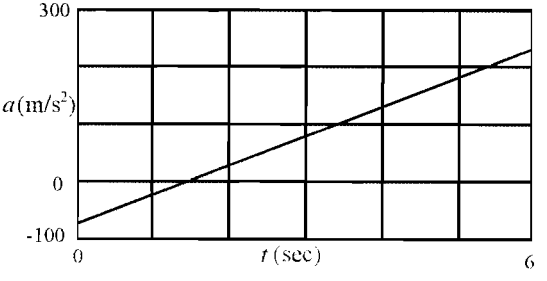
سینماتیک ذرات

۲-۱.

$$v = 25t^2 - 80t - 200$$
$$a = \frac{dv}{dt} = 50t - 80$$

} به نمودارها رجوع کنید





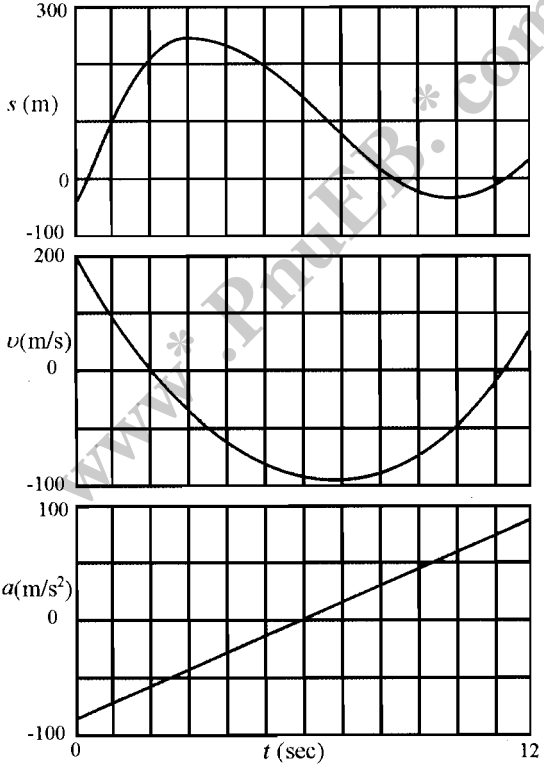
$a = 0 \quad : \quad 50t - 80 = 0 \quad \Rightarrow \quad t = 1.6\,\text{sec}$

$t = 1.6$ sec در : $v = 25(1.6)^2 - 80(1.6) - 200 = -264$ m/s

۲-۲.

$$\left.\begin{aligned} s &= 2t^3 - 40t^2 + 200t - 50 \\ v &= \frac{ds}{dt} = 6t^2 - 80t + 200 \\ a &= \frac{dv}{dt} = 12t - 80 \end{aligned}\right\}$$ به نمودارها رجوع کنید

$$v = 0 \; : \; 6t^2 - 80t + 200 = 0 \quad \Rightarrow \quad t = \frac{80 \pm \sqrt{80^2 - 4(6)(200)}}{2(6)} \quad \Rightarrow \quad t = 3.33\,\text{s}\,,\ 10\,\text{s}$$



۳-۲.

$$v = 2 - 4t + 5t^{\frac{3}{2}}$$

$$a = \frac{dv}{dt} = -4 + \frac{15}{2}t^{\frac{1}{2}}$$

$$\frac{ds}{dt} = 2 - 4t + 5t^{\frac{3}{2}}$$

$$\int_{s_0=3}^{s} ds = \int_0^t \left(2 - 4t + 5t^{\frac{3}{2}}\right) dt$$

$$s = 3 + 2t - 2t^2 + 2t^{\frac{5}{2}}$$

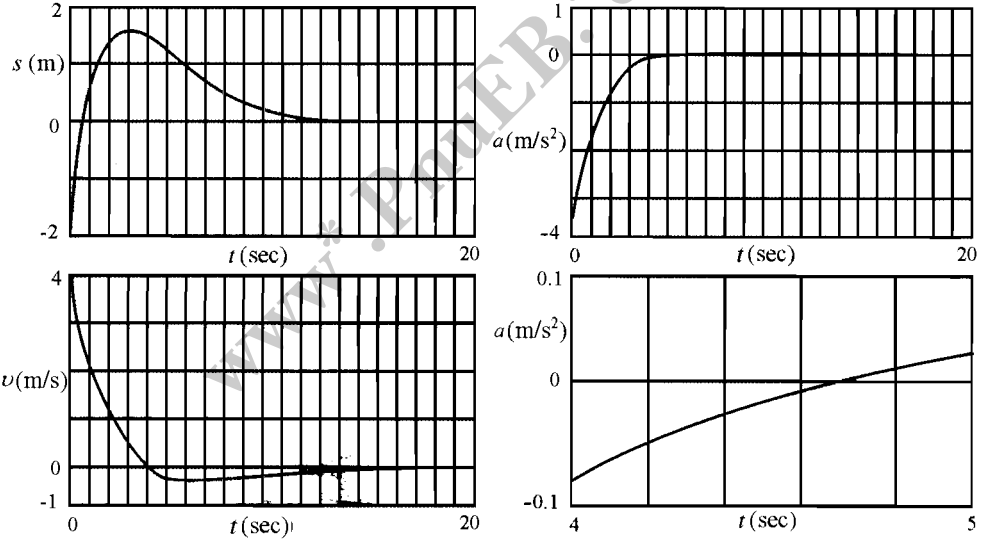
$t = 3\,\text{s}$ در :
$$\begin{cases} s = 22.2\,\text{m} \\ v = 15.98\,\text{m/s} \\ a = 8.99\,\text{m/s}^2 \end{cases}$$

$$s = (-2 + 3t)\, e^{-0.5t}$$

$$v = \frac{ds}{dt} = 3e^{-0.5t} + (-2 + 3t)(-0.5)\, e^{-0.5t} = (4 - 1.5t)\, e^{-0.5t}$$

$$a = \frac{dv}{dt} = -1.5e^{-0.5t} + (4 - 1.5t)(-0.5)\, e^{-0.5t} = (-3.5 + 0.75t)\, e^{-0.5t}$$

$$a = 0 : (-3.5 + 0.75t)\, e^{-0.5t} = 0 \quad \Rightarrow \quad t = 4.67\,\text{s}$$



۵-۲

$$a = \frac{dv}{dt} = 4t - 30$$

$$\int_{v_0=3}^{v} dv = \int_0^t (4t - 30)\, dt \quad \Rightarrow \quad v = 3 - 30t + 2t^2 \text{ m/s}$$

$$\frac{ds}{dt} = 3 + 2t^2 - 30t$$

$$\int_{s_0=-5}^{s} ds = \int_0^t (3 + 2t^2 - 30t)\, dt \quad \Rightarrow \quad s = -5 + 3t - 15t^2 + \frac{2}{3}t^3 \text{ m}$$

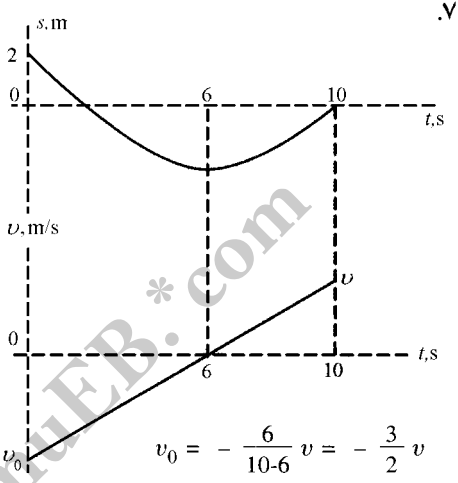
۲-۶.

$$\dot{s} = 40 - 3t^2$$

$$\Delta s = \int_2^4 (40 - 3t^2)\,dt = (40t - t^3)\Big|_2^4 = 96 - 72 = 24\ \text{m}$$

۲-۷.

مساحت زیر منحنی $\Delta s = v - t$



$$0 - 2 = -\frac{3}{2} v \left(\frac{6}{2}\right) + \frac{1}{2} v\,(4)$$

$$v = 0.8\ \text{m/s}$$

$$a = \frac{dv}{dt} = \frac{0.8}{4} = 0.2\ \text{m/s}^2$$

$$v_0 = -\frac{6}{10-6} v = -\frac{3}{2} v$$

۲-۸.

$$y = v_0 t + \frac{1}{2} a t^2 \Rightarrow y = 30t - \frac{1}{2} 9.81\, t^2$$

برای $y = -60$ m : $-60 = 30t - \frac{9.81}{2} t^2 \Rightarrow 4.905\, t^2 - 30t - 60 = 0$

$$\Rightarrow t = \frac{30 \pm \sqrt{(30)^2 + 4(4.905)(60)}}{2(4.905)} = 7.70\ \text{s}$$

$$\dot{y} = 0 \Rightarrow v^2 = v_0^2 + 2ay \Rightarrow y = h = \frac{0 - 30^2}{-2(9.81)} = 45.9\ \text{m}$$

۲-۹.

$$a = v\frac{dv}{ds} = 10(-3) = -30\ \text{m/s}^2$$

۲-۱۰.

برای شتاب ثابت : $s = \frac{1}{2} a t^2 \Rightarrow t = \left(\frac{2s}{a}\right)^{\frac{1}{2}} = \left(\frac{2(30000)}{1.5\,(9.81)}\right)^{\frac{1}{2}} = 63.95$

$$v = \sqrt{2as} = \sqrt{2(1.5)(9.81)(30000)} = 940\ \text{m/s}$$

۲-۱۱. برای شتاب ثابت : $v^2 = v_0^2 + 2as$

$$\left[\frac{300(1000)}{3600}\right]^2 = 0^2 + 2a(100) \Rightarrow a = 34.7\ \text{m/s}^2$$

$$a = \frac{34.7}{9.81} = 3.54\,g$$

۲-۱۲.
$$\left.\begin{aligned} v^2 &= v_0^2 + 2as \\ a &= \frac{g}{6} \end{aligned}\right\} \Rightarrow v^2 = 2^2 + 2\left(\frac{9.81}{6}\right)(5) \Rightarrow v = 4.51\ \text{m/s}$$

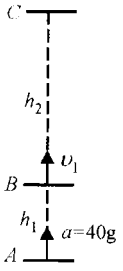
۲-۱۳.
$$C \text{ تا } B : t = \frac{10}{2} = 5\,\text{s}$$

$$v = v_1 + at \Rightarrow 0 = v_1 - 9.81(5) \Rightarrow v_1 = 49.0\ \text{m/s}$$

$$v^2 = v_1^2 + 2as \Rightarrow 0 = (49.0)^2 + 2(-9.81)h_2 \Rightarrow h_2 = 122.6\ \text{m}$$

$$B \text{ تا } A : v^2 = v_0^2 + 2as \Rightarrow (49.0)^2 = 0 + 2(40)(9.81)h_1 \Rightarrow h_1 = 3.07\ \text{m}$$

$$h = h_1 + h_2 = 125.7\ \text{m}$$



۲-۱۴. در زمان شتاب مثبت:
$$v = v_0 + at : \frac{22}{3.6} = 0 + \frac{9.81}{4}t_a \Rightarrow t_a = 2.49\ \text{s}$$

توجه داشته باشید که زمان شتاب منفی با زمان شتاب مثبت یکی است $t_d = t_a$

$$v^2 = v_0^2 + 2a\Delta s : \left(\frac{22}{3.6}\right)^2 = 0^2 + 2\,\frac{9.81}{4}\Delta s_a \Rightarrow \Delta s_a = 7.61\ \text{m} = \Delta s_d$$

در زمان سرعت ثابت:
$$\Delta s_c = 350 - \Delta s_a - \Delta s_d = 335\ \text{m}$$

$$\Delta s_c = v_c t_c \Rightarrow 335 = \frac{22}{3.6}t_c \Rightarrow t_c = 54.8\,\text{s}$$

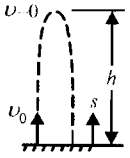
زمان کل $t = t_c + t_a + t_d = 59.8\,\text{s}$

۲-۱۵.
$$v^2 = v_0^2 + 2a(s - s_0)$$

در اوج : $0^2 = 200^2 + 2(-9.81)h \Rightarrow h = 2040\ \text{m}$

$$v = v_0 + at$$

در موقعیت شلیک : $-200 = 200 - 9.81t \Rightarrow t = 40.8\,\text{s}$



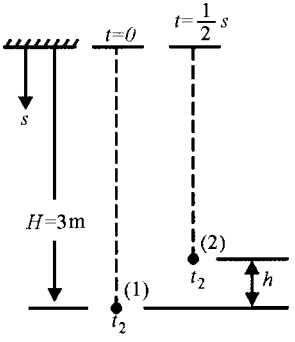
۲-۱۶.

$$s = s_0 + v_0 t + \frac{1}{2} g t^2$$

گوی 1 : $H = \frac{1}{2} g t_2^2$

گوی 2 : $(H - h) = \frac{1}{2} g (t_2 - \frac{1}{2})^2$

$H = 3\,\text{m}$, $g = 9.81\,\text{m/s}^2$ $\xrightarrow[\text{از دو معادله فوق}]{t_2 \text{ حذف}}$ $h = 2.61\,\text{m}$



۲-۱۷.

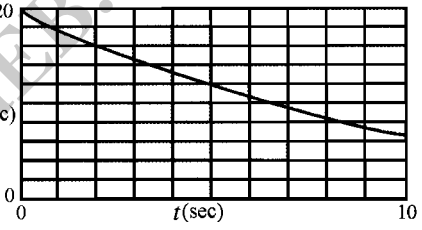
$$v = 20\, e^{-\frac{t}{10}}$$

$$a = \dot{v} = -2\, e^{-\frac{t}{10}}$$

وقتی $t = 10\,\text{s}$: $a = -2\, e^{-\frac{10}{10}} = -0.736\,\text{m/s}^2$

$$v = \frac{ds}{dt} = 20 e^{-\frac{t}{10}} \Rightarrow \int_0^s ds = \int_0^{10} 20\, e^{-\frac{t}{10}}\, dt$$

$$\Rightarrow \quad s = -200\, e^{-\frac{t}{10}} \Big|_0^{10} = 126.4\,\text{m}$$



۲-۱۸.

$$v = \frac{ds}{dt} = 16 \sin \frac{\pi t}{6}$$

$$\int_8^s ds = 16 \int_0^t \sin \frac{\pi t}{6}\, dt$$

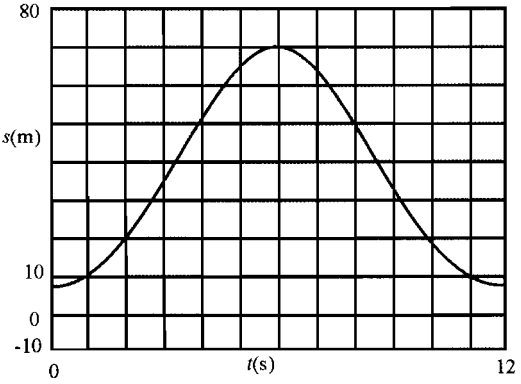
$$\Rightarrow \quad s = 8 + 16 \cdot \frac{6}{\pi} \left(-\cos \frac{\pi t}{6} \right) \Big|_0^t$$

$$= 8 + \frac{96}{\pi} \left[1 - \cos \frac{\pi t}{6} \right]$$

$\cos \frac{\pi t}{6} = -1$ است که $s = s_{\max}$ وقتی

یعنی $t = 6\,\text{s}$ باشد.

$$s_{\max} = 8 + \frac{96}{\pi} [1 - (-1)] = 69.1\,\text{mm}$$



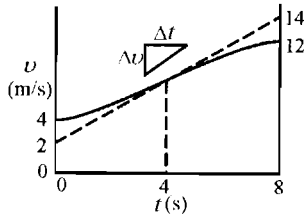
۲-۱۹.

$$a_{AV} = \frac{\Delta v}{\Delta t} = \frac{12 - 4}{8} = 1.00\,\text{m/s}^2$$

$$a_4 = \frac{dv}{dt} = \frac{14 - 2}{8} = 1.50\,\text{m/s}^2$$

$$\Delta a = a_4 - a_{AV} = 1.50 - 1.00 = 0.5\,\text{m/s}^2$$

$$\Delta s = \int v\,dt = \frac{14 + 2}{2}(8) = 64\,\text{m}$$



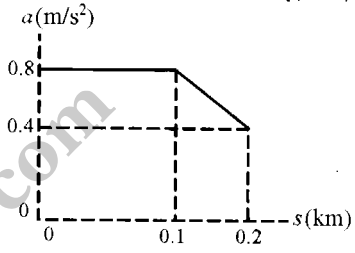
۲-۲۰.

$$a = v\frac{dv}{ds}$$

$$\int_{v_0}^{v} v\,dv = \int_0^{200} a\,ds = \text{سطح زیر منحنی } a - s$$

$$\frac{v^2}{2} - \frac{(40 / 3.6)^2}{2} = 0.8(100) + 0.6(100)$$

$$\Rightarrow v = 20.1\,\text{m/s} \quad \text{یا} \quad 72.3\,\text{km/h}$$



۲-۲۱.

$$\Delta v = \int a\,dt = \text{سطح زیر منحنی } a - t$$

$$t = 4\,\text{s} \quad \text{برای} : v_4 - 100 = -4(9.81)\frac{4 - 2}{2} \Rightarrow v_4 = 60.8\,\text{m/s}$$

$$t = 8\,\text{s} \quad \text{برای} : v_8 - 60.8 = -4(9.81)(6 - 4) \Rightarrow v_8 = -17.72\,\text{m/s}$$

۲-۲۲.

$$v^2 = v_0^2 + 2a(s - s_0)$$

$$0 = 4^2 + 2\left(\frac{-9.81}{4}\right)(s) \Rightarrow s = 3.26\,\text{m}$$

$$v = v_0 + at : \quad 0 = 4 + \left(-\frac{9.81}{4}\right)t_{up} \Rightarrow t_{up} = 1.631\,\text{s}$$

$$t = 2t_{up} = 2(1.631) = 3.26\,\text{s}$$

۲-۲۳.

$$v_0 = \frac{100}{3.6} = 27.8\,\text{m/s}$$

$$a = -g\sin\theta = -9.81\,\sin\left[\tan^{-1}\frac{6}{100}\right] = -0.588\,\text{m/s}^2$$

(الف) $v = v_0 + at = 27.8 - 0.588\,(10) = 21.9\,\text{m/s}$

(ب) $v^2 = v_0^2 + 2a(s - s_0) = 27.8^2 + 2(-0.588)(100) \Rightarrow v = 25.6\,\text{m/s}$

۲-۲۴.

$$a = \frac{1}{2}\frac{d(v^2)}{ds} = \frac{1}{2}\frac{900 - 2500}{400 - 100} = -\frac{8}{3}\ \text{m/s}^2 \quad (\text{ثابت})$$

$$\Delta v = \int a\,dt \quad \Rightarrow \quad v - 50 = -\frac{8}{3}t$$

در نقطه B : $30 - 50 = -\frac{8}{3}t \quad \Rightarrow \quad t = 7.50\,\text{s}$

$$\Delta s = \int v\,dt = \int_{5.5}^{7.5}\left(50 - \frac{8}{3}t\right)dt = 65.3\,\text{m}$$

۲-۲۵.

$$s_{\text{car}} = vt = \frac{120}{3.6}t$$

$$s_{\text{cycle}} = v_{\text{av}}\,t_1 + v_{\max}\,t_2 = \frac{1}{2}\frac{150}{3.6}t_1 + \frac{150}{3.6}t_2$$

که در آن : $t_1 = \frac{v_{\max}}{a} = \frac{150}{(3.6)(6)} = 6.945$

$$t_2 = t - 6.94 - 2$$

$$s_{\text{car}} = s_{\text{cycle}} \quad \Rightarrow \quad \frac{120}{3.6}t = \frac{75}{3.6}6.94 + \frac{150}{3.6}(t - 8.94)$$

$$\Rightarrow \quad 30t = 820.8 \quad \Rightarrow \quad t = 27.36\,\text{s}$$

$$s = \frac{120}{3.6}(27.36) = 912\,\text{m}$$

۲-۲۶.

$$a = v\frac{dv}{dx} \quad \Rightarrow \quad \int a\,dx = \int_0^0 v\,dv \quad \Rightarrow \quad \frac{1}{2}v^2 = -\int a\,dx$$

بازه (m)	$a\,\Delta x\ (m^2/s^2)$
0 – 0.1	$-6\,g\,(0.1)$
0.1 – 0.2	$-6.1\,g\,(0.1)$
0.2 – 0.3	$-6.4\,g\,(0.1)$
0.3 – 0.4	$-6.9\,g\,(0.1)$
0.4 – 0.5	$-7.6\,g\,(0.1)$
0.5 – 0.6	$-8.1\,g\,(0.1)$
0.6 – 0.7	$-8.9\,g\,(0.1)$
0.7 – 0.8	$-9.5\,g\,(0.1)$
	$\Sigma = -58.4\ \text{m}^2/\text{s}^2$

$$\frac{1}{2}v^2 \cong -\sum a\,\Delta x = -(-58.4) \quad \Rightarrow \quad v = 10.80\,\text{m/s} \quad \text{یا} \quad 10.80(3.6) = 38.9\,\text{km/h}$$

۲-۲۷.

$$v = 20 - \frac{2}{3}s \quad \Rightarrow \quad \frac{dv}{ds} = -\frac{2}{3}$$

توجه داشته باشید که وقتی $s = 15\,\text{m}$ باشد $v = 10\,\text{m/s}$ است.

$$a = v\frac{dv}{ds} = 10\left(-\frac{2}{3}\right) = -6.67\,\text{m/s}^2$$

$$v = \frac{ds}{dt} \quad \Rightarrow \quad \int_0^t dt = \int_0^{30}\frac{ds}{v} = \int_0^{30}\frac{ds}{20 - \frac{2}{3}s}$$

$$\Rightarrow t = -\frac{3}{2}\ln\left(20 - \frac{2}{3}s\right)\Big|_0^{30} = -\frac{3}{2}\ln\frac{0}{20} \to \infty$$

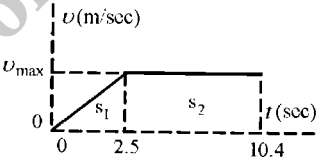
۲-۲۸.

$$s_1 = \frac{1}{2}(2.5)\,v_{max}$$

$$s_2 = (10.4 - 2.5)\,v_{max}$$

$$s_1 + s_2 = (1.25 + 7.9)\,v_{max} = 100$$

$$\Rightarrow v_{max} = 10.93\ \text{m/s}$$

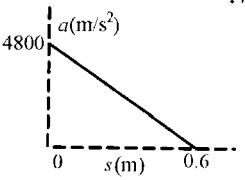


۲-۲۹.

$$\int_0^v v\,dv = \int_0^s a\,ds$$

$$\frac{1}{4}v^2 = a - s \text{ سطح زیر منحنی} = \frac{1}{2}(0.6)(4800)$$

$$\Rightarrow v = 53.7\ \text{m/s}$$



۲-۳۰.

$$v = \frac{ds}{dt}$$

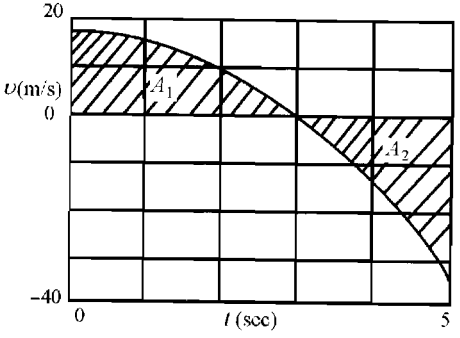
$$\Delta s_1 = A_1 = \int_0^3 (18 - 2t^2)\,dt = 36\,m$$

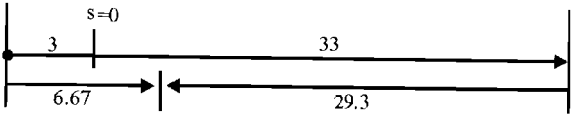
$$\Delta s_2 = A_2 = \int_3^5 (18 - 2t^2)\,dt = -29.3\,m$$

$$D = \Delta s_1 + |\Delta s_2| = 36 + 29.3 = 65.3\,\text{m}$$

$$\Delta s = \Delta s_1 + \Delta s_2 = 6.67\,\text{m}$$

$$s_5 = -3 + 6.67 = 3.67\,\text{m}$$



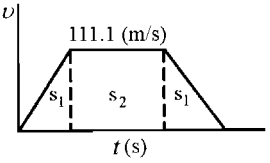


۲-۳۱.

$$400\ \text{km/h} \approx \frac{400}{3.6} = 111.1\ \text{m/s}$$

$$v^2 = 2as \quad \Rightarrow \quad s_1 = \frac{(111.1)^2}{2(0.6)(9.81)} = 1049\ \text{m}$$

$$\Rightarrow \quad s_2 = 1000 - 2(1049) = 7903\ \text{m}$$



$$\left.\begin{aligned} t_1 &= \frac{v}{a} = \frac{111.1}{0.6(9.81)} = 18.88\ \text{s} \\ t_2 &= \frac{s_2}{v} = \frac{7903}{111.1} = 71.13\ \text{s} \end{aligned}\right\} \Rightarrow t = 2t_1 + t_2 = 2(18.88) + 71.13 = 108.9\ \text{s} = 1.81\ \text{min}$$

۲-۳۲.

$\int_0^v v\,dv = \frac{1}{2}v^2$ برابر است با a–s سطح زیر منحنی

$$\text{سطح}\Big|_0^{200\text{m}} = \frac{3+6}{2}(100) + \frac{6+4}{2}(100) = 950\ \text{m}^2/\text{s}^2$$

$$\frac{1}{2}v^2 = 950 \quad \Rightarrow \quad v = 43.6\ \text{m/s}$$

$$\frac{dv}{ds} = \frac{a}{v} = \frac{4}{43.6} = 0.0918\ \text{s}^{-1}$$

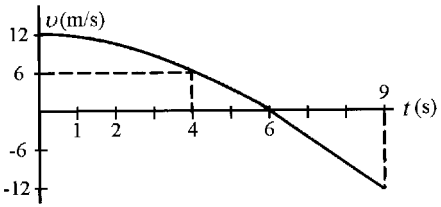
۲-۳۳.

$$0 < t < 4\ \text{s} \quad : a = -\frac{3t}{4}$$

$$a = \frac{dv}{dt} \quad \Rightarrow \quad \int_{12}^{v} dv = -\int_0^t \frac{3t}{4}\,dt \quad \Rightarrow \quad v = 12 - \frac{3}{8}t^2 \quad \Rightarrow \quad v_4 = 6\ \frac{\text{m}}{\text{s}}$$

$$4 < t < 9\ \text{s} \ : \ a = -3\ \text{m/s}^2 = \text{ثابت}$$

$$v = v_4 + a\Delta t = 6 - 3(t-4) = 18 - 3t \quad \Rightarrow \quad v_9 = -9\ \text{m/s}$$



$$0 < t < 4\text{s} \ : \ dx = v\,dt \quad \Rightarrow \quad \int_0^{x_4} dx = \int_0^4 \left(12 - \frac{3}{8}t^2\right) dt \quad \Rightarrow \quad s_4 = 40\ \text{m}$$

$$4 < t < 9\text{s} \ : \ x_6 - x_4 = \int_4^6 v\,dt = \frac{1}{2}(6-4)6 = 6\ \text{m}$$

$$\Delta x = 40 + 6 = 46\ \text{m}$$

۲-۳۴.

حالت ۱ : $a = -kv$

$$-kv = \frac{dv}{dt} \Rightarrow -k\int_0^t dt = \int_{v_0}^{v} \frac{dv}{v} \Rightarrow v = v_0 e^{-kt}$$

$$\frac{ds}{dt} = v_0 e^{-kt} \Rightarrow \int_{s_0=0}^{s} ds = v_0 \int_{t_0=0}^{t} e^{-kt}\, dt \Rightarrow s = \frac{v_0}{k}(1 - e^{-kt})$$

حالت ۲ : $a = -kt$

$$-kt = \frac{dv}{dt} \Rightarrow -k\int_0^t t dt = \int_{v_0}^{v} dv \Rightarrow v = v_0 - \frac{1}{2}kt^2$$

$$\frac{ds}{dt} = v_0 - \frac{1}{2}kt^2 \Rightarrow \int_0^s ds = \int_0^t (v_0 - \frac{1}{2}kt^2)\, dt \Rightarrow s = v_0 t - \frac{1}{6}kt^3$$

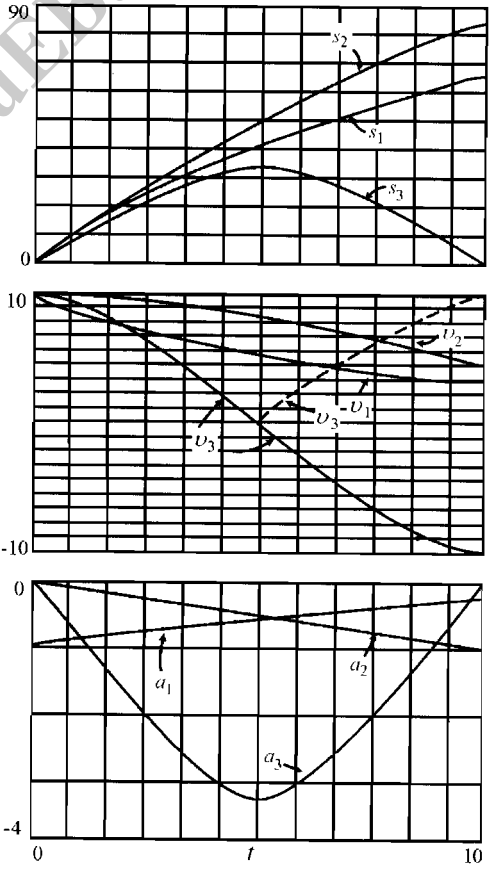
حالت ۳ : $a = -ks$

$$-ks = v\frac{dv}{ds} \Rightarrow -k\int_0^s s ds = \int_{v_0}^{v} v\, dv \Rightarrow v = \pm\sqrt{v_0^2 - ks^2}$$

$$\frac{ds}{dt} = \pm\sqrt{v_0^2 - ks^2} \Rightarrow \int_0^s \frac{ds}{\sqrt{v_0^2 - ks^2}} = \int_0^t dt$$

$$\Rightarrow \frac{1}{\sqrt{k}} \sin^{-1}\left(\frac{\sqrt{k}}{v_0} s\right) = t$$

$$\Rightarrow s = \frac{v_0}{k} \sin(\sqrt{k})\, t$$



۲-۳۵. $0 < t < 10\,\text{s} \;:\; a = 6 - kt \quad , \quad k = \frac{6}{10}\ \text{m/s}^3$

$$a = \frac{dv}{dt} = 6\left(1 - \frac{t}{10}\right)$$

$$\int_0^v dv = \int_0^t 6\left(1 - \frac{t}{10}\right)dt \quad \Rightarrow \quad v = 6t - \frac{3}{10}t^2$$

$$v_{10} = 6(10) - \frac{3}{10}(10)^2 = 30\ \text{m/s}$$

$$v = \frac{ds}{dt} \quad \Rightarrow \quad s_{10} = \int_0^{10}\left(6t - \frac{3}{10}t^2\right)dt = 200\,\text{m}$$

$$t > 10\ \text{s} \;:\; \Delta s = v_{10}\,\Delta t \quad \Rightarrow \quad \Delta t = \frac{400 - 200}{30} = 6.67$$

$$t = 10 + \Delta t = 16.67\,\text{s}$$

۲-۳۶.
$$v dv = a dx \;\Rightarrow\; \int_0^x dx = \int_{v_0}^{v} \frac{v\,dv}{-c_1 - c_2 v^2}$$

$$\Rightarrow\; x = \frac{-1}{2C_2}\ln\left(C_1 + C_2 v^2\right)\Big|_{v_0}^{v} = \frac{1}{2C_2}\ln\frac{C_1 + C_2 v_0^2}{C_1 + C_2 v^2}$$

وقتی $v = 0$ $\Rightarrow$ $x = D = \frac{1}{2C_2}\ln\left(1 + \frac{C_2}{C_1}v_0^2\right)$

۲-۳۷.
الف) ثابت $= 9.81\ \text{m/s}^2 = g_0$

$$v^2 = v_0^2 + 2a(s - s_0) \;\Rightarrow\; v^2 = 0^2 + 2(9.81)(800\times1000) \;\Rightarrow\; v = 3960\,\text{m/s}$$

ب) $$a = -g_0\frac{R^2}{r^2} = v\frac{dv}{dr} \Rightarrow -g_0 R^2\int_{R+h}^{R}\frac{dr}{r^2} = \int_{v_0=0}^{v} v\,dv$$

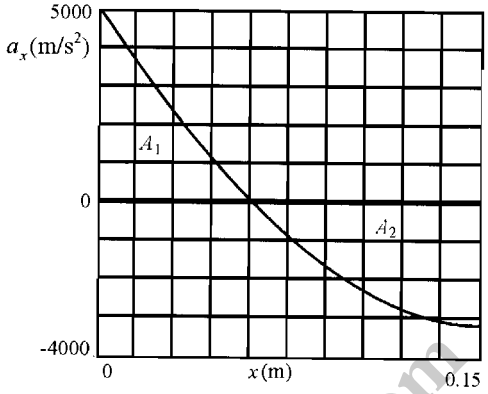
$$\Rightarrow\; -g_0 R^2\left(-\frac{1}{r}\right)\Big|_{R+h}^{R} = \frac{1}{2}v^2\Big|_0^v \quad \Rightarrow \quad v = \sqrt{\frac{2g_0 R h}{R + h}}$$

$$\Rightarrow\; v = \sqrt{\frac{2(9.81)(6371\times1000)(800\times1000)}{(6371 + 800)\times1000}} = 3730\,\text{m/s}$$

۲-۳۸. سطح زیر منحنی $a - x$ برابر است با: $\int a_x\,dx = \int v\,dv = \Delta\left(\frac{1}{2}v^2\right)$

چون در $x = 0$ و $x = 150$ ، سرعت $v = 0$ می‌باشد، لذا $A_1 = A_2 = \frac{1}{2}v_{max}^2$

طبق نمودار: $A_1 = A_2 \approx 160\ \text{m}^2/\text{s}^2$ بنابراین $v_{max} = \sqrt{2(160)} \approx 18\ \text{m/s}$

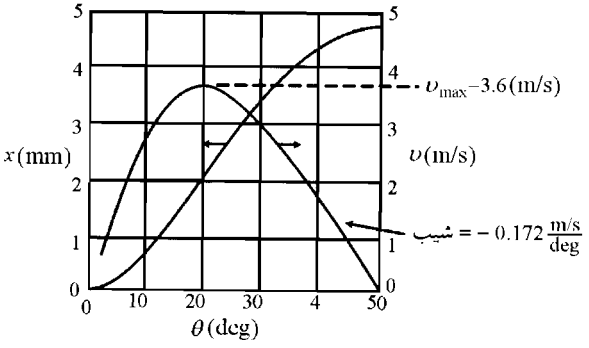


۲-۳۹.

سرعت $v = \left(\text{شیب } \frac{\text{mm}}{\text{deg}}\right)\left(\frac{1\ \text{m}}{1000\ \text{mm}}\right)\left(\frac{24000\ \text{deg}}{1\ \text{s}}\right) = 24\ \text{m/s}$

شتاب $a = \left(\text{شیب } \frac{\text{m/s}}{\text{deg}}\right)\left(\frac{24000\ \text{deg}}{1\ \text{s}}\right) = 24000\ \text{m/s}^2$

θ, deg	شیب $\frac{\text{mm}}{\text{deg}}$	v $\frac{\text{m}}{\text{s}}$
0	0	0
5	0.076	1.82
10	0.112	2.69
15	0.137	3.29
20	0.147	3.53
25	0.147	3.53
30	0.130	3.12
35	0.102	2.45
40	0.071	1.70
45	0.036	0.86
50	0	0



شتاب $a = -0.1172 \times 24000 \approx -4100\ \text{m/s}^2$

۲-۴۰.

$$a = -kv = \frac{dv}{dt} \Rightarrow -k\int_0^t dt = \int_{v_0}^{v}\frac{dv}{v} \Rightarrow v = v_0\, e^{-kt}$$

با شرایط داده شده : $1 = 4\, e^{-k(2)}$, $k = 0.693\ s^{-1} \Rightarrow v = v_0\, e^{-0.693\,t}$

وقتی $v = \frac{v_0}{10} \Rightarrow \frac{v_0}{10} = v_0\, e^{-0.693\,T} \Rightarrow T = 3.32\ s$

همچنین : $a = -kv = v\frac{dv}{ds} \Rightarrow -k\int_{s=0}^{s} ds = \int_{v_0}^{v} dv \Rightarrow v = v_0 - ks$

با شرایط داده شده : $\frac{v_0}{10} = v_0 - kD$, $k = 0.693\ s^{-1}$, $v_0 = 4\ m/s$

$$\Rightarrow \frac{4}{10} = 4 - 0.696\,D \Rightarrow D = 5.19\ m$$

(ملاحظه کنید که T مستقل از v_0 است اما D مستقل نیست)

۲-۴۱.

الف) ثابت $a = 2\ m/s^2$

$$v = \frac{250}{3.6} = 49.6\ m/s$$

$$v^2 - v_0^2 = 2a(s - s_0) \Rightarrow 69.4^2 - 0^2 = 2(2)\,s \Rightarrow s = 1206\ m$$

ب) $a = a_0 - kv^2 = v\frac{dv}{ds} \Rightarrow \int_0^s ds = \int_0^v \frac{v\,dv}{a_0 - kv^2}$

$$\Rightarrow s = -\frac{1}{2k}\ln(a_0 - kv^2)\Big|_0^v = \frac{-1}{2k}\ln\left[\frac{a_0 - kv^2}{a_0}\right]$$

$$\Rightarrow s = -\frac{1}{2(4)(10^{-5})}\ln\left[\frac{2 - 4(10^{-5})(69.4)^2}{2}\right] = 1268\ m$$

۲-۴۲.

$$v\,dv = a\,dx \Rightarrow \int_0^v v\,dv = \int_0^x \frac{k}{(L-x)^2}\,dx$$

$$\frac{v^2}{2} = \frac{k}{L-x}\Big|_0^x \Rightarrow v^2 = \frac{2kx}{L(L-x)}$$

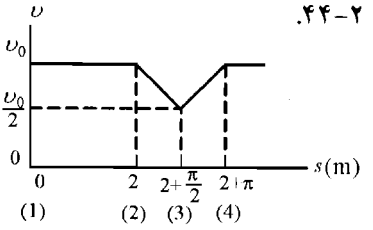
برای $x = L - \frac{D}{2}$: $v = 2\sqrt{\frac{k(L - \frac{D}{2})}{LD}}$

۲-۴۳.

$$a = g - cy^2 = v\frac{dv}{dy} \Rightarrow \int_0^{y_m}(g - cy^2)\,dy = \int_{v_0}^{0} v\,dv$$

$$\Rightarrow \left(gy - c\frac{y^3}{3}\right)\Big|_0^{y_m} = \frac{v^2}{2}\Big|_{v_0}^{0} \Rightarrow gy_m - c\frac{y_m^2}{3} = -\frac{v_0^2}{2} \Rightarrow c = \frac{3v_0^2 + 6gy_m}{2y_m^3}$$

۲-۴۴. زمان $t_{\frac{1}{2}}$ لازم برای $\frac{1}{2}$ حلقه را محاسبه کنید.



موقتاً، فرض کنید v_0 سرعت حرکت مستقیم باشد.

زمان رفتن از (1) به (2) $t_{1\text{-}2} = \dfrac{2}{v_0}$

برای $t_{2\text{-}3}$: $\dfrac{dv}{ds} = -c$, $a_t = v\dfrac{dv}{ds} = -cv = \dfrac{dv}{dt}$

$$\Rightarrow -c\int_{t_2}^{t_3} dt = \int_{v_2}^{v_3}\frac{dv}{v} \Rightarrow -ct_{2\text{-}3} = \ln\left(\frac{v_0/2}{v_0}\right) \Rightarrow t_{2\text{-}3} = \frac{1}{c}\ln 2$$

به همین ترتیب : $t_{3\text{-}4} = \dfrac{1}{c}\ln 2$

$$c = \frac{v_0/2}{\pi/2} = \frac{v_0}{\pi} \quad , \quad t_{1/2} = t_{1\text{-}2} + t_{2\text{-}3} + t_{3\text{-}4} = \frac{6.36}{v_0}$$

زمان کل حلقه $t = 2\,t_{1/2} = \dfrac{12.71}{v_0} = \dfrac{12.71}{0.25} = 50.8\,\text{s}$

۲-۴۵.

$$a = -kv^2$$

$$v\,dv = a\,dx \Rightarrow \int_{v_0}^{v}\frac{v\,dv}{-kv^2} = \int_0^x dx \Rightarrow x = \frac{-1}{k}\ln v\Big|_{v_0}^{v} \Rightarrow x = \frac{1}{k}\ln\frac{v_0}{v}$$

وقتی : $x = D = \dfrac{1}{k}\ln 2 = \dfrac{0.693}{k}$ $\quad v = \dfrac{v_0}{2}$

$$v = \frac{dx}{dt} \quad kx = \ln\frac{v_0}{v} \Rightarrow v = v_0\,e^{-kx} \Rightarrow \frac{dx}{v_0\,e^{-kx}} = dt \quad \text{یا} \quad \int_0^t dt = \frac{1}{v_0}\int_0^x e^{kx}\,dx$$

$$\Rightarrow t = \frac{1}{v_0}\frac{1}{k}e^{kx}\Big|_0^x = \frac{1}{kv_0}[e^{kx} - 1]$$

برای $x = D$: $e^{kx} = 2 \Rightarrow t = \dfrac{1}{k\,v_0}[2 - 1] \Rightarrow t = \dfrac{1}{k\,v_0}$

۲-۴۶.

الف) $(g_m)_0 = 1.620\ \text{m/s}^2 =$ ثابت

$$v^2 = v_0^2 + 2(g_m)_0(s - s_0) \Rightarrow v^2 = 0^2 + 2(1.620)(1200\times1000) \Rightarrow v = 1972\ \text{m/s}$$

ب) $a = -(g_m)_0\dfrac{R_m^2}{r^2} = v\dfrac{dv}{dr}$

$$\Rightarrow -(g_m)_0 R_m^2\int_{R_m+h}^{R_m}\frac{dr}{r^2} = \int_{v_0=0}^{v} v\,dv$$

$$\Rightarrow -(g_m)_0 R_m^2\left(-\frac{1}{r}\right)\Big|_{R_m+h}^{R_m} = \frac{1}{2}v^2\Big|_0^v$$

$$\Rightarrow v = \sqrt{\frac{2(g_m)_0 R_m h}{R_m + h}} = \sqrt{\frac{2(1.620)(1738\times1000)(1200\times1000)}{(1738\times1000) + (1200\times1000)}} = 1517\ \text{m/s}$$

۲-۴۷.

$$a = v\frac{dv}{ds} = 0.981 - 0.013\,v^2$$

$$\Rightarrow \int_0^{v_B} \frac{v\,dv}{0.981 - 0.013\,v^2} = \int_0^{200} ds \quad \Rightarrow \quad \frac{1}{2(-0.013)} \ln(0.981 - 0.013\,v^2)\Big|_0^{v_B} = 200$$

$$\Rightarrow \ln\left[\frac{0.981 - 0.013\,v_B{}^2}{0.981}\right] = 200(2)(-0.013) \quad \Rightarrow \quad v_B = 8.66\,\text{m/s}$$

۲-۴۸.

$$a = \frac{dv}{dt} \quad \Rightarrow \quad \int_0^v \frac{dv}{g - kv} = \int_0^t dt \quad \Rightarrow \quad -\frac{1}{k}\ln(g - kv)\Big]_0^v = t$$

$$\Rightarrow kt = \ln\frac{g}{g - kv} \quad \Rightarrow \quad \frac{g}{g - kv} = e^{kt} \quad \Rightarrow \quad v = \frac{g}{k}(1 - e^{-kt})$$

$$v = \frac{dy}{dt} \quad \Rightarrow \quad \int_0^y dy = \frac{9}{k}\int_0^t (1 - e^{-kt})\,dt \quad \Rightarrow \quad y = \frac{g}{k}\left(t + \frac{1}{k}e^{-kt}\right)$$

$$\Rightarrow y = \frac{g}{k}\left[t - \frac{1}{k}(1 - e^{-kt})\right]$$

۲-۴۹.

بالا رفتن : $a_u = -g - kv^2 = v\dfrac{dv}{dy}$

$$\Rightarrow \int_0^h dy = -\int_{v_0}^0 \frac{v\,dv}{g + kv^2} \quad \Rightarrow \quad h = -\frac{1}{2k}\ln\left[g + kv^2\right]_{v_0}^0 = \frac{1}{2k}\ln\left[\frac{g + kv_0{}^2}{g}\right]$$

$$\Rightarrow h = \frac{1}{2(0.006)}\ln\left[\frac{9.81 + 0.006\,(30)^2}{9.81}\right] = 36.5\,\text{m}$$

پائین آمدن : $a_d = -g + kv^2 = v\dfrac{dv}{dy}$

$$\Rightarrow \int_h^0 dy = \int_0^{v_f} \frac{v\,dv}{-g + kv^2} \quad \Rightarrow \quad -h = \frac{1}{2k}\ln\left[-g + kv^2\right]_0^{v_f} = \frac{1}{2k}\ln\left[\frac{g - kv_f^2}{g}\right]$$

$$\Rightarrow v_f = \sqrt{\frac{g}{k}(1 - e^{-2kh})} = \sqrt{\frac{9.81}{0.006}\left(1 - e^{-2(0.066)(36.5)}\right)} = 24.1\ \text{m/s}$$

۲-۵۰.

بالا رفتن : $a_u = -g - kv^2 = \dfrac{dv}{dt} \quad \Rightarrow \quad \displaystyle\int_0^{t_u} dt = -\int_{v_0}^0 \frac{dv}{g + kv^2}$

$$\Rightarrow t_u = \frac{1}{\sqrt{gk}}\tan^{-1}\left(\frac{v\sqrt{gk}}{g}\right)\Big|_0^{v_0} = \frac{1}{\sqrt{gk}}\tan^{-1}\left(v_0\sqrt{\frac{k}{g}}\right)$$

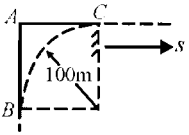
$$\Rightarrow t_u = \frac{1}{\sqrt{9.81(0.066)}} \tan^{-1}\left(30\sqrt{\frac{0.006}{9.81}}\right) = 2.63\,\text{s}$$

پائین آمدن : $a_d = -g + kv^2 = \frac{dv}{dt} \Rightarrow \int_0^{t_d} dt = \int_0^{v_f} \frac{dv}{-g + kv^2}$

$$\Rightarrow t_d = \frac{1}{\sqrt{gk}} \tanh^{-1}\left(\frac{v\sqrt{gk}}{g}\right)\Big|_0^{v_f} = \frac{1}{\sqrt{gk}} \tanh^{-1}\left(v_f\sqrt{\frac{k}{g}}\right)$$

$$\Rightarrow t_d = \frac{1}{\sqrt{9.81(0.066)}} \tanh^{-1}\left(24.1\sqrt{\frac{0.006}{9.81}}\right) = 2.83\,\text{s}$$

۲-۵۱. ابتدا زمان حرکت شتابدار B را محاسبه می‌کنیم:



$$v = v_0 + at$$

$$\frac{100}{3.6} = \frac{40}{3.6} + 0.1(9.81)\,t \Rightarrow t = 17\,\text{s}$$

مسافت طی شده توسط A در این مدت: $d_A = \frac{100}{3.6}(17) = 472.2\,\text{m}$

جابجایی بعد از نقطهٔ C : $s_A = 472.2 - 100 = 372.2\,\text{m}$

مسافت طی شده توسط B در مدت $17\,\text{s}$:

$$d_B = v_0 t + \frac{1}{2}at^2 = \frac{40}{3.6}(17) + \frac{1}{2}(0.1)(9.81)(17)^2 = 330.6\,\text{m}$$

جابجایی بعد از نقطهٔ C : $s_B = 330.6 - \frac{\pi(100)}{2} = 173.6\,\text{m}$

بنابراین A به اندازهٔ $s_A - s_B = 372.2 - 173.6 = 198.6\,\text{m}$ از B در مسیر مستقیم جلوتر خواهد بود.

۲-۵۲. برای B ، سطح زیر نمودار $v_f - v_0 = a - t$

$$\frac{(100 - 40)}{3.6} = 0.1(9.81)\,t_1 + \frac{1}{2}(0.1)(9.81)(5) \Rightarrow t_1 = 14.5\,\text{s}$$

B در زمان $t_2 = 14.5 + 5 = 19.5\,\text{s}$ به سرعت $100\,\frac{\text{km}}{\text{h}}$ می‌رسد.

سرعت B در زمان t_1 $v_1 = \frac{40}{3.6} + (0.1)\,9.81\,(14.5) = 25.3\,\text{m/s}$

تغییرات شتاب در زمان $t_1 < t < t_2$ برابر است: $a = 3.82 - 0.196\,t$

$$\int_{v_1=25.3}^{v} dv = \int_{t_1=14.5}^{t} (3.82 - 0.196t)\,dt \Rightarrow v = -9.48 + 3.82t - 0.098\,t^2$$

$$\int_{s_1}^{s_2} ds = \int_{t_1=14.5}^{t_2=19.5} (-9.48 + 3.82t - 0.098t^2)\,dt \Rightarrow (s_2 - s_1) = 134.7\,\text{m}$$

مسافت طی شده توسط B در مدت $19.5\,\text{s}$:

$$d_B = \frac{40}{3.6}(14.5) + \frac{1}{2}(0.1)(9.81)(14.5)^2 + 134.7 = 398.9\,\text{m}$$

جابجایی بعد از نقطهٔ C : $s_B = 398.9 - \frac{\pi(100)}{2} = 241.8\,\text{m}$

مسافت طی شده توسط A در مدت 19.5 s:

$$d_A = \frac{100}{3.6}(19.5) = 541.6\,\text{m}$$

جابجایی بعد از نقطهٔ C :

$$s_A = 541.6 - 100 = 441.6\,\text{m}$$

بنابراین A به اندازهٔ $s_A - s_B = 441.6 - 241.8 = 199.7\,\text{m}$ از B در مسیر مستقیم جلوتر خواهد بود.
(خیلی بیشتر از 198.6 m در مسئله ۲-۵۱ نیست.)

۲-۵۳.

$$a = \frac{d^2x}{dt^2} = Kt - k^2x \Rightarrow \frac{d^2x}{dt^2} + kx^2 = Kt$$

یک معادله دیفرانسیل خطی درجه دو است که جواب آن عبارتست از:

$$x = x_h + x_p = A\sin kt + B\cos kt + \frac{K}{k^2}t$$

شرایط اولیه : $x(0) = B = 0$

$$\dot{x}(0) = kA + \frac{K}{k^2} = 0 \Rightarrow A = -\frac{K}{k^3}$$

بنابراین : $x = \frac{K}{k^3}(kt - \sin kt)$

۲-۵۴. برای شتابی به صورت $a = -g - kv^2$ از نتایج مسئله‌های ۲-۴۹ و ۲-۵۰ استفاده می‌کنیم.

$$\begin{cases} t_u = \dfrac{1}{\sqrt{gk}}\tan^{-1}\left(v_0\sqrt{\dfrac{k}{g}}\right) \\ h = \dfrac{1}{2k}\ln\left[\dfrac{g + kv_0^2}{g}\right] \end{cases}$$

برای اعداد داده شده:

$$t_u = \frac{1}{\sqrt{9.81\,(0.0005)}}\tan^{-1}\left(120\sqrt{\frac{0.0005}{9.81}}\right) = 10.11\,\text{s}$$

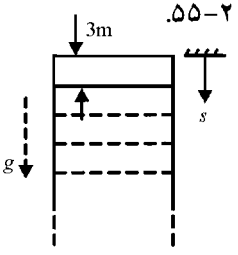
$$h = \frac{1}{2(0.0005)}\ln\left[\frac{9.81 + 0.0005\,(120)^2}{9.81}\right] = 550\,\text{m}$$

$$y = y_0 + v_{y_0}t$$

$$0 = 550 - 4t_d \Rightarrow t_d = 137.6\,\text{s}$$

زمان پرواز $t = t_u + t_d = 10.11 + 137.6 = 147.7\,\text{s}$

۲-۵۵.



$$s = \cancel{s_0}^{\,0} + \cancel{v_0}^{\,0}t + \frac{1}{2}gt^2$$

وقتی $s = 3\,\text{m}$ باشد:

$$3 = \frac{1}{2}(9.81)\,t_{3\text{m}}^2 \Rightarrow t_{3\text{m}} = 0.782\,\text{s}$$

زمان عبور گوی از طبقه اول: $t_1 = t_{3\text{m}} - t_{0\text{m}} = 0.782 - 0 = 0.782\,\text{m}$

طبقه دهم : $27 = \frac{1}{2}(9.81)\, t_{27m}{}^2 \Rightarrow t_{27m} = 2.346\,s$

$30 = \frac{1}{2}(9.81)\, t_{30m}{}^2 \Rightarrow t_{30m} = 2.473\,s$

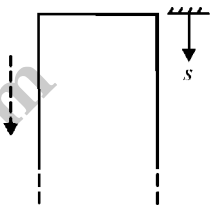
$t_{10} = t_{30m} - t_{27m} = 2.473 - 2.346 = 0.1269\ s$

طبقه صدم : $297 = \frac{1}{4}(9.81)\, t_{297m}{}^2 \Rightarrow t_{297m} = 7.7814\,s$

$300 = \frac{1}{2}(9.81)\, t_{300m}{}^2 \Rightarrow t_{300m} = 7.8206\,s$

$t_{100} = t_{300m} - t_{297m} = 7.8206 - 7.7814 = 0.0392\,s$

۲-۵۶.



$$a = -g - kv^2 = \frac{dv}{dt}$$

$$\int_0^t dt = \int_0^v \frac{dv}{g - kv^2}$$

به پیوست ج - ۱۰ رجوع کنید $\Rightarrow$ $t_d = \frac{1}{\sqrt{gk}} \tanh^{-1} \sqrt{\frac{k}{g}}\, v \Big|_0^v = \frac{1}{\sqrt{gk}} \tanh^{-1} \sqrt{\frac{k}{g}}\, v$

$$\Rightarrow v = \frac{ds}{dt} = \sqrt{\frac{g}{k}} \tanh\left(\sqrt{gk}\ t\right) \Rightarrow \int_0^s ds = \sqrt{\frac{g}{k}} \int_0^t \tanh\left(\sqrt{gk}\right) t\, dt$$

$$\Rightarrow s = \frac{1}{k} \ln \cosh \sqrt{gk}\ t \Rightarrow t = \frac{\cosh^{-1}(e^{sk})}{\sqrt{gk}} = \frac{\cosh^{-1}(e^{0.016\ s})}{0.396}$$

$s(m)$	$t(sec)$
0	0
3	0.7887
27	2.5193
30	2.6760
297	13.7503
300	13.8715

$t_1 = t_{3m} - t_{0m} = 0.7887\,s$

$t_{10} = t_{30m} - t_{27m} = 0.1567\,s$

$t_{100} = t_{300m} - t_{297m} = 0.1212\,s$

۲-۵۷. ◀

$$\frac{dv}{dt} = ke^{-bt} - cv - g \Rightarrow \frac{dv}{dt} + cv = ke^{-bt} - g$$

برای شکل استاندارد حل معادلهٔ دیفرانسیل خطی درجه اول:

$$e^{-\int c dt} = e^{-ct}$$

$$v = Ae^{-ct} + e^{-ct} \int (ke^{-bt} - g)\, e^{ct}\, dt$$

$$= Ae^{-ct} + \frac{k}{c - b} e^{-bt} - \frac{g}{c}$$

وقتی $t = 0$ و $v = 0$ باشد:

$$0 = A + \frac{k}{c - b} - \frac{g}{c} \Rightarrow A = \frac{g}{c} - \frac{k}{c - b}$$

بنابراین:

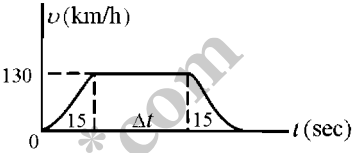
$$v = \left(\frac{g}{c} - \frac{k}{c - b}\right) e^{-ct} + \frac{k}{c - b} e^{-bt} - \frac{g}{c}$$

یا

$$v = \frac{g}{c}(e^{-ct} - 1) + \frac{k}{c - b}(e^{-bt} - e^{-ct})$$

۲-۵۸. ◀

تابع درجه ۳ : $v = a + bt + ct^2 + dt^2$



$$v(0) = a = 0$$

$$\frac{dv}{dt} = b + 2ct + 3dt^2$$

$$\left.\frac{dv}{dt}\right|_{t=0} = 0 \Rightarrow b = 0 \quad , \quad \left.\frac{dv}{dt}\right|_{t=15} = 30c + 675d = 0$$

$$v(15) = c(15)^2 + d(15)^3 = \frac{130}{3.6}$$

از حل معادلات قبل خواهیم داشت: $c = 0.4815 \text{ m/s}^3 \quad , \quad d = -0.0214 \text{ m/s}^4$

$$\Rightarrow v = 0.4815\,t^2 - 0.0214\,t^3 \qquad (\text{برای } 0 \le t \le 15\,\text{s})$$

$$s = \int_0^t (0.4815\,t^2 - 0.0214\,t^3)\,dt = 0.1605\,t^3 - 0.0054\,t^4$$

این مسافت طی حرکت کندشونده نیز طی می‌شود. $s(15) = 271 \text{ m}$

مسافت طی حرکت با سرعت ثابت : $3200 - 2(271) = 2658 \text{ m}$

$$\Delta s = v\,\Delta t \; : \; 2658 = \frac{130}{3.6}\Delta t \Rightarrow \Delta t = 73.6\,\text{s}$$

کل زمان حرکت بین ایستگاهها $t = 73.6 + 2(15) = 103.6\,\text{s}$

$$a = \frac{dv}{dt} = 0.963\,t - 0.0642\,t^2$$

$$\frac{da}{dt} = 0.963 - 0.1284\,t = 0 \Rightarrow t = 7.5\,\text{s}$$

$$a_{max} = 0.963(7.5) - 0.0642(7.5)^2 = 3.61 \text{ m/s}^2$$

۲-۵۹.

$$\mathbf{v}_{av} = \frac{\Delta \mathbf{r}}{\Delta t} = \frac{(5.1\,\mathbf{i} + 0.4\,\mathbf{j}) - 5\,\mathbf{i}}{0.02} = 5\,\mathbf{i} + 20\,\mathbf{j} \text{ m/s}$$

$$v_{av} = \sqrt{5^2 + 20^2} = 20.6 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{20}{5}\right) = 76.0°$$

۲-۶۰.

$$\mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{(-0.1\mathbf{i} + 1.8\mathbf{j}) - (0.1\mathbf{i} + 2\mathbf{j})}{0.1} = -2\mathbf{i} - 2\mathbf{j}$$

$$a_{av} = \sqrt{2^2 + 2^2} = 2.83 \text{ m/s}^2$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-2}{-2}\right) = 225°$$

۲-۶۱.

$x = t^2 - 4t + 20$ $\quad$ $y = 3\sin 2t$

$\dot{x} = 2t - 4$ $\quad$ $\dot{y} = 6\cos 2t$

$\ddot{x} = 2$ $\quad$ $\ddot{y} = -12\sin 2t$

در $t = 3$ s :

$\dot{x} = 2$ mm/s $\quad$ $\dot{y} = 5.67$ mm/s

$\ddot{x} = 2$ mm/s^2 $\quad$ $\ddot{y} = 3.35$ mm/s^2

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{2^2 + 5.76^2} = 6.10 \text{ mm/s}$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{2^2 + 3.35^2} = 3.90 \text{ mm/s}^2$$

$$\mathbf{v} = 2\mathbf{i} + 5.76\mathbf{j} \text{ mm/s} \quad , \quad \mathbf{a} = 2\mathbf{i} + 3.35\mathbf{j} \text{ mm/s}^2$$

$$\theta = \cos^{-1}\frac{\mathbf{v}.\mathbf{a}}{va} = \cos^{-1}\left(\frac{2(2) + 5.76(3.35)}{(6.10)(3.90)}\right) = 11.67°$$

۲-۶۲.

$$x = 20 + \frac{1}{4}t^2 \quad , \quad \dot{x} = \frac{1}{2}t \quad , \quad \ddot{x} = \frac{1}{2} \text{ mm/s}^2$$

$$y = 15 - \frac{1}{6}t^3 \quad , \quad \dot{y} = -\frac{1}{2}t^2 \quad , \quad \ddot{y} = -t \text{ mm/s}^2$$

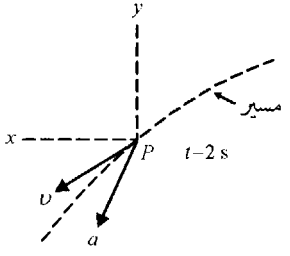
$t = 2$ s

$\dot{x} = 1$ mm/s

$\dot{y} = -2$ mm/s

$\ddot{x} = \frac{1}{2}$ mm/s^2

$\ddot{y} = -2$ mm/s^2



$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{1^2 + (-2)^2} = 2.24 \text{ mm/s}$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{\left(\frac{1}{2}\right)^2 + (-2)^2} = 2.06 \text{ mm/s}^2$$

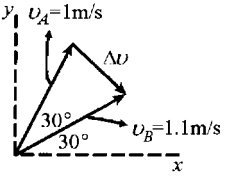
۲-۶۳.

$$v = \dot{s} = \frac{t}{2} \quad , \quad v_A = \frac{2}{2} = 1\ \text{m/s} \quad , \quad v_B = \frac{2.2}{2} = 1.1\ \text{m/s}$$

$$\Delta v_x = v_{Bx} - v_{Ax} = 1.1\cos 30° - 1.0\cos 60° = 0.453\ \text{m/s}$$

$$\Delta v_y = v_{By} - v_{Ay} = 1.1\sin 30° - 1.0\sin 60° = -0.316\ \text{m/s}$$

$$\Delta v = \sqrt{0.453^2 + 0.316^2} = 0.552\ \text{m/s}$$



شتاب متوسط برابر است با:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{0.552}{0.20} = 2.76\ \text{m/s}^2$$

$$\mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{0.453\mathbf{i} - 0.316\mathbf{j}}{0.20} = 2.26\mathbf{i} - 1.580\mathbf{j}\ \text{m/s}^2$$

۲-۶۴.

$$\begin{cases} x = 3\cos 4t \quad , \quad \dot{x} = -12\sin 4t \quad , \quad \ddot{x} = -48\cos 4t \\ y = 2\sin 4t \quad , \quad \dot{y} = 8\cos 4t \quad , \quad \ddot{y} = -32\sin 4t \end{cases}$$

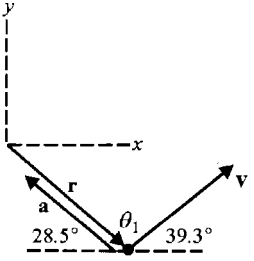
در زمان $t = 1.4\ \text{s}$:

$$\begin{cases} x = 2.33\ \text{m} \quad , \quad \dot{x} = 7.58\ \text{m/s} \quad , \quad \ddot{x} = -37.2\ \text{m/s}^2 \\ y = -1.263\ \text{m} \quad , \quad \dot{y} = 6.20\ \text{m/s} \quad , \quad \ddot{y} = 20.2\ \text{m/s}^2 \end{cases}$$

$$r = 2.65\ \text{m} \quad , \quad v = 9.79\ \text{m/s} \quad , \quad a = 42.4\ \text{m/s}^2$$

$$\theta_1 = \cos^{-1}\left[\frac{\mathbf{a}.\mathbf{v}}{av}\right] = \cos^{-1}\left[\frac{-37.2\ (7.58) + 20.0\ (6.20)}{42.4\ (9.79)}\right]$$

$$\Rightarrow \quad \theta_1 = 112.2°$$



$\theta_2 = \cos^{-1}\left[\frac{\mathbf{a}.\mathbf{r}}{ar}\right] = 180°$: بطور مشابه

۲-۶۵.

$$x = 2t^3 - 3t \quad , \quad \dot{x} = 6t^2 - 3 \quad , \quad \ddot{x} = 12t$$

$$\ddot{y} = 4t \quad , \quad \dot{y} = 2t^2 + k_1 : \ 4 = 0 + k_1 \ \Rightarrow \ k_1 = 4\ \text{m/s} \ \Rightarrow \ \dot{y} = 2t^2 + 4$$

$$y = \frac{2}{3}t^3 + 4t + k_2 : 0 = 0 + 0 + k_2 \ \Rightarrow \ k_2 = 0 \ \Rightarrow \ y = \frac{2}{3}t^3 + 4t$$

$t = 2\,s$ در :

$\dot{x} = 6(2)^3 - 3 = 21\ m/s$

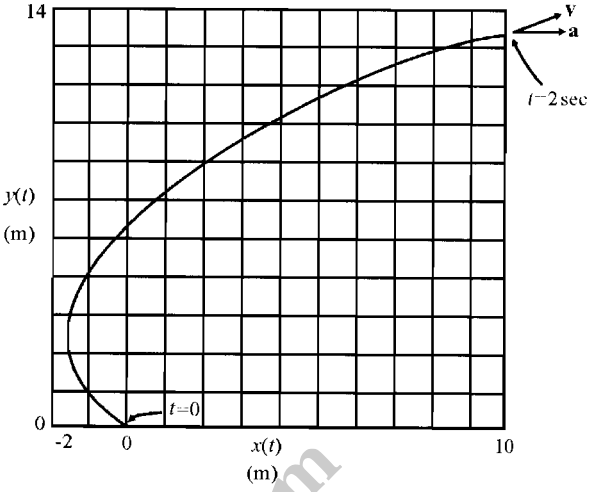
$\dot{y} = 2(2)^2 + 4 = 12\ m/s$

$\ddot{x} = 12(2) = 24\ m/s^2$

$\ddot{y} = 4(2) = 8\ m/s^2$

$v = \sqrt{21^2 + 12^2} = 24.2\ m/s$

$a = \sqrt{24^2 + 8^2} = 25.3\ m/s^2$



۲-۶۶. طبق مثال ۲-۶:

$$R = 2s = \frac{u^2 \sin 2\theta}{g} \quad , \quad h = \frac{u^2 \sin^2\theta}{2g}$$

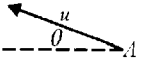
با حذف u خواهیم داشت:

$$R = \frac{(2gh / \sin^2\theta)\sin 2\theta}{g} = \frac{(2gh / \sin^2\theta)(2\sin\theta\cos\theta)}{g} = 4h\cot\theta$$

۲-۶۷. طبق مثال ۲-۶:

$$2s = \frac{u^2 \sin 2\theta}{g} = \frac{2(u\cos\theta)(u\sin\theta)}{g}$$

$2s = 7.5\ m$, $u\cos\theta = 10\ m/s$, $u\sin\theta = v_y$



$$v_y = \frac{2sg}{2u\cos\theta} = \frac{7.5(9.81)}{2(10)} = 3.68\ m/s$$

$$h = \frac{u^2\sin^2\theta}{2g} = \frac{v_y^2}{2g} = \frac{(3.68)^2}{2(9.81)} = 0.690\ m$$

۲-۶۸. محورهای $x - y$ را در موقعیت اولیهٔ G قرار دهید.

$$x = x_0 + v_{x_0} t \ : \ 1 = (v_0\cos\theta)\,t$$

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \ : \ 1.06 = (v_0\sin\theta)\,t - \frac{1}{2}(9.81)\,t^2$$

$$v_y = v_{y_0} - gt \ : \ 0 = v_0\sin\theta - 9.81\,t$$

از حل همزمان معادلات فوق:

$t = 0.46\ s$

$v_0 = 5.04\ m/s$

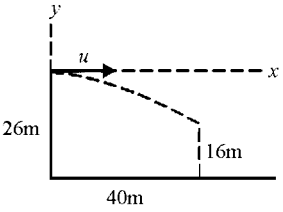
$\theta = 64.7°$

۲-۶۹.

$a_y = -g$

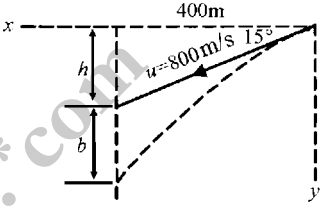
$y = 0 - \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(26 - 16)}{9.81}} = 1.428\ \text{s}$

$x = ut \Rightarrow u = \frac{40}{1.428} = 28.0\ \text{m/s}$



۲-۷۰.

$h = 400 \tan 15° = 107.2\ \text{m}$



طبق مثال ۲-۶ معادله مسیر عبارتست از:

$y = x \tan\theta + \frac{g x^2}{2u^2} \sec^2\theta$

$b = \frac{g x^2}{2u^2} \sec^2\theta$

$x = 400\ \text{m}$ در : $b = \frac{9.81(400)^2}{2(800)^2} \sec^2 15° = 1.314\ \text{m}$

۲-۷۱. سرعت‌های افقی جیپ و پرتابه برابرند (30 km/h = 8.33 m/s) بنابراین پرتابه صرفنظر از هر سرعت عمودی جیپ به جیپ برمی‌گردد.

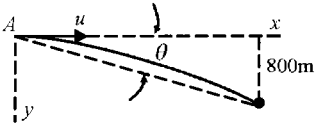
y جهت در : $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$

موقع برخورد : $0 = 0 + 20t - \frac{1}{2}(9.81)\, t^2 \Rightarrow t = 4.08\ \text{s}$

$s = vt = 8.33\ (4.08) = 34.0\ \text{m}$

۲-۷۲.

$u = \frac{1000}{3.6} = 278\ \text{m/s}$



y جهت : $y = v_{y_0} t + \frac{1}{2} g t^2$

$800 = 0 + \frac{1}{2}(9.81)\, t^2 \Rightarrow t = 12.77\ \text{s}$

x جهت : $x = v_{x_0} t + \frac{1}{2} a_x t^2$

$= 278\ (12.77) + \frac{1}{2}(9.81)(12.77)^2 = 3950\ \text{m}$

$\theta = \tan^{-1} \frac{800}{3950} = 11.46°$

۲-۷۳. از دستگاه مختصات x-y شکل استفاده کنید.

الف) $v_0 = 14\ \text{m/s}$

$x = x_0 + v_{x_0} t$ (در دیوار چپ) : $9 = 0 + 14\cos 60^\circ\, t \Rightarrow t = 1.28\,\text{s}$

$y = y_0 + v_{y_0} - \frac{1}{2} g t^2$: $y = 14\ \sin 60^\circ\ (1.28) - \frac{1}{2}(9.81)(1.28)^2 = 7.48\,\text{m}$

(به دیوار برخورد می‌کند)

پاسخ : $(x\ ,\ y) = (9\ ,\ 7.48)\text{m}$

ب) $v_0 = 18\ \text{m/s}$

راه‌حل قسمت قبل را تکرار کنید تا بیابید وقتی $x = 9\,\text{m}$ باشد، $y = 10.7\,\text{m}$ بنابراین آب از دیوار چپ عبور می‌کند.

$x = x_0 + v_{x_0} t$ (در دیوار راست) : $15 = 0 + 18\cos 60^\circ\, t \Rightarrow t = 1.667\,\text{s}$

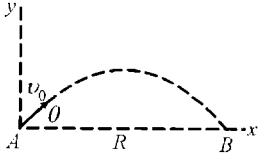
به همین صورت : $y = 12.3\,\text{m}$ (در $t = 1.667$)

بنابراین آب از کل ساختمان عبور می‌کند. پس $y = 0$

$y = y_0 + v_{y_0} - \frac{1}{2} g t^2$ $(y = 0\ ,\ y_0 = 1.5\,\text{m})$ $\Rightarrow t = 3.27\,\text{s}$

$x = x_0 + v_{x_0} t = 0 + 18\cos 60^\circ\ (3.27) = 29.4\,\text{m}$

۲-۷۴.



$v_{x_0} = v_0 \cos\theta$

$v_{y_0} = v_0 \sin\theta$

$x = x_0 + v_{x_0} t$ (در نقطه B) : $R = 0 + (v_0 \cos\theta)\, t_f$ (1)

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ (در نقطه B) : $0 = 0 + (v_0 \sin\theta)\, t_f - \frac{g}{2} t_f^2$ (2)

(2) $\Rightarrow t_f = 0\ ,\ \frac{2 v_0 \sin\theta}{g}$ ($t = 0$ زمان پرتاب است)

(1) $\Rightarrow R = (v_0 \cos\theta)\left(\frac{2 v_0 \sin\theta}{g}\right) = \frac{v_0^2 \sin 2\theta}{g}$

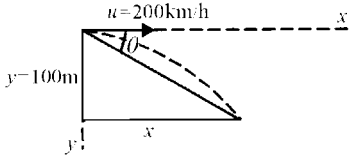
$\frac{dR}{d\theta} = 0 \Rightarrow \frac{v_0^2}{g} 2\cos 2\theta = 0 \Rightarrow \theta = 45^\circ$

$R_{\max} = \frac{v_0^2 \sin(2\times 45)}{g} = \frac{v_0^2}{g}$

۷۵-۲.

حرکت عمودی: $a_y = g = 9.81\ \text{m/s}^2$

$y = \frac{1}{2} g t^2$



حرکت افقی: $a_x = 0$

$x = ut$

$\Rightarrow y = \frac{g x^2}{2u^2} \Rightarrow x = v\sqrt{\frac{2y}{g}}$

$x = \frac{200}{3.6}\sqrt{\frac{2(100)}{9.81}} = 251\ \text{m}$

$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{100}{251} = 21.7°$

۷۶-۲.

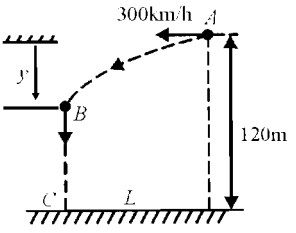
$t = 0$: لحظه رها کردن بسته در نقطه A

t_B ، t_C : زمان بسته در نقاط B و C

از A تا B : $y = \frac{1}{2} g t_B^2$ (1)

از B تا C : $(120 - y) = 1.8\ (t_C - t_B)$ (2)

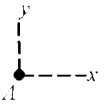
$t_C = 37\ \text{s}$



(2) ، (1) حل $\Rightarrow t_B = 3.49\ \text{s}$ ، $y = 59.7\ \text{m}$

$L = \frac{300}{3.6}(3.49) = 291\ \text{m}$

۷۷-۲.



الف) $v_0 = 42\ \text{m/s}$ ، $\theta = 8°$

$x = x_0 + v_{x_0} t$

(در نقطه B) : $60 = 0 + (42\cos 8°)t \Rightarrow t = 1.443\ \text{s}$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$

(در نقطه B) : $-(2.3 - h) = 0 + 42\sin 8°\ (1.443) - \frac{1}{2}(9.81)(1.443)^2 \Rightarrow h = 0.525\ \text{m}$

ب) $v_0 = 36\ \text{m/s}$ ، $\theta = 12°$

$x = x_0 + v_{x_0} t$

(در نقطه B) : $60 = 0 + (36\cos 12°)t \Rightarrow t = 1.704\ \text{s}$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$

(در نقطه B) : $-(2.3 - h) = 0 + 36\sin 12°\ (1.704) - \frac{1}{2}(9.81)(1.704)^2 \Rightarrow h = 0.813\ \text{m}$

(در بیس‌بال زمان حرکت توپ در هوا مهم است. مسیرهای کم ارتفاع بهتر هستند.)

۲-۷۸. مبدأ دستگاه مختصات x–y را در نقطه A قرار دهید.

$x = x_0 + v_{x_0} t$ (در نقطه B) : $800 + s\cos 20° = (120\cos 40°)t$ (1)

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ (در نقطه B) : $-s\sin 20° = (120\sin 40°)t - \frac{9.81}{2} t^2$ (2)

حل همزمان (1) و (2) $\Rightarrow$ $s = 1057\,\text{m}$, $t = 19.50\,\text{s}$

۲-۷۹. مبدأ دستگاه مختصات y – x را در نقطه A قرار دهید.

$v_{x_0} = 25\cos\theta$

$v_{y_0} = 25\sin\theta$

$v_y^2 = v_{y_0}^2 - 2g(y - y_0)$ $\xrightarrow{\text{در سقف پرواز}}$ $0^2 = (25\sin\theta)^2 - 2(9.81)(5 - 0) \Rightarrow \theta = 23.3°$

برد $R = \frac{v_0^2 \sin 2\theta}{g} = \frac{25^2 \sin(2\times 23.3°)}{9.81} = 46.4\,\text{m}$

۲-۸۰.

$a_x = 0$: $x = v_{x_0} t \Rightarrow 11.7 = v t_B$

$a_y = g$: $y = v_{y_0} t + \frac{1}{2} g t^2$

در نقطه B : $2.55 - 1.05 = 0 + \frac{9.81}{2} t_B^2$

$\Rightarrow t_B = 0.553\,\text{s}$

$\Rightarrow v = \frac{11.7}{0.553} = 21.2\,\text{m/s}$

در نقطه C : $2.55 = \frac{1}{2}(9.81)\, t_C^2 \Rightarrow t_C = 0.721\,s$

$s + 11.7 = 21.2\,(0.721) \Rightarrow s = 3.55\,\text{m}$

۲-۸۱. طبق مثال ۲-۶:

برد افقی : $2\,\text{s} = \frac{u^2 \sin 2\theta}{g}$

$\Rightarrow 16(10^3) = \frac{u^2 \sin 90°}{9.81} \Rightarrow u = 396\,\text{m/s}$

ارتفاع اوج : $h = \frac{u^2 \sin^2\theta}{2g} \Rightarrow H = h + 600$

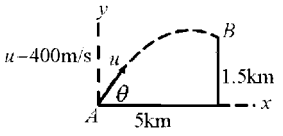
$= \frac{(396)^2 \sin^2 45°}{2(9.81)} + 600 = 4600\,\text{m}$

$y = u t \sin\theta - \frac{1}{2} g t^2 \Rightarrow -600 = 396\,(0.7071)\, t - \frac{1}{2}(9.81)\, t^2$

$\Rightarrow t^2 - 57.11\, t - 122.3 = 0 \Rightarrow t = 59.18\,\text{s}$

$x = u t \cos\theta \Rightarrow R = 396\,(59.18)\cos 45° = 16579\,\text{m}$ یا $R = 16.58\,\text{km}$

۲-۸۲. طبق مثال ۲-۶ معادله مسیر چنین است:

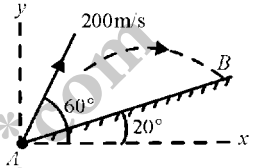


$$y = x\tan\theta - \frac{gx^2}{2u^2}\sec^2\theta$$
$$= x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$$

جایگذاری مقادیر $\Rightarrow 1500 = 5000\tan\theta - \frac{9.81(5000)^2}{2(400)^2}(1 + \tan^2\theta)$

$\Rightarrow \tan^2\theta - 6.524\tan\theta + 2.957 = 0 \Rightarrow \theta_1 = 26.1^\circ \quad , \quad \theta_2 = 80.6^\circ$

۲-۸۳.



$$\begin{cases} v_{x_0} = 200\cos 60^\circ = 100\ \text{m/s} \\ v_{y_0} = 200\sin 60^\circ = 173.2\ \text{m/s} \end{cases}$$

$t_f =$ زمان حرکت پرتابه

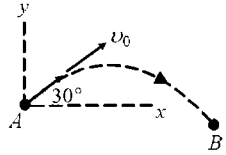
$x = x_0 + v_{x_0}t$ (B در نقطه) : $R\cos 20^\circ = 100\,t_f \quad (1)$

$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$ (B در نقطه) : $R\sin 20^\circ = 173.2\,t_f - \frac{9.81}{2}t_f^2 \quad (2)$

$(1) : t_f = 0.00940\,R$

$(2) : R\sin 20^\circ = 173.2(0.00940\,R) - \frac{9.81}{2}(0.00940\,R)^2 \Rightarrow R = 2970\ \text{m}$

۲-۸۴. محورهای مختصات $x - y$ را در A قرار دهید. (هدف نقطهٔ B)



$x_B = (v_0\cos 30^\circ)\,t$

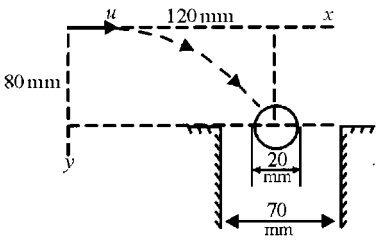
$y_B = (v_0\sin 30^\circ)\,t - \frac{1}{2}gt^2$

$x_B = 3.8\ \text{m} \quad , \quad y_B = -0.3\ \text{m} \Rightarrow v_0 = 6.15\ \text{m/s} \quad , \quad t = 0.713\ \text{s}$

$x_B = 4.1\ \text{m} \quad , \quad y_B = -0.1\ \text{m} \Rightarrow v_0 = 6.68\ \text{m/s} \quad , \quad t = 0.709\ \text{s}$

بنابراین محدودهٔ سرعت $6.15 \le v_0 \le 6.68$ m/s خواهد بود.

۲-۸۵.



$x = ut \quad , \quad y = \frac{1}{2}gt^2$

$x = u\sqrt{\frac{2y}{g}} \Rightarrow u = x\sqrt{\frac{g}{2y}}$

$x_{\max} = 120 + \frac{70}{2} - \frac{20}{2} = 145\ \text{mm}$

$$x_{min} = 120 - \frac{70}{2} + \frac{20}{2} = 95 \text{ mm}$$

$$u_{max} = 0.145\sqrt{\frac{9.81}{2(0.080)}} = 1.135 \text{ m/s}$$

$$u_{min} = 0.095\sqrt{\frac{9.81}{2(0.080)}} = 0.744 \text{ m/s}$$

۲-۸۶. دستگاه مختصات x-y را در A قرار دهید.

$$x_B = (11 \cos\theta)\, t$$

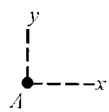
$$y_B = (11 \sin\theta)\, t - \frac{9.81}{2} t^2$$

$x_B = 12 \text{ m} \;,\; y_B = -0.650 \text{ m}$ (بالای دیرک) $\Rightarrow \theta = 31.7°$ یا $\theta = 55.2°$

$x_B = 12 \text{ m} \;,\; y_B = -1 \text{ m}$ (پائین دیرک) $\Rightarrow \theta = 28.8°$ یا $\theta = 56.4°$

محدوده : $28.8° \le \theta \le 31.7°$ یا $55.2° \le \theta \le 56.4°$

۲-۸۷. مبدأ دستگاه مختصات را در نقطه پرتاب در نظر بگیرید:



$$v_{x_0} = v_0 \sin\theta = 12 \sin\theta$$

$$v_{y_0} = v_0 \cos\theta = 12 \cos\theta$$

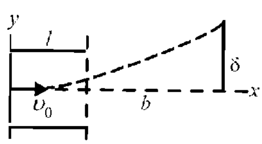
$v_y = v_{y_0} - gt$ (در پایان حرکت)

$$-12 \cos\theta = 12 \cos\theta - 9.81\, t_f \Rightarrow t_f = 2.45 \cos\theta$$

$v_x = v_{x_0} - 0.4\, t$ (در پایان حرکت)

$$-12 \sin\theta = 12 \sin\theta - 0.4\,(2.45 \cos\theta) \Rightarrow \tan\theta = 0.041 \Rightarrow \theta = 2.33°$$

۲-۸۸. زمان عبور از بین صفحات برابر است با:



$$t = \frac{l}{v_0}$$

$v_x = v_0 =$ ثابت

$$v_y = a_y\, t = \frac{eE}{m} \frac{l}{v_0}$$

به محض خروج از صفحات و انحراف در این نقطه:

$$y = \frac{1}{2} a_y\, t^2 = \frac{eE}{2m} \frac{l^2}{v_0^2}$$

$$\frac{\delta - y}{b} = \frac{v_y}{v_x} \Rightarrow \delta = \frac{eE}{2m}\frac{l^2}{v_0^2} + b\,\frac{eE}{m}\frac{l}{v_0} / v_0$$

$$\Rightarrow \delta = \frac{eEl}{mv_0^2}\left(\frac{l}{2} + b\right)$$

۲-۸۹. براساس نتایج مثال ۲-۶:

$$h = \frac{u^2 \sin^2\theta}{2g} \quad , \quad s = \frac{u^2 \sin 2\theta}{2g}$$

$$\frac{h}{s} = \frac{\sin^2\theta}{\sin 2\theta} = \frac{1}{2}\tan\theta$$

$$\theta = \tan^{-1}\frac{2h}{s} = \tan^{-1}\frac{2(500)}{400} = 68.2°$$

$$u^2 = \frac{2gh}{\sin^2\theta} = \frac{2(9.81)(0.5)}{(0.9285)^2} = 11.38\ \frac{m^2}{s^2} \Rightarrow u = 3.37\ m/s$$

$$v = u\cos\theta = \text{ثابت} \Rightarrow v = 3.37\ (0.3714) = 1.253\ m/s$$

۲-۹۰. دستگاه مختصات را در نقطه پرتاب قرار دهید.

$$x = x_0 + v_{x_0} t \Rightarrow 20 = (40\cos\theta)\, t_f \quad (1)$$

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \Rightarrow -1.8 = 0 + (-40\sin\theta)\, t_f - \frac{9.81}{2} t_f^2 \quad (2)$$

$$(1) : t_f = \frac{1}{2\cos\theta}$$

$$(2) : -1.8 = -40\sin\left(\frac{1}{2\cos\theta}\right) - \frac{9.81}{2}\left(\frac{1}{2\cos\theta}\right)^2$$

$$\left(\frac{1}{\cos^2\theta} = \tan^2\theta + 1\right) \Rightarrow 1.266\tan^2\theta + 20\tan\theta - 0.574 = 0 \Rightarrow \theta = 1.640°$$

$$d = 20\tan 1.640 = 0.573\ m$$

$$h = (2.2 + 0.6) - (0.573 + 1) = 1.227\ m$$

۲-۹۱.

$$y = R\sin\theta = -u t\cos\theta + \frac{1}{2} g t^2$$

$$x = R\cos\theta = u t\sin\theta$$

$$t \text{ حذف} \Rightarrow R\sin\theta = -R\frac{\cos\theta}{\sin\theta}\cos\theta + \frac{1}{2} g \left(\frac{R\cos\theta}{u\sin\theta}\right)^2$$

$$\Rightarrow \frac{1}{\sin\theta} = \frac{g}{2}\frac{R}{u^2\tan^2\theta} \Rightarrow R = \frac{2u^2\tan^2\theta}{g\sin\theta} = \frac{2u^2}{g}\tan\theta\sec\theta$$

۲-۹۲. دستگاه مختصات x-y را در نقطه پرتاب قرار دهید.

$$x = x_0 + v_{x_0} t$$

$$(\text{در حلقه}) : 4 = 0 + (v_0\cos 50°)\, t_f \Rightarrow t_f = \frac{6.22}{v_0}$$

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$$

$$(\text{در حلقه}) : 0.9 = 0 + (v_0\sin 50°)\left(\frac{6.22}{v_0}\right) - \frac{9.81}{2}\left(\frac{6.22}{v_0}\right)^2 \Rightarrow v_0 = 7.01\ m/s$$

۲-۹۳. از دستگاه مختصات شکل استفاده کنید. در غیاب سطح دایروی $C - B$، بـرد روی سطح افقی برابر خواهد بود با:

$$\frac{v_0^2}{g}\sin 2\theta = \frac{68^2}{9.81}\sin(2\times 30^\circ) = 408.2\ \text{m} > 300\ \text{m}$$

لذا نقطه برخورد خارج از B خواهد بود.

$$x = x_0 + v_{x_0}t \quad\Rightarrow\quad x = (68\cos 30^\circ)t \qquad (1)$$

$$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2 \quad\Rightarrow\quad y = (68\sin 30^\circ)t - \frac{9.81}{2}t^2 \qquad (2)$$

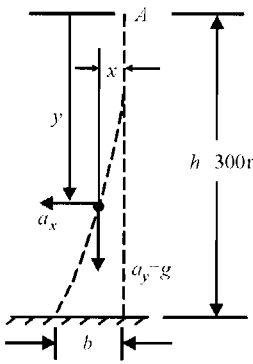
قیدهای سطح : $(x-300)^2 + (y-150)^2 = 150^2 \qquad (3)$

از حل کامپیوتری معادلات (1) تا (3) خواهیم داشت:

$$x = 373\ \text{m} \quad , \quad y = 18.75\ \text{m} \quad , \quad t = 6.33\ \text{s}$$

(پاسخ‌ها نشان می‌دهند که نقطه برخورد روی سطح دایروی خواهد بود نه بالاتر از C.)

۲-۹۴.



$$a_y = g \quad , \quad v_g = gt \quad , \quad y = \frac{1}{2}gt^2$$

$$a_x = 2v_y\omega\cos\gamma = 2gt\omega\cos\gamma$$

$$v_x = gt^2\omega\cos\gamma$$

$$x = \frac{1}{3}gt^3\omega\cos\gamma$$

وقتی $y = h \quad\Rightarrow\quad t = \sqrt{\frac{2h}{g}}$

بنابراین : $x = b = \frac{1}{3}g\left(\frac{2h}{g}\right)^{\frac{3}{2}}\omega\cos\gamma \quad\Rightarrow\quad b = \frac{2\sqrt{2}}{3}h\sqrt{\frac{h}{g}}\,\omega\cos\gamma$

برای $g = 9.793\ \text{m/s}^2$ ، $\omega = 0.7292(10^{-4})\ \text{rad/s}$ ، $h = 300\ \text{m}$ خواهیم داشت:

$$b = \frac{2\sqrt{2}}{3}(300)\sqrt{\frac{300}{9.793}}(0.7292)(10^{-4})\cos 30^\circ = 0.0989\ \text{m} = 98.9\ \text{mm}$$

۲-۹۵.

$$\left.\begin{aligned} \mathbf{a} &= -k\mathbf{v} - g\mathbf{j} \\ a_x\mathbf{i} + a_y\mathbf{j} &= -k(v_x\mathbf{i} + v_y\mathbf{j}) - g\mathbf{j} \end{aligned}\right\} \Rightarrow \begin{aligned} a_x &= -kv_x \\ a_y &= -kv_y - g \end{aligned}$$

$$\mathbf{x} : a_x = \frac{dv_x}{dt} = -kv_x$$

$$\int_{v_{x_0}}^{v_x} \frac{dv_x}{dt} = -\int_0^t k\,dt \quad \Rightarrow \quad v_x = v_{x_0}\,e^{-kt} \qquad \text{یا} \qquad v_x = (v_0\cos\theta)\,e^{-kt}$$

$$v_x = \frac{dx}{dt} = v_{x_0}\,e^{-kt}$$

$$\int_0^x dx = \int_0^t v_{x_0}\,e^{-kt}\,dt \quad \Rightarrow \quad x = \frac{v_{x_0}}{k}\,[1 - e^{-kt}] = \frac{v_0\cos\theta}{k}\,[1 - e^{-kt}]$$

$$\mathbf{y}: a_y = \frac{dv_y}{dt} = -k\,v_y - g$$

$$\int_{v_{y_0}}^{v_y} \frac{dv_y}{k\,v_y + g} = -\int_0^t dt \quad \Rightarrow \quad v_y = \left[v_{y_0} + \frac{g}{k}\right] e^{-kt} - \frac{g}{k}$$

$$\Rightarrow \quad v_y = \left[v_0\sin\theta + \frac{g}{k}\right] e^{-kt} - \frac{g}{k}$$

$$v_y = \frac{dy}{dt} = \left[v_{y_0} + \frac{g}{k}\right] e^{-kt} - \frac{g}{k}$$

$$\int_0^y dy = \int_0^t \left\{\left[v_{y_0} + \frac{g}{k}\right] e^{-kt} - \frac{g}{k}\right\} dt$$

$$\Rightarrow \quad y = \frac{1}{k}\left[v_0\sin\theta + \frac{g}{k}\right][1 - e^{-kt}] - \frac{g}{k}\,t$$

سرعت نهایی $(t \to \infty)$: $v_x \to 0$

$$v_y \to -\frac{g}{k}$$

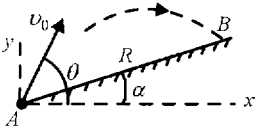
◀ ۹۶-۲.

$$x = x_0 + v_{x_0}\,t$$

(B در نقطه) : $R\cos\alpha = (v_0\cos\theta)\,t_f$

$$y = y_0 + v_{y_0}\,t - \frac{1}{2}\,gt^2$$

(B در نقطه) : $R\sin\alpha = (v_0\sin\theta)\,t_f - \frac{1}{4}\,g\,t_f^2$



x رابطه : $t_f = \dfrac{R\cos\alpha}{v_0\cos\alpha}$

y رابطه : $R\sin\alpha = (v_0\sin\theta)\left(\dfrac{R\cos\alpha}{v_0\cos\theta}\right) - \dfrac{1}{2}\,g\left(\dfrac{R\cos\alpha}{v_0\cos\theta}\right) \Rightarrow R = \dfrac{2v_0^2\cos^2\theta}{g\cos\alpha}(\tan\theta - \tan\alpha)$

$$\frac{dR}{d\theta} = 0 = \frac{4v_0^2\cos\theta(-\sin\theta)}{g\cos\alpha}(\tan\theta - \tan\alpha) + \frac{2v_0^2\cos^2\theta}{g\cos\alpha}\,\frac{1}{\cos^2\theta} = 0$$

$$\Rightarrow \quad \frac{2v_0^2}{g\cos\alpha}[2\cos\theta\sin\theta(\tan\alpha - \tan\theta) + 1] = 0$$

$$\Rightarrow \quad 2\cos\theta\sin\theta\left(\tan\alpha - \frac{\sin\theta}{\cos\theta}\right) + 1 = 0$$

$$\Rightarrow \quad (2\cos\theta\sin\theta)\tan\alpha - 2\sin^2\theta + 1 = 0$$

$$\Rightarrow \sin 2\theta \tan\alpha - 2\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right) + 1 = 0$$

$$\Rightarrow \sin 2\theta \tan\alpha + \cos 2\theta = 0 \quad \Rightarrow \quad \tan 2\theta = -\frac{1}{\tan\alpha}$$

$$\Rightarrow 2\theta = \tan^{-1}\left(\frac{-1}{\tan\alpha}\right) = 180^\circ - \tan^{-1}\left(\frac{1}{\tan\alpha}\right) = 180^\circ - (90^\circ - \alpha) = 90^\circ + \alpha$$

$$\Rightarrow \theta = \frac{90^\circ + \alpha}{2}$$

مقادیر خاص :
$$\begin{cases} \alpha = 0 \Rightarrow \theta = 45^\circ \\ \alpha = 30 \Rightarrow \theta = 60^\circ \\ \alpha = 45 \Rightarrow \theta = 67.5^\circ \end{cases}$$

۹۷-۲.

$a_t =$ ثابت ، $v = \cancel{v_0}^{0} + a_t t \Rightarrow a_t = \frac{v}{t} = \frac{100/3.6}{10} = 2.78\ \text{m/s}^2$

$v_8 = 2.78(8) = 22.2\ \text{m/s}$

$a_n = \frac{v^2}{\rho} = \frac{22.2^2}{80} = 6.17\ \text{m/s}^2$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2.78^2 + 6.17^2} = 6.77\ \text{m/s}^2$

۹۸-۲.

a_1 : سرعت افزایش می‌یابد، بدون انحنای مسیر

a_2 : سرعت افزایش می‌یابد، اتومبیل به چپ گردش می‌کند.

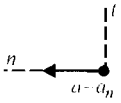
a_3 : سرعت ثابت، اتومبیل به چپ گردش می‌کند.

a_4 : سرعت کاهش می‌یابد، اتومبیل به چپ گردش می‌کند.

a_5 : سرعت کاهش می‌یابد، بدون انحنای مسیر

a_6 : سرعت کاهش می‌یابد، اتومبیل به راست گردش می‌کند.

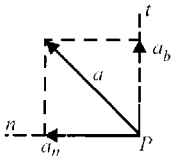
۹۹-۲.



الف) $a_n = \frac{v^2}{\rho} = \frac{1.2^2}{0.6} = 2.4\ \text{m/s}^2$

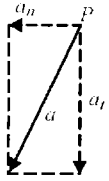
$a_t = 0$

$a = \sqrt{a_n^2 + a_t^2} = 2.4\ \text{m/s}^2$



ب) $a_n = 2.4\ \text{m/s}^2 \qquad a_t = 2.4\ \text{m/s}^2$

$a = \sqrt{2.4^2 + 2.4^2} = 3.39\ \text{m/s}^2$



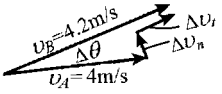
ج) $a_n = 2.4\ \text{m/s}^2 \qquad a_t = -4.8\ \text{m/s}^2$

$a = \sqrt{2.4^2 + 4.8^2} = 5.37\ \text{m/s}^2$

۲-۱۰۰.

$$\Delta\theta = (25 - 15)\frac{\pi}{180} = 0.1745 \text{ rad}$$

$$v_{av} = \frac{4 + 4.2}{2} = 4.1 \text{ m/s}$$



$$a_n = \frac{\Delta v_n}{\Delta t} = \frac{v_{av}(\Delta\theta)}{\Delta t} = \frac{4.1(0.1745)}{2.62 - 2.4} = 3.25 \text{ m/s}^2$$

$$a_t = \frac{\Delta v_t}{\Delta t} = \frac{4.2 - 4}{0.22} = 0.909 \text{ m/s}^2$$

۲-۱۰۱.

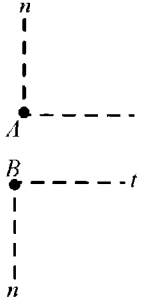
$$v = v_0 + a_t t: \quad \frac{50}{3.6} = \frac{100}{3.6} + 12\,a_t \Rightarrow a_t = -1.157 \text{ m/s}^2$$

$$a = \sqrt{a_n^2 + a_t^2} \quad : \quad 2 = \sqrt{1.157^2 + a_n^2} \Rightarrow a_n = 1.631 \text{ m/s}^2$$

$$v_6 = v_0 + a_t t = \frac{100}{3.6} - 1.157(6) = 20.8 \text{ m/s}$$

$$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{v^2}{a_n} = \frac{20.8^2}{1.631} = 266 \text{ m}$$

۲-۱۰۲.



در نقطه A : $a_n = \frac{v_A^2}{\rho_A} \Rightarrow 0.4(9.81) = \frac{v_A^2}{120 - 0.6} \Rightarrow v_A = v = 21.6 \text{ m/s}$

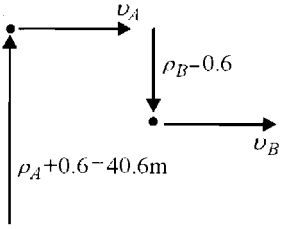
در نقطه B : $a_n = \frac{v_B^2}{\rho_B} \Rightarrow 0.25(9.81) = \frac{21.7^2}{\rho_B + 0.6} \Rightarrow v_B = 190.4 \text{ m}$

۲-۱۰۳.

$$v_A = \frac{50}{3.6} = 13.89 \text{ m/s} \quad , \quad v_B = \frac{100}{3.6} = 27.8 \text{ m/s}$$

$$v_B = v_A + a_t t \Rightarrow a_t = \frac{27.8 - 13.89}{10} = 1.389 \text{ m/s}^2$$

در نقطه A : $a_n = \frac{v^2}{\rho} = \frac{13.89^2}{40.6} = 4.75 \text{ m/s}^2$



$$a_A = \sqrt{4.75^2 + 1.389^2} = 4.95 \text{ m/s}^2$$

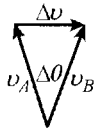
در نقطه B : $a_A = a_B = 4.95 \text{ m/s}^2$

$$4.95 = \sqrt{1.389^2 + a_n^2} \Rightarrow a_n = 4.75$$

$$a_n = \frac{27.8^2}{\rho_B - 0.6} \Rightarrow \rho_B = 163.0 \text{ m}$$

۲-۱۰۴.

$$v_A = v_B = v = 2\ \text{m/s}$$



$$\Delta v = 2v \sin\frac{\Delta\theta}{2} = 4\sin\frac{\Delta\theta}{2}\ \text{m/s}$$

$$\Delta t = \frac{r\Delta\theta}{v} = \frac{0.8\Delta\theta}{2} = 0.4\Delta\theta$$

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{4\sin\frac{\Delta\theta}{2}}{0.4\Delta\theta} = 5\,\frac{\sin\frac{\Delta\theta}{2}}{\frac{\Delta\theta}{2}}$$

	$\Delta\theta°$	$\frac{\Delta\theta°}{2}$	$\frac{\Delta\theta}{2}$ rad	$\sin\frac{\Delta\theta}{2}$	a_{av} m/s^2	٪ اختلاف
(الف)	30°	15°	0.262	0.259	4.94	1.1
(ب)	15°	7.5°	0.1309	0.1305	4.99	0.3
(ج)	5°	2.5°	0.0436	0.0436	4.998	0.03

۲-۱۰۵.

$$a_n = \frac{v^2}{\rho} \Rightarrow v = \sqrt{a_n\rho} = \sqrt{0.8\,g\rho}$$

$$v_A = \sqrt{0.8\,g\rho_A} = \sqrt{0.8(9.81)(85)} = 25.8\ \text{m/s}$$

$$v_B = \sqrt{0.8\,g\rho_B} = \sqrt{0.8(9.81)(200)} = 39.6\ \text{m/s}$$

مسیر BB مزیت قابل توجهی دارد.

۲-۱۰۶.

$$v = v_0 + a_t t = 0 + 1.8(5) = 9\ \text{m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{9^2}{40} = 2.025\ \text{m/s}^2$$

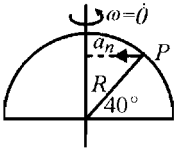
$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(1.8)^2 + (2.025)^2} = 2.71\ \text{m/s}^2$$

۲-۱۰۷.

$$a = a_n = r\dot{\theta}^2 = R\cos\gamma\,\dot{\theta}^2$$

$$= \frac{12.742(10^6)}{2}\cos 40°\,(0.729\times10^{-4})^2$$

$$= 0.0259\ \text{m/s}^2$$



۲-۱۰۸.

$$a_n = g = \frac{v^2}{\rho} = \frac{\left(\frac{27955}{3.6}\right)^2}{(6371 + 240)(10^3)} = 9.12\,\text{m/s}^2$$

مقایسه: $g = g_0 \left(\frac{R}{R+h}\right)^2 = 9.821 \left(\frac{6371}{6371+240}\right)^2 = 9.12\,\text{m/s}^2$ ✓

۲-۱۰۹.

شعاع سیاره مشتری $R = \frac{142984}{2}(10^3) = 7.15(10^7)\,\text{m}$

$\rho = R + h = 7.15(10^7) + 10^6\text{m} = 7.25(10^7)\text{m}$

طبق قانون گرانش: $a_n = g = g_0 \frac{R^2}{(R+h)^2} = g_0 \frac{R^2}{\rho^2} = 24.85 \frac{[7.15(10^7)]^2}{[7.25(10^7)]^2} = 24.2\,\text{m/s}^2$

$a_n = \frac{v^2}{\rho} \Rightarrow v^2 = a_n \rho = (24.2)(7.25)(10^7) \Rightarrow v = 41900\,\text{m/s}$

۲-۱۱۰.

برای P_1: $a_n = \frac{v^2}{r} \Rightarrow v = \sqrt{0.1(40)} = 2\,\text{m/s}$

$a_t = \sqrt{a_n^2 + a_t^2} = \sqrt{40^2 + 30^2} = 50\,\text{m/s}^2$

برای P_2: $a_n = \frac{v^2}{r} = \frac{2^2}{0.05} = 80\,\text{m/s}^2$

$a_2 = \sqrt{a_n^2 + a_t^2} = \sqrt{80^2 + 30^2} = 85.4\,\text{m/s}^2$

۲-۱۱۱.

$a_n = v\dot{\beta} = g \Rightarrow \dot{\beta} = \frac{9.79}{\frac{800(10^3)}{3.6}} = 0.04406\,\text{rad/s}$

یا $\dot{\beta} = 0.04406 \left(\frac{180}{\pi}\right) = 2.52 \frac{\text{deg}}{\text{s}}$

۲-۱۱۲.

برای ایستگاه فضایی $a_n = r\dot{\theta}^2$ (که در آن $a_n = 9.81\,\text{m/s}^2$)

بنابراین: $9.81 = (75 + 6)\,\dot{\theta}^2 \Rightarrow \dot{\theta} = 0.348\,\text{rad/s}$

$N = 0.348 \left(\frac{60}{2\pi}\right) = 3.32\,\text{rev/min}$

۲-۱۱۳.

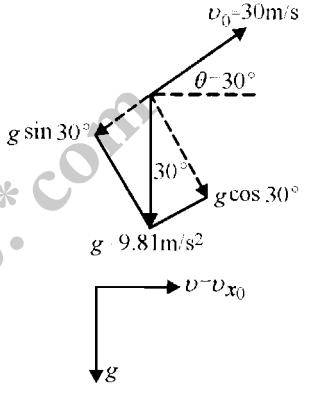
$$a_t = \frac{v_f - v_i}{\Delta t} = \frac{6 - 3}{2} = 1.5\ \text{m/s}^2$$

در وسط فاصله زمانی $v = 4.5\ \text{m/s}^2$

$$a_{P_1} = \sqrt{a_t^2 + a_n^2} = \sqrt{1.5^2 + \left(\frac{4.5^2}{0.06}\right)^2} = 338\ \text{m/s}^2 \quad (= 34.4\,\text{g}!)$$

$$a_{P_2} = a_t = 1.5\ \text{m/s}^2$$

۲-۱۱۴.



(الف) $a_n = g\cos 30° = \dfrac{v^2}{\rho}$

$$\rho = \frac{30^2}{9.81\cos 30°} = 105.9\ \text{m}$$

$$\dot{v} = -g\sin 30° = -4.91\ \text{m/s}^2$$

(ب) $a_n = g = \dfrac{v^2}{\rho} \Rightarrow \rho = \dfrac{(30\cos 30°)^2}{9.81} = 68.8\ \text{m}$

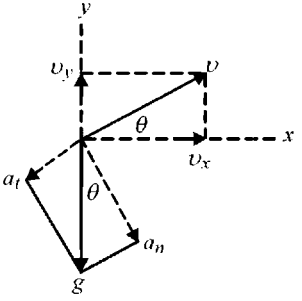
$$\dot{v} = 0$$

۲-۱۱۵. زمان t_{up} برای رسیدن به اوج از رابطهٔ زیر به دست می‌آید:

$$v_y = v_{y_0} - gt: \quad 0 = 30\sin 30° - 9.81\,t_{up} \Rightarrow t_{up} = 1.53\ \text{s}$$

بنابراین $t = 1\ \text{s}$ قبل از رسیدن به اوج و $t = 2.5\ \text{s}$ بعد از اوج است.

(الف) $t = 1\ \text{s}$



$$v_x = 30\cos 30° = 25.98\ \text{m/s}$$

$$v_y = 30\sin 30° - 9.81(1) = 5.19\ \text{m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = 26.49\ \text{m/s}$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} = 11.3°$$

$$a_n = g\cos\theta = 9.81\cos 11.3° = 9.62\ \text{m/s}^2$$

$$\rho = \frac{v^2}{a_n} = \frac{26.49^2}{9.62} = 73.0\ \text{m}$$

$$a_t = -g\sin\theta = -9.81\sin 11.3° = -1.922\ \text{m/s}^2$$

(ب) $t = 2.5\,\text{s}$

$v_x = 30\cos 30° = 25.98\ \text{m/s}$

$v_y = 30\sin 30° - 9.81(2.5) = -9.525\ \text{m/s}$

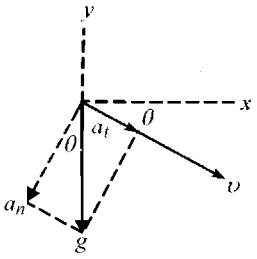
$v = \sqrt{v_x^2 + v_y^2} = 27.67\ \text{m/s}$

$\theta = \tan^{-1}\dfrac{v_y}{v_x} = 20.13°$

$a_n = g\cos\theta = 9.81\cos 20.13° = 9.21\ \text{m/s}^2$

$\rho = \dfrac{v^2}{a_n} = \dfrac{27.67^2}{9.21} = 83.1\ \text{m}$

$a_t = +g\sin\theta = +9.81\sin 20.13° = 3.38\ \text{m/s}^2$



۲-۱۱۶. $a_t = -0.6\ \text{m/s}^2$ (ثابت)

$v_B^2 = v_A^2 + 2a_t s = 16^2 - 2(0.6)(120) \Rightarrow v_B = 10.58\ \text{m/s}$

$a_n = \dfrac{v_B^2}{\rho} = \dfrac{10.58^2}{60} = 1.867\ \text{m/s}^2$

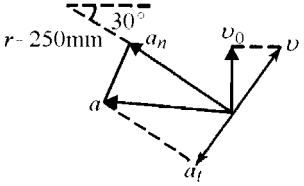
$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.6^2 + 1.867^2} = 1.961\ \text{m/s}^2$

۲-۱۱۷.

$v = \dfrac{v_0}{\cos 30°} = \dfrac{2}{\cos 30°} = 2.31\ \text{m/s}$

$a_n = \dfrac{v^2}{r} = \dfrac{2.31^2}{0.250} = 21.3\ \text{m/s}^2$

$a_t = -a_n\tan 30° = -12.32\ \text{m/s}^2$

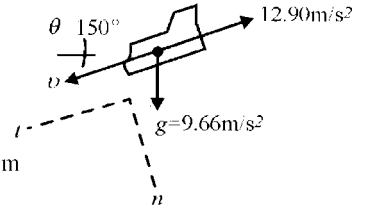


۲-۱۱۸.

$\dot{v} = a_t = 9.66\sin 1.5° - 12.90 = -12.65\ \text{m/s}^2$

$a_n = g\cos\theta = 9.66\cos 1.5° = 9.657\ \text{m/s}^2$

$a_n = \dfrac{v^2}{\rho} \Rightarrow \rho = \dfrac{v^2}{a_n} = \dfrac{(15450/3.6)^2}{9.657} \Rightarrow \rho = 1907\ \text{km}$



۲-۱۱۹.

$a_n = 0.8g = \dfrac{v^2}{\rho} \Rightarrow v = \sqrt{0.8g\rho}$

اتومبیل A : $v_A = \sqrt{0.8(9.81)(88)} = 26.3\ \text{m/s}$

B اتومبیل : $v_B = \sqrt{0.8(9.81)(72)} = 23.8\,\text{m/s}$

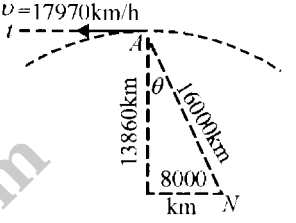
$$t_A = \frac{s_A}{v_A} = \frac{\pi(88)}{26.3} = 10.52\,\text{s} \quad , \quad t_B = \frac{s_B}{v_B} = \frac{\pi(72) + 2(16)}{23.8} = 10.86\,\text{s}$$

اتومبیل A مسابقه را خواهد برد.

۲-۱۲۰.

شتاب از A به سمت N برابر است با: $a = g = g_0\left(\frac{R}{R+h}\right)^2$

$$= 9.821\left(\frac{6371}{16000}\right)^2 = 1.557\,\text{m/s}^2$$



$$a_n = 1.557\cos\theta = 1.557(0.866) = 1.348\,\text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{(17970)^2}{1.348(10^{-3})(3600)^2} = 18480\,\text{km}$$

۲-۱۲۱.

$$v_{x=15} = \sqrt{2g(1.8)} = \sqrt{3.6g}\ \text{m/s}$$

$$a_n = \frac{v^2}{\rho} \ , \ \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{[1 + (0.016\,x)^2]^{\frac{3}{2}}}{0.016}$$

$$\rho_{x=15} = 67.9\,\text{m}$$

$$a_n = \frac{3.6(9.81)}{67.9} = 0.520\,\text{m/s}^2$$

۲-۱۲۲.

$$y = \frac{x^2}{16} \Rightarrow \frac{dy}{dx} = \frac{x}{8} \ , \ \frac{d^2y}{dx^2} = \frac{1}{8}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + \left(\frac{x}{8}\right)^2\right]^{\frac{3}{2}}}{\frac{1}{8}}$$

$$\rho_{y=1} = \frac{[1 + (0.5)^2]^{\frac{3}{2}}}{\frac{1}{8}} = 11.17\,\text{m}$$

$$v = r\dot{\theta} = \frac{0.75}{2} \cdot \frac{120 \times 2\pi}{60} = 1.5\pi\,\text{m/s}$$

$$a_t = \dot{v} = 0 \quad , \quad a = a_n = \frac{v^2}{\rho} = \frac{(1.5\pi)^2}{11.17} = 1.986\,\text{m/s}^2$$

۲-۱۲۳. $a_n = \frac{v^2}{r} = \frac{4^2}{0.120} = 133.3\ \text{m/s}^2$

$a_t = -a_n \operatorname{ctn} 4^\circ = -133.3 \operatorname{ctn} 4^\circ = -1907\ \text{m/s}^2$

(با فرض a_t ثابت): $v_f = v_i + a_t t \Rightarrow 0 = 4 - 1907\, t \Rightarrow t = 2.10(10^{-3})\ \text{s}$

۲-۱۲۴. $y - 0.1 = k x^2 \quad , \quad k = -\frac{1}{0.1} = -10\ \text{m}^{-1}$

$y = 0.1 - 10 x^2\ m$

$\dot{x} = 0.15\ \text{m/s} \quad , \quad \ddot{x} = 0$

$\dot{y} = -20 x \dot{x} \quad , \quad \ddot{y} = -20 \dot{x}^2$

$a = a_y = \ddot{y} = -20(0.15)^2 = 0.45\ \text{m/s}^2$

$\frac{dy}{dx} = -20 x = -20(0.06) = -1.2\ \text{m/s}$ ($x = 60$ mm در)

$\theta = \tan^{-1}(1.2) = 50.2^\circ$

$a_n = a \cos\theta = 0.45 \cos 50.2^\circ = 0.288\ \text{m/s}^2$

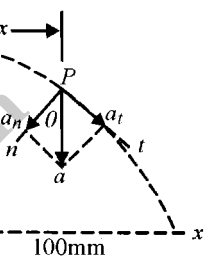
$a_t = a \sin\theta = 0.45 \sin 50.2^\circ = 0.346\ \text{m/s}^2$

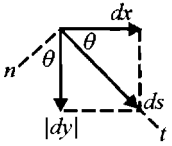
$x = 60$ mm برای : $\dot{y} = -20(0.06)(0.15) = -0.18\ \text{m/s}$

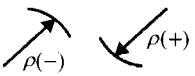
$v = \sqrt{0.15^2 + 0.18^2} = 0.234\ \text{m/s}$

$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{0.234^2}{0.288} = 0.1906\ \text{m}$

مقایسه : $\rho_{xy} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{[1 + (-1.2)^2]^{\frac{3}{2}}}{-20} = 0.1906\ \text{m}$ ✓







۲-۱۲۵. $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \overset{C \text{ در نقطه}}{\Rightarrow} 0 = 50 + 0 - \frac{1}{2}(9.81) t^2 \Rightarrow t = 3.193\ \text{s}$

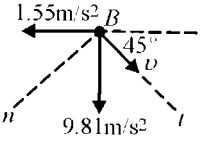
$x = x_0 + v_{x_0} t + \frac{1}{2} a_x t^2 \overset{C \text{ در نقطه}}{\Rightarrow} 40 = 0 + 15(3.193) + \frac{1}{2} a_x (3.193)^2 \Rightarrow a_x = -1.55\ \text{m/s}^2$

B در نقطه : $\begin{cases} v_x = v_{x_0} + a_x t \Rightarrow v_x = 15 - 1.55\, t \\ v_y = v_{y_0} - g t \Rightarrow |v_y| = 9.81\, t \end{cases}$

قرار می‌دهیم : $v_x = |v_y| \Rightarrow t = 1.32\ \text{s}$

بنابراین در نقطه B : $v_x = |v_y| = 12.95\ \text{m/s}$

سرعت در نقطه B $v = 12.95\sqrt{2} = 18.3\ \text{m/s}$



$a_n = 9.81\cos 45° + 1.55\cos 45° = 8\ \text{m/s}^2$

$a_n = \dfrac{v^2}{\rho} \Rightarrow \rho = \dfrac{18.3^2}{8} = 41.8\ \text{m}$

۲-۱۲۶.

$a_t =$ ثابت $\Rightarrow v_C^2 = v_A^2 + 2a_t\,\Delta s_{A-C}$

$v_A = \dfrac{250}{3.6}\ \text{m/s}$, $v_C = \dfrac{200}{3.6}\ \text{m/s} \Rightarrow a_t = \dfrac{(200)^2 - (250)^2}{(3.6)^2\ 2\ (300)} = -2.89\ \text{m/s}^2$

$v_B^2 = v_A^2 + 2a_t\Delta s_{A-B} = \left(\dfrac{250}{3.6}\right)^2 + 2(-2.89)(150) = 3954\left(\dfrac{\text{m}}{\text{s}}\right)^2 \Rightarrow v_B = 62.9\ \text{m/s}$

در نقطه B : $a_n = \dfrac{v_B^2}{\rho} = \dfrac{3954}{500} = 7.91\ \text{m/s}^2$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(2.89)^2 + (7.91)^2} = 8.42\ \text{m/s}^2$

۲-۱۲۷.

$$\begin{cases} x = -4t^2 + 14t + 4\ \text{m} \\ \dot{x} = -8t + 14\ \text{m/s} \\ \ddot{x} = -8\ \text{m/s}^2 \end{cases} \qquad \begin{cases} y = 6t + 1\ \text{m} \\ \dot{y} = 6\ \text{m/s} \\ \ddot{y} = 0 \end{cases}$$

در $t = 1\ \text{s}$:

$$\begin{cases} x = 14\ \text{m} \\ \dot{x} = 6\ \text{m/s} \\ \ddot{x} = -8\ \text{m/s}^2 \end{cases} \qquad \begin{cases} y = 7\ \text{m} \\ \dot{y} = 6\ \text{m/s} \\ \ddot{y} = 0 \end{cases}$$

$\mathbf{e}_t = \dfrac{\mathbf{v}}{v} = \dfrac{6\mathbf{i} + 6\mathbf{j}}{\sqrt{2(36)}} = 0.707\mathbf{i} + 0.707\mathbf{j}$

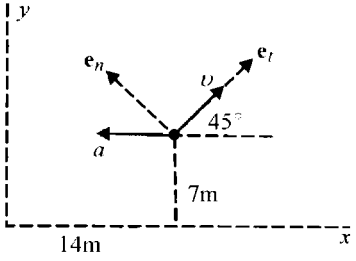
$\mathbf{e}_n \perp \mathbf{e}_t$: $\mathbf{e}_n = -0.707\mathbf{i} + 0.707\mathbf{j}$

$a_t = \mathbf{a}\cdot\mathbf{e}_t = -8\mathbf{i}\cdot\left(\dfrac{\sqrt{2}}{2}\mathbf{i} + \dfrac{\sqrt{2}}{2}\mathbf{j}\right) = -5.66\ \text{m/s}^2$

$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{6^2 + 6^2} = 8.49\ \text{m/s}$

$a_n = \mathbf{a}\cdot\mathbf{e}_n = -8\mathbf{i}\cdot\left(-\dfrac{\sqrt{2}}{2}\mathbf{i} + \dfrac{\sqrt{2}}{2}\mathbf{j}\right) = 5.66\ \text{m/s}^2$

$\rho = \dfrac{v^2}{a_n} = \dfrac{8.49^2}{5.66}\ 12.73\ \text{m}$



۲-۱۲۸.

$x = 16 - 12t + 4t^2$ $\quad y = 2 + 15t - 3t^2$

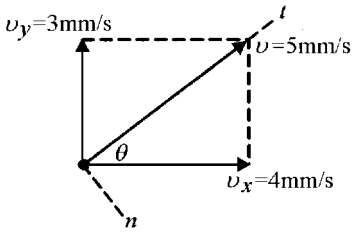
$\dot{x} = 8t - 12$ $\quad \dot{y} = 15 - 6t$

$\ddot{x} = 8$ $\quad \ddot{y} = -6$

در $t = 2\,s$:

$\dot{x} = 4\,mm/s$ $\quad \dot{y} = 3\,mm/s$

$\ddot{x} = 8\,mm/s^2$ $\quad \ddot{y} = -6\,mm/s^2$

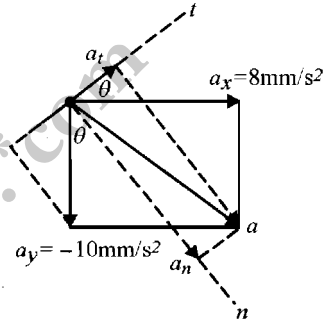


$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$

$$a_t = 8\cos\theta - 6\sin\theta = 8\left(\frac{4}{5}\right) - 6\left(\frac{3}{4}\right) = 2.8\,mm/s^2$$

$$a_n = 6\cos\theta + 8\sin\theta = 6\left(\frac{4}{5}\right) + 8\left(\frac{3}{5}\right) = 9.6\,mm/s^2$$

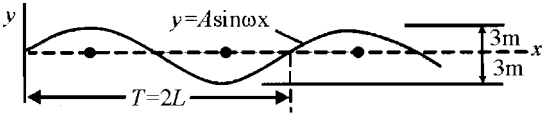
$$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{v^2}{a_n} = \frac{5^2}{9.6} = 2.60\,mm$$



۲-۱۲۹. ◀

$$y = A\sin\omega x \;,\; A = 3\,m \;,\; \omega = \frac{2\pi}{T}$$

شعاع مارپیچ $\rho = \dfrac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$



$$\frac{dy}{dx} = A\omega\cos\omega x \;,\; \frac{d^2y}{dx^2} = -A\omega^2\sin\omega x$$

$\dfrac{d\rho}{dx} = 0$ قرار می‌دهیم تا نشان دهیم که $|\rho|$ در $x = \dfrac{T}{4}$ و $x = \dfrac{3T}{4}$ مینیمم است.

$$\rho_{min} = \frac{\left[1 + \left\{A\frac{2\pi}{T}\cos\left(\frac{2\pi}{T}\cdot\frac{T}{4}\right)\right\}^2\right]}{A\left(\frac{2\pi}{T}\right)^2\sin\left(\frac{2\pi}{T}\cdot\frac{T}{4}\right)} = \frac{T^2}{4\pi^2 A}$$

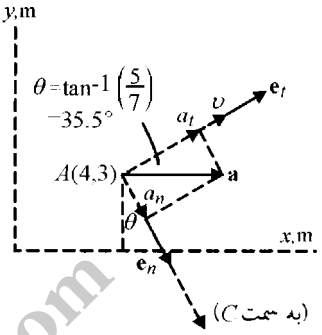
$$a_n = \frac{v^2}{\rho} \Rightarrow 0.7(9.81) = \frac{\left(\frac{80}{3.6}\right)^2}{\frac{T^2}{(4\pi^2 . 3)}} \Rightarrow T = 92.3\,m = 2L \Rightarrow L = 46.1\,m$$

۲-۱۳۰.

$$\begin{cases} x = 2t^2 + 3t - 1 \;,\; \dot{x} = 4t + 3 \;,\; \ddot{x} = 4 \\ y = 5t - 2 \;,\; \dot{y} = 5 \;,\; \ddot{y} = 0 \end{cases}$$

در $t = 1\ \text{s}$:

$$\begin{cases} x = 4\ \text{m} \;,\; \dot{x} = 7\ \text{m/s} \;,\; \ddot{x} = 4\ \text{m/s}^2 \\ y = 3\ \text{m} \;,\; \dot{y} = 5\ \text{m/s} \;,\; \ddot{y} = 0 \end{cases}$$



$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{7^2 + 5^2} = 8.60\ \text{m/s}$$

$$\mathbf{e}_t = \cos 33.5°\,\mathbf{i} + \sin 35.5°\,\mathbf{j} = 0.814\,\mathbf{i} + 0.581\,\mathbf{j}$$

$$\mathbf{e}_n \perp \mathbf{e}_t \Rightarrow \mathbf{e}_n = 0.581\,\mathbf{i} - 0.814\,\mathbf{j}$$

$$a_n = \mathbf{a} \cdot \mathbf{e}_n = 4(0.581) = 2.32\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{v^2}{a_n} = \frac{8.60^2}{2.32} = 31.8\ \text{m}$$

$$x_C = x_A + \rho \sin\theta = 4 + 31.8 \sin 35.5° = 22.5\ \text{m}$$

$$y_C = y_A - \rho \cos\theta = 3 - 31.8 \cos 35.5° = -22.9\ \text{m}$$

۲-۱۳۱.

$$v_r = \dot{r} = 40\ \text{mm/s}$$

$$v_\theta = r\dot{\theta} = 300(0.1) = 30\ \text{mm/s}$$

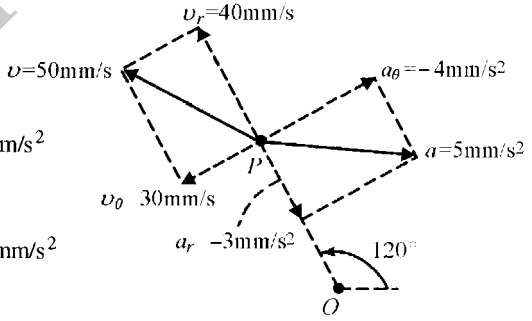
$$v = \sqrt{40^2 + 30^2} = 50\ \text{mm/s}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 300(0.1)^2 = -3\ \text{mm/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$= 300(-0.04) + 2(40)(0.1) = -4\ \text{mm/s}^2$$

$$a = \sqrt{3^2 + 4^2} = 5\ \text{mm/s}^2$$



۲-۱۳۲.

$$\mathbf{v} = \dot{r}\mathbf{e}_r . r\dot{\theta}\mathbf{e}_\theta = 0.5\,\mathbf{e}_r + (2+7)\left(5\frac{\pi}{180}\right)\mathbf{e}_\theta = 0.5\,\mathbf{e}_r + 0.785\,\mathbf{e}_\theta$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\,\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\mathbf{e}_\theta$$

$$= \left[-1.2 - 9\left(5\frac{\pi}{180}\right)^2\right]\mathbf{e}_r + \left[9\left(2\frac{\pi}{180}\right) + 2(0.5)\left(5\frac{\pi}{180}\right)\right]\mathbf{e}_\theta$$

$$= -1.26\,\mathbf{e}_r + 0.4\,\mathbf{e}_\theta\ \text{m/s}^2$$

۲-۱۳۳.

$$v_r = \dot{r} = 0.15\,\text{m/s}$$

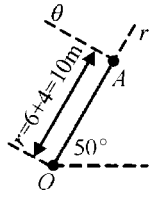
$$v_\theta = r\dot{\theta} = 10\left(2\frac{\pi}{180}\right) = 0.349\,\text{m/s}$$

$$v = \sqrt{0.15^2 + 0.349^2} = 0.38\,\text{m/s}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 10\left(2\frac{\pi}{180}\right)^2 = -0.0122\,\text{m/s}^2$$

$$a_\theta = r\ddot{\theta} - 2\dot{r}\dot{\theta} = 0 + 2(0.15)\left(2\frac{\pi}{180}\right) = 0.0105\,\text{m/s}^2$$

$$a = \sqrt{0.0122^2 + 0.0105^2} = 0.01607\,\text{m/s}^2$$



۲-۱۳۴.

موقعیت	r	$\dot{r}$	$\ddot{r}$	θ	$\dot{\theta}$	$\ddot{\theta}$
A	+	−	+	+	+	+
B	+	0	+	+	+	0
C	+	+	+	+	+	−

نکات: ۱- $r \geq 0$ همیشه، طبق تعریف

۲- $\dot{r}$ با بررسی شکل تعیین می‌شود.

۳- $\ddot{r}$ از رابطهٔ $a_r = \ddot{r} - r\dot{\theta}^2 = 0$ به دست می‌آید.

۴- $\theta \geq 0$ طبق تعریف و از روی شکل

۵- $\dot{\theta} > 0$ در اینجا با بررسی مشخص می‌شود.

۶- $\ddot{\theta}$ از رابطه $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$ به دست می‌آید.

۲-۱۳۵.

$$\mathbf{v}_A = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta = v\mathbf{e}_r + l\Omega\mathbf{e}_\theta$$

$$\mathbf{a}_A = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} - 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

$$= l\Omega^2\mathbf{e}_r + 2v\Omega\mathbf{e}_\theta$$

$$\mathbf{v}_B = 4v\mathbf{e}_r + 2l\Omega\mathbf{e}_\theta$$

$$\mathbf{a}_B = -2l\Omega^2\mathbf{e}_r + 8v\Omega\mathbf{e}_\theta$$

۲-۱۳۶.

$$r = \frac{t^3}{3}\ ,\quad \theta = 4\cos\frac{\pi t}{6}$$

در $t = 2\ \text{s}$:

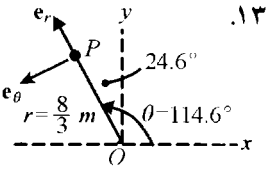
$$r = \frac{8}{3}\ \text{m}\ ,\quad \theta = 2\ \text{rad}\ (114.6°)$$

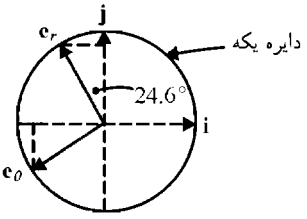
$$\mathbf{e}_r = \mathbf{j}\cos 24.6° - \mathbf{i}\sin 24.6°$$

$$= -0.416\,\mathbf{i} + 0.908\,\mathbf{j}$$

$$\mathbf{e}_\theta = -\mathbf{i}\cos 24.6° - \mathbf{j}\sin 24.6°$$

$$= -0.909\,\mathbf{i} - 0.416\,\mathbf{j}$$





۲-۱۳۷.

$$\mathbf{a} = [\ddot{r} - r\dot{\theta}^2]\,\mathbf{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\,\mathbf{e}_\theta$$

$$\ddot{r} = \ddot{\theta} = 0\ ,\qquad \dot{\theta} = \Omega\ ,\qquad r = l$$

$$a = \sqrt{(l\Omega)^2 + (2\dot{l}\Omega)^2} = \Omega\sqrt{l^2\Omega^2 + 4\dot{l}^2}$$

$$0.011 = 0.05\sqrt{[4.2(0.05)]^2 + 4\dot{l}^2} \Rightarrow \dot{l} = 0.0328\ \text{m/s} = 32.8\ \text{mm/s}$$

۲-۱۳۸.

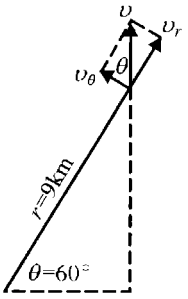
$$v_\theta = r\dot{\theta} = 9(10^5)(0.02) = 180\ \text{m/s}$$

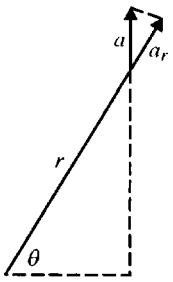
$$v = \frac{v_\theta}{\cos 60°} = \frac{180}{0.5} = 360\ \text{m/s}$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$= 21 - 9(10^3)(0.02)^2 = 17.4\ \text{m/s}^2$$

$$a = \frac{a_r}{\sin 60°} = \frac{17.5}{0.87} = 20.1\ \text{m/s}^2$$





۲-۱۳۹. شتاب در تمام جهات صفر است. بنابراین:

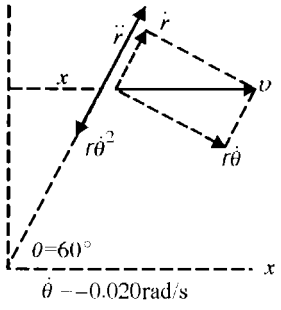
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 \Rightarrow \ddot{r} = r\dot{\theta}^2$$

$$r = \frac{h}{\sin\theta} = \frac{10}{\frac{\sqrt{3}}{2}} = 11.55\ \text{km}$$

$$\ddot{r} = 11.55\,(-0.020)^2 = 0.00462\ \text{km/s}^2 = 4.62\ \text{m/s}^2$$

$$v = \frac{|r\dot{\theta}|}{\sin\theta} = \frac{h\dot{\theta}}{\sin^2\theta} = \frac{|10(-0.020)|}{\left(\frac{\sqrt{3}}{2}\right)^2} = 0.267\ \text{km/s}$$

یا $v = 0.267(3600) = 960\ \text{km/h}$



۲-۱۴۰. $\dot{\mathbf{r}} \neq v \quad , \quad \dot{r} \neq \mathbf{v} \quad , \quad \ddot{\mathbf{r}} \neq a \quad , \quad \ddot{r} \neq \mathbf{a}$

زیرا کمیت‌های اسکالر هرگز با برداری برابر نیستند.

$v \neq (v_r = \dot{r})$ زیرا $\dot{r} \neq v$

$a \neq \ddot{r}$ زیرا $\ddot{r}$ فقط یکی از بخش‌های a_r (در نتیجه a) می‌باشد.

$\mathbf{v} = \dot{\mathbf{r}}$ زیرا $\dot{\mathbf{r}} \neq \dot{r}\mathbf{e}_r$ شامل یک مؤلفهٔ $\mathbf{e}_n$ نیز می‌باشد.

$\mathbf{a} = \ddot{\mathbf{r}}$ زیرا $\ddot{\mathbf{r}} \neq \ddot{r}\mathbf{e}_r$ شامل یک مؤلفهٔ دیگر $\mathbf{e}_r$ و نیز یک مؤلفهٔ $\mathbf{e}_\theta$ می‌باشد.

$\dot{\mathbf{r}} = \mathbf{v}$ زیرا $\dot{\mathbf{r}} \neq r\dot{\theta}\mathbf{e}_\theta$ شامل یک مؤلفهٔ $\mathbf{e}_r$ نیز می‌باشد.

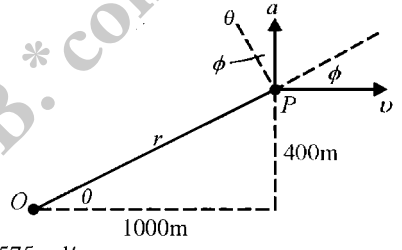
۲-۱۴۱.

$$r = \sqrt{1000^2 + 400^2} = 1077 \text{ m}$$

$$\theta = \tan^{-1}\frac{400}{1000} = 21.8°$$

$$v = \frac{600}{3.6} = 166.7 \text{ m/s}$$

$$a = a_n = \frac{v^2}{\rho} = \frac{166.7^2}{1200} = 23.1 \text{ m/s}^2$$



$$v_r = \dot{r} = v\cos\theta = 166.7\cos 21.8° = 154.7 \text{ m/s}$$

$$v_\theta = r\dot{\theta} = -166.7\sin 21.8° = 1077\dot{\theta} \Rightarrow \dot{\theta} = -0.0575 \text{ rad/s}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \Rightarrow 23.1\sin 21.8° = \ddot{r} - 1077(-0.0575)^2 \Rightarrow \ddot{r} = 12.15 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \Rightarrow 23.1\cos 21.8° = 1077\ddot{\theta} + 2(154.7)(-0.0575) \Rightarrow \ddot{\theta} = 0.0365 \text{ rad/s}$$

۲-۱۴۲.

$$\theta = 0.4 + 0.12t + 0.06t^3 \qquad r = 0.8 - 0.1t - 0.05t^2$$

$$\dot{\theta} = 0.12 + 0.18t^2 \qquad \dot{r} = -0.1 - 0.1t$$

$$\ddot{\theta} = 0.36t \qquad \ddot{r} = -0.1$$

در $t = 2$ s :

$$\begin{cases} \theta = 1.12 \text{ rad} \\ \dot{\theta} = 0.84 \text{ rad/s} \\ \ddot{\theta} = 0.72 \text{ rad/s}^2 \end{cases} \qquad \begin{cases} r = 0.4 \text{ m} \\ \dot{r} = -0.3 \text{ m/s} \\ \ddot{r} = -0.1 \text{ m/s}^2 \end{cases}$$

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta = -0.3\mathbf{e}_r + 0.4(0.84)\mathbf{e}_\theta = -0.3\mathbf{e}_r + 0.336\mathbf{e}_\theta \text{ m/s}$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta = [-0.1 - 0.4(0.84)^2]\mathbf{e}_r + [0.4(0.72) + 2(-0.3)(0.84)]\mathbf{e}_\theta$$

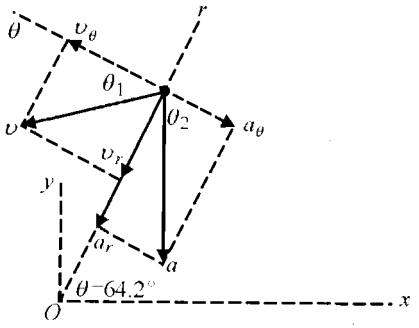
$$= -0.382\mathbf{e}_r - 0.216\mathbf{e}_\theta \text{ m/s}^2$$

$$\theta_1 = \tan^{-1}\frac{|v_r|}{|v_\theta|} = 41.8°$$

$$\alpha = 90° + \theta + \theta_1 = 195.9°$$

$$\theta_2 = \tan^{-1}\frac{|a_r|}{|a_\theta|} = 60.5°$$

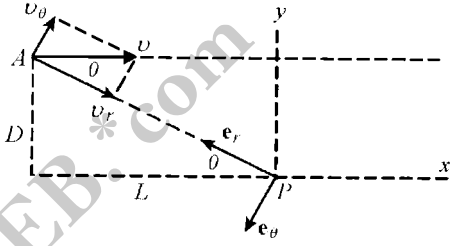
$$\beta = -90° + \theta - \theta_2 = -86.4°$$



۲-۱۴۳.

$$v' = |v_r| = v\cos\theta = v\frac{L}{\sqrt{L^2 + D^2}}$$

$$v' = 115\frac{150}{\sqrt{150^2 + 6^2}} = 114.91 \text{ km/h}$$



ضریب $\cos\theta$ مبنای بیان این جمله است که از نظر سینماتیک رادار می‌تواند سرعتی برابر یا کمتر را اندازه بگیرد نه بیشتر. همانطور که ملاحظه می‌شود با این وجود اتکا به حد سرعت (بدون توجه به $\cos\theta$) بهترین راهکار می‌باشد!

۲-۱۴۴.

$$r = r_0 \cosh Kt$$

$$\dot{r} = r_0 K \sinh Kt$$

$$\ddot{r} = r_0 K^2 \cosh Kt$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = r_0 K^3 \cosh Kt - (r_0 \cosh Kt) K^2 = 0$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2 r_0 K \sinh Kt\,(K) = 2 r_0 K^2 \sinh Kt$$

از طرفی $\cosh^2 Kt - \sinh^2 Kt = 1 \Rightarrow \sinh Kt = \sqrt{\cosh Kt - 1} = \sqrt{\left(\frac{r}{r_0}\right)^2 - 1}$

وقتی $r = R$: $\sinh Kt = \sqrt{\left(\frac{R}{r_0}\right)^2 - 1}$

بنابراین در لحظهٔ ترک پروانه:

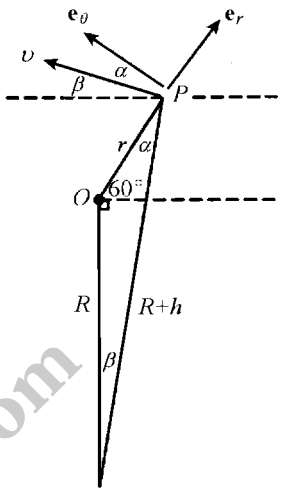
$$a = a_\theta = 2 r_0 K^2 \sqrt{\left(\frac{R}{r_0}\right)^2 - 1} = 2K^2\sqrt{R^2 - r_0^2}$$

۲-۱۴۵.

$$\frac{\sin 150^\circ}{R+h}=\frac{\sin\alpha}{R}\Rightarrow\frac{\sin 150^\circ}{6371+240}=\frac{\sin\alpha}{6371}\Rightarrow\alpha=28.8^\circ$$

$$\alpha+\beta+150^\circ=180^\circ\Rightarrow\beta=1.200^\circ$$

$$v_r=\dot r=-3742\ \text{m/s}=-v\sin\alpha\Rightarrow v=\frac{3742}{\sin 28.8^\circ}=7767\ \text{m/s}$$



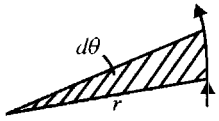
از آنجا که v تقریباً در نقطهٔ O موازی افق است ($\beta=1.200^\circ$)، می‌توان با صرف‌نظر از β (فرض یک زمین تخت) به جواب تقریباً نزدیکی دست یافت ($v=7767$ m/s)

۲-۱۴۶.

$$dA=\frac{1}{2}\,r\,d\theta\,(r)=\frac{1}{2}r^2\,d\theta$$

$$\dot A=\frac{dA}{dt}=\frac{1}{2}r^2\frac{d\theta}{dt}=\frac{1}{2}r^2\dot\theta=\text{ثابت}$$

$$a_\theta=\frac{1}{r}\frac{d}{dt}(r^2\dot\theta)=0$$



۲-۱۴۷.

$$r=b-c\cos\theta\Rightarrow\dot r=c\dot\theta\sin\theta\ ,\quad \ddot r=c\dot\theta^2\cos\theta$$

$$a_r=\ddot r-r\dot\theta^2=c\dot\theta^2\cos\theta-(b-c\cos\theta)\dot\theta^2=(2c\cos\theta-b)\dot\theta^2$$

$$a_\theta=r\ddot\theta+2\dot r\dot\theta=0+2c\dot\theta^2\sin\theta$$

$$a=\sqrt{a_r^2+a_\theta^2}=\omega^2\sqrt{4c^2-4bc\cos\theta+b^2}\quad(\omega=\dot\theta\ \text{که درآن})$$

۲-۱۴۸.

$$v=\frac{90}{3.6}=25\ \text{m/s}\ ,\quad a=0.5\ \text{m/s}^2$$

$$v_r=\dot r=v\sin 45^\circ=25\frac{\sqrt2}{2}=17.68\ \text{m/s}$$

$$v_\theta=r\dot\theta=-25\cos 45^\circ=400\,\dot\theta$$

$$\Rightarrow\dot\theta=-0.0442\ \text{rad/s}$$

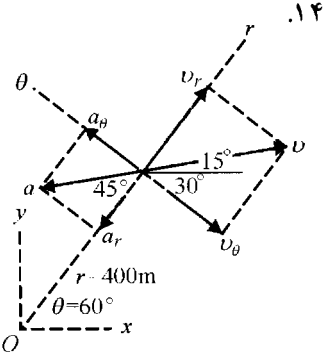
$$a_r=\ddot r-r\dot\theta^2$$

$$\Rightarrow-0.5\cos 45^\circ=\ddot r-400(-0.0442)^2$$

$$\Rightarrow\ddot r=0.428\ \text{m/s}^2$$

$$a_\theta=r\ddot\theta+2\dot r\dot\theta$$

$$\Rightarrow 0.5\sin 45^\circ=400\ddot\theta+2(17.68)(-0.0442)\Rightarrow\ddot\theta=0.00479\ \text{rad/s}^2$$



۲-۱۴۹.

$r = 0.75 + 0.5 = 1.25\,\text{m}$, $\dot{r} = 0.2\,\text{m/s}$, $\ddot{r} = -0.3\,\text{m/s}^2$

$\theta = 30°$, $\dot{\theta} = 0.1745\,\text{rad/s}$, $\ddot{\theta} = 0$

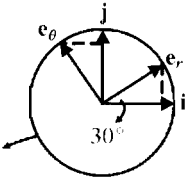
$\mathbf{v} = v_r\mathbf{e}_r + v_\theta\mathbf{e}_\theta = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta = 0.2\,\mathbf{e}_r + 1.25(0.1745)\,\mathbf{e}_\theta = 0.2\,\mathbf{e}_r + 0.218\,\mathbf{e}_\theta\,\text{m/s}$

$v = \sqrt{v_r^2 + v_\theta^2} = 0.296\,\text{m/s}$

$\mathbf{a} = a_r\mathbf{e}_r + a_\theta\mathbf{e}_\theta = (\ddot{r} - r\dot{\theta}^2)\,\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\mathbf{e}_\theta$

$= [-0.3 - 1.25(0.1745)^2]\,\mathbf{e}_r + [1.25(0) + 2(0.2)(0.1745)]\,\mathbf{e}_\theta$

$= -0.338\,\mathbf{e}_r + 0.0698\,\mathbf{e}_\theta$



$a = \sqrt{a_r^2 + a_\theta^2} = 0.345\,\text{m/s}^2$

$\mathbf{e}_r = \mathbf{i}\cos 30° + \mathbf{j}\sin 30°$

$\mathbf{e}_\theta = -\mathbf{i}\sin 30° + \mathbf{j}\cos 30°$

$\mathbf{v} = 0.2[\mathbf{i}\cos 30° + \mathbf{j}\sin 30°] + 0.218[-\mathbf{i}\sin 30° + \mathbf{j}\cos 30°] = 0.064\,\mathbf{i} + 0.289\,\mathbf{j}\,\text{m/s}$

$\mathbf{a} = -0.338[\mathbf{i}\cos 30° + \mathbf{j}\sin 30°] + 0.0698[-\mathbf{i}\sin 30° + \mathbf{j}\cos 30°] = -0.328\,\mathbf{i} - 0.1086\,\mathbf{j}\,\text{m/s}^2$

۲-۱۵۰.

$h = x\tan\theta$

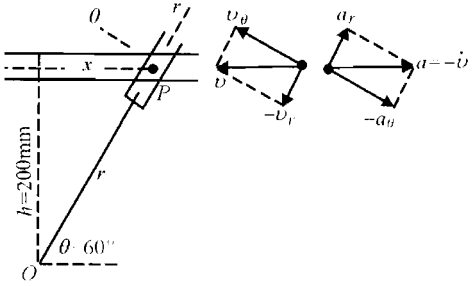
$0 = \dot{x}\tan\theta + x\dot{\theta}\sec^2\theta$

$= \dot{x}\tan\theta + (h\,\text{ctn}\,\theta)\,\dot{\theta}\sec^2\theta$

$v = -\dot{x} = h\dot{\theta}\,\text{ctn}\,\theta\sec^2\theta \,/\, \tan\theta$

$= h\dot{\theta}\csc^2\theta = 200(2)\left(\frac{2}{\sqrt{3}}\right)^2 = 533\,\text{mm/s}$

$v_r = -v\cos\theta = -533\left(\frac{1}{2}\right) = -267\,\text{mm/s}$



$-a = \dot{v} = h\dot{\theta}^2\csc\theta\,(-\,\text{ctn}\,\theta\csc\theta)\,\dot{\theta} = -2h\dot{\theta}^2\,\text{ctn}\,\theta\csc^2\theta$

$a = 2(200)2^2\left(\frac{1}{\sqrt{3}}\right)\left(\frac{2}{\sqrt{3}}\right)^2 = 1232\,\text{mm/s}^2$

$a_r = a\cos\theta = 1232\left(\frac{1}{2}\right) = 616\,\text{mm/s}^2$

به همین ترتیب مقادیر $v_r = \dot{r}$ و $\ddot{r}$ از $r = h\csc\theta$ به دست می‌آیند.

۲-۱۵۱.

$\dot{\theta} = 2.20\left(\frac{\pi}{180}\right) = 0.0384\,\text{rad/s}$

$v_r = \dot{r} = 110\,\text{m/s}$

$v_\theta = r\dot{\theta} = 3600(0.0384) = 138.24$ m/s

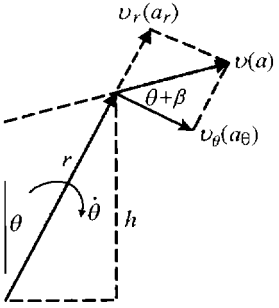
$v = \sqrt{v_r^2 + v_\theta^2} = 176.66$ m/s

$30 + \beta = \tan^{-1}\dfrac{110}{138.24} = 38.5° \Rightarrow \beta = 8.5°$

$h = r\cos 30° = 3600\cos 30° = 3118$ m

$a_r = \ddot{r} - r\dot{\theta}^2 = 6 - 3600(0.0384)^2 = 0.691$ m/s^2

$a = \dfrac{a_r}{\sin(\theta+\beta)} = \dfrac{0.691}{\sin 38.5°} = 1.11$ m/s^2



$a_\theta = \dfrac{a_r}{\tan(\theta+\beta)} = r\ddot{\theta} + 2\dot{r}\dot{\theta} \Rightarrow \dfrac{0.691}{\tan 38.5°} = 3600\ddot{\theta} + 2(110)(0.0384) \Rightarrow \ddot{\theta} = -2.10(10^{-3})$ rad/s^2

۲-۱۵۲.

$r = \overline{OA} = 300$ mm

$v_r = \dot{r} = v\cos 30° = 2\cos 30° = 1.732$ m/s

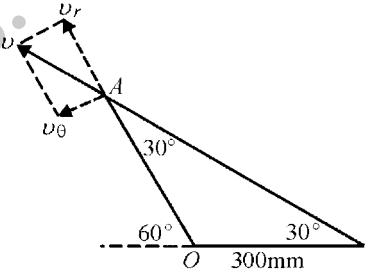
$v_\theta = r\dot{\theta} \Rightarrow \dot{\theta} = \dfrac{v_\theta}{r} = \dfrac{v\sin 30°}{r} = \dfrac{2(0.5)}{0.3} = 3.33$ rad/s

$a = 0 \Rightarrow a_r = \ddot{r} - r\dot{\theta}^2 = 0$

$\Rightarrow \ddot{r} = r\dot{\theta}^2 = 0.300(3.33)^2 = 3.33$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$\Rightarrow \ddot{\theta} = -\dfrac{2\dot{r}\dot{\theta}}{r} = -\dfrac{2(1.732)(3.33)}{0.300} = -38.5$ rad/s^2



۲-۱۵۳.

$x = R + s\cos\alpha = R + \left(s_0 + v_0 t + \dfrac{1}{2}at^2\right)\cos\alpha$

$= R + \dfrac{1}{2}at^2\cos\alpha$

$y = s\sin\alpha = \dfrac{1}{2}at^2\sin\alpha$

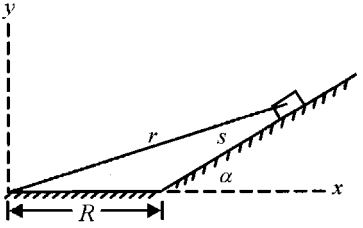
$r = \sqrt{x^2 + y^2}$

$= \sqrt{\left(R + \dfrac{1}{2}at^2\cos\alpha\right)^2 + \left(\dfrac{1}{2}at^2\sin\alpha\right)^2}$

$= \sqrt{R^2 + Rat^2\cos\alpha + \dfrac{1}{4}a^2t^4}$

$\dot{r} = \dfrac{1}{2}\left(R^2 + Rat^2\cos\alpha + \dfrac{1}{4}a^2t^4\right)^{-\frac{1}{2}}[2Rat\cos\alpha + a^2t^3]$

$= \dfrac{\frac{1}{2}at(2R\cos\alpha + at^2)}{\sqrt{R^2 + Rat^2\cos\alpha + \frac{1}{4}a^2t^4}}$



۲-۱۵۴.

$$r = 1.6 + 0.3\sin\frac{\pi t}{2}\ \text{m} \qquad \theta = \frac{\pi}{4} + \frac{\pi}{8}\sin\frac{\pi t}{2}$$

$$\dot{r} = \frac{0.3\pi}{2}\cos\frac{\pi t}{2}\ \text{m/s} \qquad \dot{\theta} = \frac{\pi^2}{16}\cos\frac{\pi t}{2}\ \text{rad/s}$$

$$\ddot{r} = -\frac{0.3\pi^2}{4}\sin\frac{\pi t}{2}\ \text{m/s}^2 \qquad \ddot{\theta} = -\frac{\pi^3}{32}\sin\frac{\pi t}{2}\ \text{rad/s}^2$$

$$v_r = \dot{r} = \frac{0.3\pi}{2}\cos\frac{\pi t}{2}$$

$$v_\theta = r\dot{\theta} = \left(1.6 + 0.3\sin\frac{\pi t}{2}\right)\left(\frac{\pi^2}{16}\cos\frac{\pi t}{6}\right)$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{0.3\pi^2}{4}\sin\frac{\pi t}{2} - \left(1.6 + 0.3\sin\frac{\pi t}{6}\right)\left(\frac{\pi^2}{16}\cos\frac{\pi t}{2}\right)^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \left(1.6 + 0.3\sin\frac{\pi t}{2}\right)\left(\frac{-\pi^3}{32}\sin\frac{\pi t}{2}\right) + 2\left(\frac{0.3\pi}{2}\cos\frac{\pi t}{2}\right)\left(\frac{\pi^2}{16}\cos\frac{\pi t}{2}\right)$$

در $t = 1\ \text{s}$:

$$\left.\begin{aligned} v_r &= 0 \\ v_\theta &= 0 \end{aligned}\right\} \quad v = \sqrt{v_r^2 + v_\theta^2} = 0$$

$$\left.\begin{aligned} a_r &= -0.740\ \text{m/s}^2 \\ a_\theta &= -1.841\ \text{m/s}^2 \end{aligned}\right\} \quad a = \sqrt{0.740^2 + 1.841^2} = 1.984\ \text{m/s}^2$$

در $t = 2\ \text{s}$:

$$\left.\begin{aligned} v_r &= -0.471\ \text{m/s} \\ v_\theta &= 0.987\ \text{m/s} \end{aligned}\right\} \quad v = 1.094\ \text{m/s}$$

$$\left.\begin{aligned} a_r &= -0.609\ \text{m/s}^2 \\ a_\theta &= 0.581\ \text{m/s}^2 \end{aligned}\right\} \quad a = 0.842\ \text{m/s}^2$$

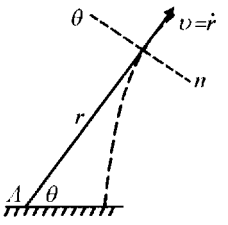
۲-۱۵۵. خط شعاعی r باید برای $\dot{\theta} = 0$ مماس بر مسیر باشد. بنابراین جهت θ + در قسمت مثبت جهت نرمال منحنی قرار دارد.

$$r = 10.5\ \text{km} \quad , \quad \dot{r} = 480\ \text{m/s}$$

$$\dot{\theta} = 0 \quad , \quad \ddot{\theta} = -0.00720\ \text{rad/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 10500(-0.00720) + 2(480)(0) = -75.6\ \text{m/s}^2$$

$$-a_\theta = a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{v^2}{-a_\theta} = \frac{480^2}{75.6} = 3048\ \text{m}$$



۲-۱۵۶.

$$r = \overline{BD} = 2R\sin\frac{\theta}{2} \quad , \quad \dot{r} = R\dot{\theta}\cos\frac{\theta}{2} \quad , \quad \ddot{r} = -\frac{R}{2}\dot{\theta}^2\sin\frac{\theta}{2}$$

برای $\theta = 30°$:

$$\begin{cases} r = 2(375)\sin 15° = 194.1\ \text{mm} \\ \dot{r} = 375(4)\cos 15° = 1448.9\ \text{mm/s} \\ \ddot{r} = -\dfrac{375}{2}(4)^2\sin 15° = -776.5\ \text{mm/s}^2 \end{cases}$$

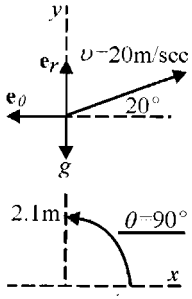
$$a_r = \ddot{r} - r\dot{\theta}^2 = -776.5 - 194.1(4)^2 = 3882.1\ \text{mm/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1448.9)(4) = 11591.2\ \text{mm/s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = 12224\ \text{mm/s}^2 = 12.22\ \text{m/s}^2$$

۲-۱۵۷.

$r = 2.1\ \text{m}$



$$\dot{r} = v_r = 20\sin 20° = 6.84\ \text{m/s}$$

$$v_\theta = r\dot{\theta} = -20\cos 20° = -18.79\ \text{m/s}$$

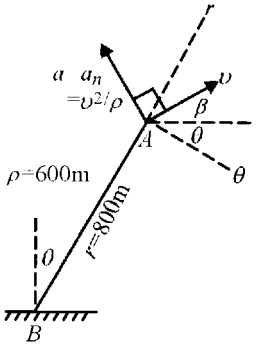
$$\dot{\theta} = \frac{v_\theta}{r} = \frac{-18.79}{2.1} = -8.95\ \text{rad/s}$$

$$a_r = -g = \ddot{r} - r\dot{\theta}^2 \Rightarrow -9.81 = \ddot{r} - 2.1(-8.95)^2 \Rightarrow \ddot{r} = 158.4\ \text{m/s}^2$$

$$a_\theta = 0 = r\ddot{\theta} + 2\dot{r}\dot{\theta} \Rightarrow 2.1\ddot{\theta} + 2(6.84)(-8.95) = 0 \Rightarrow \ddot{\theta} = 58.3\ \text{rad/s}^2$$

۲-۱۵۸.

$$\theta = \beta = 30° \quad , \quad v = \frac{400}{3.6} = 111.1\ \text{m/s}$$



$$v_r = v\sin(\beta + \theta) = 111.1\sin 60° = 96.2\ \text{m/s}$$

$$v_\theta = r\dot{\theta} = v\cos(\beta + \theta) = 111.1\cos 60° = 55.6\ \text{m/s}$$

$$\dot{r} = v_r = 96.2\ \text{m/s}$$

$$\dot{\theta} = \frac{v_\theta}{r} = \frac{55.6}{800} = 0.0694\ \text{rad/s}$$

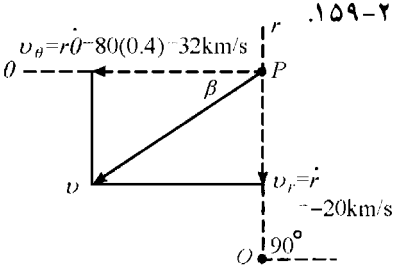
$$a_r = \ddot{r} - r\dot{\theta}^2 = a_n\cos 60° = \frac{(111.1)^2}{600}\cos 60 = 10.29\ \text{m/s}^2$$

$$a_\theta = r\ddot{\theta} = 2\dot{r}\dot{\theta} = -a_n\sin 60° \Rightarrow \ddot{\theta} = -\frac{(111.1)^2}{600}\frac{1}{800}\frac{\sqrt{3}}{2} - \frac{2(96.2)(0.0694)}{800} = -0.0390\ \text{rad/s}^2$$

۲-۱۵۹.

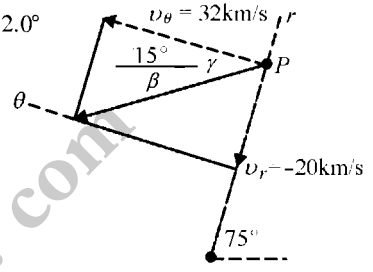
الف) $v = \sqrt{v_\theta^2 + v_r^2} = \sqrt{32^2 + 20^2} = 37.7\,\text{km/s}$

$\beta = \tan^{-1}\dfrac{|v_r|}{v_\theta} = \tan^{-1}\dfrac{20}{32} = 32.0°$



ب) v_r و v_θ تغییری نمی‌کنند. $\Rightarrow v = 37.7\,\text{km/s}$, $\gamma = 32.0°$

$\beta = \gamma - 15° = 32.0 - 15° = 17.01°$



۲-۱۶۰.

$r = \sqrt{200^2 + 100^2} = 224\,\text{m}$

$\theta = \tan^{-1}\dfrac{100}{200} = 26.6°$

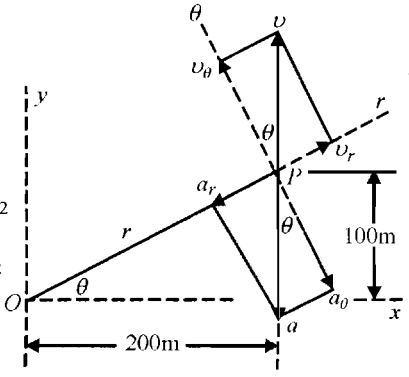
$v_r = \dot{r} = v\sin\theta = 15\sin 26.6° = 6.71\,\text{m/s}$

$v_\theta = r\dot{\theta} = 15\cos 26.6° = 224\,\dot{\theta} \Rightarrow \dot{\theta} = 0.06\,\text{rad/s}$

$a = -g - kv^2 = -9.81 - 0.01(15)^2 = -12.06\,\text{m/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 \Rightarrow -12.06\sin 26.6° = \ddot{r} - 224(0.06)^2$

$\Rightarrow \ddot{r} = -4.59\,\text{m/s}^2$



$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \Rightarrow -12.06\cos 26.6° = 224\,\ddot{\theta} + 2(6.71)(0.06)$

$\Rightarrow \ddot{\theta} = -0.0518\,\text{rad/s}^2$

۲-۱۶۱.

$r = 80 + 12\cos 4\theta\,\text{mm}$

$\dot{r} = -48\,\dot{\theta}\sin 4\theta\,\text{mm/s}$ $(\dot{\theta} = 4\pi\,\text{rad/s})$

$\ddot{r} = -192\,\dot{\theta}^2\cos 4\theta\,\text{mm/s}^2$ $(\ddot{\theta} = 0)$

$a_r = \ddot{r} - r\dot{\theta}^2$, $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2\dot{r}\dot{\theta}$

الف) $\theta = 0$: $a_r = -192(4\pi)^2\,1 - 92(4\pi)^2 = -44.8\ \text{m/s}^2$

$a_\theta = 2(0) = 0$

$\mathbf{a} = -44.8\,\mathbf{e}_r\ \text{m/s}^2$

ب) $\theta = \frac{\pi}{8}$: $a_r = -192(4\pi)^2\cos\frac{\pi}{2} - \left(80 + 12\cos\frac{\pi}{2}\right)(4\pi)^2 = -12.63\ \text{m/s}^2$

$a_\theta = 0 + 2\left(-48.4\pi\,.\,\sin\frac{\pi}{2}\right)(4\pi) = -15.16\ \text{m/s}^2$

$\mathbf{a} = -12.63\,\mathbf{e}_r - 15.16\,\mathbf{e}_\theta$

ج) $\theta = \frac{\pi}{4}$: $a_r = -192(4\pi)^2\cos\pi - (80 + 12\cos\pi)(4\pi)^2 = 19.58\ \text{m/s}^2$

$a_\theta = 0 + 2(-48.4\pi\,.\,\sin\pi)(4\pi) = 0$

$\mathbf{a} = +19.58\,\mathbf{e}_r\ \text{m/s}^2$

۲-۱۶۲.

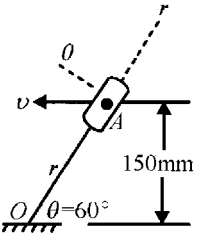
$\dot{r} = v_r = -v\cos\theta = -1.5\cos 60° = -0.75\ \text{m/s}$

$r\dot{\theta} = v\sin\theta = 1.5\sin 60° \Rightarrow \dot{\theta} = \dfrac{1.5\sin 60°}{\dfrac{0.15}{\sin 60°}} = 7.5\ \text{rad/s}$

$a = 0 \Rightarrow a_r = \ddot{r} - r\dot{\theta}^2 = 0$

$\Rightarrow \ddot{r} = r\dot{\theta}^2 = \dfrac{0.5}{\sin 60°}(7.5)^2 = 9.74\ \text{mm/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \Rightarrow \ddot{\theta} = \dfrac{-2\dot{r}\dot{\theta}}{r} = \dfrac{-2(-0.75)(7.5)}{\dfrac{0.15}{\sin 60°}} = 65\ \text{rad/s}^2$



۲-۱۶۳.

$v = \dfrac{17970}{3.6} = 4991.7\ \text{m/s} \qquad \beta = \tan^{-1}\dfrac{8000}{13800} = 30°$

$v_\theta = r\dot{\theta} \Rightarrow 4991.7\cos 30° = 15951(10^3)\,\dot{\theta}$

$\Rightarrow \dot{\theta} = 2.71(10^{-4})\ \text{rad/s}$

$v_r = \dot{r} \Rightarrow 4991.7\sin 30° = \dot{r} \Rightarrow \dot{r} = 2495.8\ \text{m/s}$

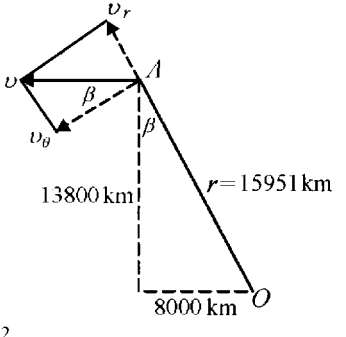
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$\Rightarrow 0 = 15951(10^3)\,\ddot{\theta} + 2(2495.8)(2.71)(10^{-4})$

$\Rightarrow \ddot{\theta} = -8.48(10^{-8})\ \text{rad/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 \Rightarrow -1.556 = \ddot{r} - 15951(10^3)(2.71\times 10^{-4})^2$

$\Rightarrow \ddot{r} = -0.388\ \text{m/s}^2$



۲-۱۶۴.

$x = x_0 + v_{x_0} t = 0 + 30\cos 30^\circ\,(0.5) = 13\ \text{m}$

$\dot{x} = v_{x_0} = 30\cos 30^\circ = 26\ \text{m/s}$

$y = y_0 + v_{y_0} - \frac{1}{2} g t^2 = 2 + 30\sin 30^\circ (0.5) - \frac{1}{2}(9.81)(0.5)^2 = 8.27\ \text{m}$

$\dot{y} = v_{y_0} - gt = 30\sin 30^\circ - 9.81(0.5) = 10.1\ \text{m/s}$

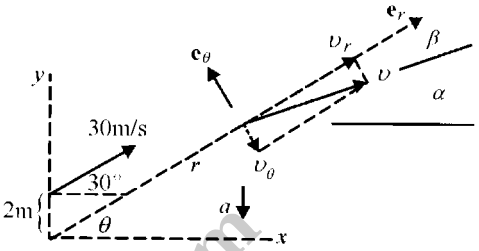
$r = \sqrt{x^2 + y^2} = 15.4\ \text{m}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{8.27}{13}\right) = 32.5^\circ$

$\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{10.1}{26}\right) = 21.2^\circ$

$\beta = \theta - \alpha = 11.3^\circ$

$v = \sqrt{\dot{x}^2 + \dot{y}^2} = 27.9\ \text{m/s}$



$v_r = v\cos\beta = 27.9\cos 11.3^\circ = 27.36\ \text{m/s} = \dot{r}$

$v_\theta = -v\sin\theta = -27.9\sin 11.3^\circ = -5.47\ \text{m/s}$

$v_\theta = r\dot{\theta} \Rightarrow -5.47 = 15.4\,\dot{\theta} \Rightarrow \dot{\theta} = -0.355\ \text{rad/s}$

$a = \ddot{y} = -9.81\ \text{m/s}^2$

$a_r = -g\sin\theta = -9.81\sin 32.5^\circ = -5.27\ \text{m/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 \Rightarrow -5.27 = \ddot{r} - 15.4(-0.355)^2 \Rightarrow \ddot{r} = -3.32\ \text{m/s}^2$

$a_\theta = -g\cos\theta = -9.81\cos 32.5^\circ = -8.27\ \text{m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \Rightarrow -8.27 = 15.4\,\ddot{\theta} + 2(27.36)(-0.355) \Rightarrow \ddot{\theta} = 0.724\ \text{rad/s}^2$

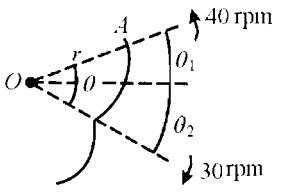
۲-۱۶۵.

$r = 100 - 75\cos\theta \quad , \quad \dot{r} = 75\dot{\theta}\sin\theta \quad , \quad \ddot{r} = 75\dot{\theta}^2\cos\theta \quad , \quad \ddot{\theta} = 0$

$\dot{\theta}_1 = \frac{40(2\pi)}{60} = \frac{4}{3}\pi\ \text{rad/s} \quad , \quad \dot{\theta}_2 = \frac{30(2\pi)}{60} = \pi\ \text{rad/s}$

$\theta = \theta_1 + \theta_2 \quad , \quad \dot{\theta} = \dot{\theta}_1 + \dot{\theta}_2 = \frac{7\pi}{3}\ \text{rad/s}$

$a_r = \ddot{r} - r\dot{\theta}_1^2 \quad , \quad a_\theta = r\ddot{\theta}_1 + 2\dot{r}\dot{\theta}_1$



$\theta = 30^\circ$ برای : $r = 100 - 75\frac{\sqrt{3}}{2} = 35.0\ \text{mm}$

$\dot{r} = 75\,\frac{7\pi}{3}\,\frac{1}{2} = 275\ \text{mm/s}$

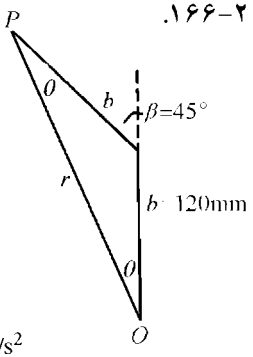
$\ddot{r} = 75\left(\frac{7\pi}{3}\right)^2\frac{\sqrt{3}}{2} = 3490\ \text{mm/s}^2$

$a_r = 3490 - 35.0\left(\frac{4\pi}{3}\right)^2 = 2875\ \text{mm/s}^2$

$a_\theta = 35.0(0) + 2(275)\left(\frac{4\pi}{3}\right) = 2303\ \text{mm/s}^2$

$a = \sqrt{a_r^2 + a_\theta^2} = 3680\ \text{mm/s}^2$ یا $3.68\ \text{m/s}^2$

۲-۱۶۶.



$2\theta = \beta = 45^\circ \Rightarrow \theta = 22.5^\circ$

$\theta = \frac{\beta}{2} \Rightarrow \dot{\theta} = \frac{\dot{\beta}}{2} = \frac{4}{2} = 2 \text{ rad/s}$

$\ddot{\theta} = \frac{\ddot{\beta}}{2} = 0$

$r = 2b\cos\theta = 2(120)\cos 22.5^\circ = 222 \text{ mm}$

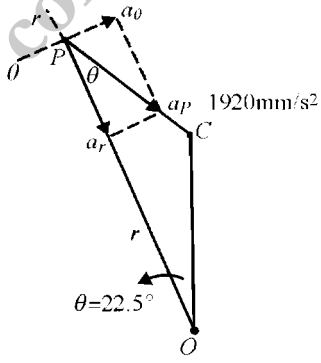
$\dot{r} = -2b\dot{\theta}\sin\theta = -2(120)(2)\sin 22.5^\circ = -183.7 \text{ mm/s}$

$\ddot{r} = -2b\dot{\theta}^2\cos\theta = -2(120)(2)^2\cos 22.5^\circ = -887 \text{ mm/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -887 - 222(2)^2 = -1774 \text{ mm/s}^2$ یا -1.774 m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 222(0) + 2(-183.7)(2) = -735 \text{ mm/s}^2$ یا -0.735 m/s^2

راه‌حل دوم :



$(a_P)_t = b\ddot{\beta} = 0$

$(a_P)_n = b\dot{\beta}^2 = 120(4)^2 = 1920 \text{ mm/s}^2$

$a_r = -1920\cos 22.5^\circ = -1774 \text{ mm/s}^2$

$a_\theta = -1920\sin 22.5^\circ = -735 \text{ mm/s}^2$

۲-۱۶۷.

$\mathbf{v} = 4\mathbf{i} - 2\mathbf{j} - \mathbf{k} \text{ m/s}$

$v = \sqrt{4^2 + 2^2 + 1^2} = 4.58 \text{ m/s}$

$a_n = a\sin 20^\circ = 8\sin 20^\circ = 2.74 \text{ m/s}^2$

$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{v^2}{a_n} = \frac{4.58^2}{2.74} = 7.67 \text{ m}$

$\dot{v} = a_t = a\cos 20^\circ = 8\cos 20^\circ = 7.52 \text{ m/s}^2$

۲-۱۶۸.

$v_{z_0} = 150\sin 60^\circ = 129.9 \text{ m/s}$

$v_{xy_0} = 150\cos 60^\circ = 75 \text{ m/s}$

$v_z = v_{z_0} - gt = 129.9 - 9.81(20) = -66.3 \text{ m/s}$

$v_{xy} = v_{xy_0} = 75 \text{ m/s}$ ثابت

$v_x = v_{xy}\cos 20^\circ = 75\cos 20^\circ = 70.5 \text{ m/s}$

$$v_y = v_{xy}\sin 20° = 75\sin 20° = 25.65\ \text{m/s}$$

$$d_{xy} = v_{xy}t = 75(20) = 1500\ \text{m}$$

$$x = d_{xy}\cos 20° = 1500\cos 20° = 1409.5\ \text{m}$$

$$y = d_{xy}\sin 20° = 1500\sin 20° = 513\ \text{m}$$

$$z = v_{z_0}t - \frac{1}{2}gt^2 = 129.9(20) - \frac{1}{2}9.81(20)^2 = 636\ \text{m}$$

$$a_x = a_y = 0 \quad , \quad a_z = -g = -9.81\ \text{m/s}^2$$

۲-۱۶۹.

$$\begin{cases} x = 30\cos 2t & , \quad y = 40\sin 2t & , \quad z = 20t + 3t^2 \\ \dot{x} = -60\sin 2t & , \quad \dot{y} = 80\cos 2t & , \quad \dot{z} = 20 + 6t \\ \ddot{x} = -120\cos 2t & , \quad \ddot{y} = -160\sin 2t & , \quad \ddot{z} = 6 \end{cases}$$

در $t = 2\ \text{s}$:

$$\begin{cases} x = -19.61\ \text{mm} & , \quad y = -30.3\ \text{mm} & , \quad z = 52\ \text{mm} \\ \dot{x} = 45.4\ \text{mm/s} & , \quad \dot{y} = -52.3\ \text{mm/s} & , \quad \dot{z} = 32\ \text{mm/s} \\ \ddot{x} = 78.4\ \text{mm/s}^2 & , \quad \ddot{y} = 121.1\ \text{mm/s}^2 & , \quad \ddot{z} = 6\ \text{mm/s}^2 \end{cases}$$

$$r = \sqrt{x^2 + y^2 + z^2} = 63.3\ \text{mm}$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} = 76.3\ \text{mm/s}$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2} = 144.4\ \text{mm/s}^2$$

$$\theta_1 = \cos^{-1}\left[\frac{\mathbf{r}\cdot\mathbf{v}}{rv}\right] = \cos^{-1}\left[\frac{-19.61(45.4) - 30.3(-52.3) + 52(32)}{(63.3)(76.3)}\right] = 60.8°$$

$$\theta_2 = \cos^{-1}\left[\frac{\mathbf{r}\cdot\mathbf{a}}{ra}\right] = \cos^{-1}\left[\frac{-19.61(78.4) - 30.3(121.1) + 52(6)}{(63.3)(144.4)}\right] = 122.4°$$

۲-۱۷۰.

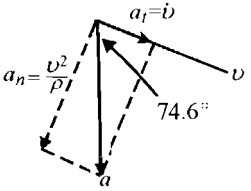
$$\mathbf{v} = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}\ \text{m/s} \Rightarrow v = \sqrt{6^2 + 3^2 + 2^2} = 7\ \text{m/s}$$

$$\mathbf{a} = 3\mathbf{i} - \mathbf{j} - 5\mathbf{k}\ \text{m/s}^2 \Rightarrow a = \sqrt{3^2 + 1^2 + 5^2} = \sqrt{35}\ \text{m/s}^2$$

$$\theta = \cos^{-1}\left[\frac{\mathbf{v}\cdot\mathbf{a}}{va}\right] = \cos^{-1}\left[\frac{6(3) - 3(-1) - 2(5)}{7\sqrt{35}}\right] = 74.6°$$

$$\dot{v} = a\cos 74.6° = 1.571\ \text{m/s}^2$$

$$a_n = a\sin 74.6° = 5.70\ \text{m/s}^2 = \frac{v^2}{\rho} \Rightarrow \rho = \frac{7^2}{5.70} = 8.59\ \text{m}$$



۲-۱۷۱.

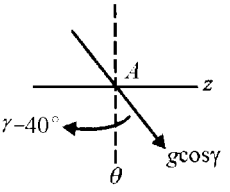
$$\left.\begin{aligned} v_\theta &= r\dot\theta \\ v_\theta &= v\cos\gamma \end{aligned}\right\} \Rightarrow \dot\theta = \frac{v}{r}\cos\gamma = \frac{15}{5}(0.766) = 2.298\ \text{rad/s}$$

$$a_\theta = g\cos^2\gamma = 9.81(0.76)^2 = 5.76\ \text{m/s}^2$$

$$a_z = g\cos\gamma\sin\gamma = 9.81(0.766)(0.6428) = 4.83\ \text{m/s}^2$$

$$a_r = \ddot r - r\dot\theta^2 = 0 - 5(2.298)^2 = -26.41\ \text{m/s}^2$$

$$a = \sqrt{\overline{26.41}^2 + \overline{5.76}^2 + \overline{4.83}^2} = 27.5\ \text{m/s}^2$$



۲-۱۷۲.

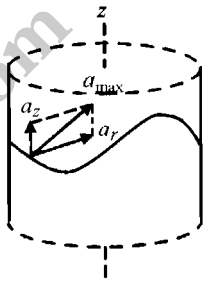
$$a_r = \ddot r - r\dot\theta^2 = 0 - r\omega^2$$

$$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 0 = 0$$

$$a_z = \frac{d^2}{dt^2}(z_0\sin 2\pi nt) = -4n^2\pi^2 z_0\sin 2\pi nt$$

$$a = \sqrt{(-r\omega^2)^2 + (-4n^2\pi^2 z_0\sin 2\pi nt)^2}$$

$$a_{max} = \sqrt{r^2\omega^4 + 16n^4\pi^4 z_0^2}$$



۲-۱۷۳.

$$v_r = \dot l\sin\beta = c\sin\beta$$

$$v_\theta = r\dot\theta = (l\sin\beta)K = Kl\sin\beta$$

$$v_z = \dot l\cos\beta = c\cos\beta$$

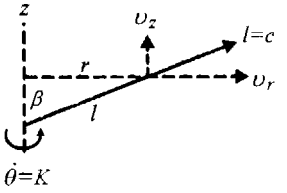
$$v = \sqrt{(c\sin\beta)^2 + (Kl\sin\beta)^2 + (c\cos\beta)^2} = \sqrt{c^2 + k^2l^2\sin^2\beta}$$

$$a_r = \ddot r - r\dot\theta^2 = 0 - l\sin\beta(K^2) = -k^2 l\sin\beta$$

$$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 2c\sin\beta(K) = 2Kc\sin\beta$$

$$a_z = \dot v_z = 0$$

$$a = \sqrt{(-Kl^2\sin\beta)^2 + (2Kc\sin\beta)^2} = K\sin\beta\sqrt{K^2l^2 + 4c^2}$$



۲-۱۷۴.

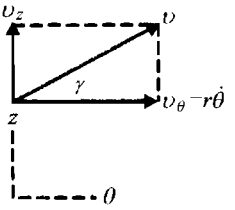
زاویه مارپیچ $\gamma = \tan^{-1}\dfrac{3}{7.2\pi} = 7.55°$

$$v = \frac{25}{3.6} = 6.94\ \text{m/s}$$

$$v_z = v\sin\gamma = 6.94\sin 7.55° = 0.912\ \text{m/s}$$

$$v_\theta = v\cos\gamma = 6.94\cos 7.55° = 6.88\ \text{m/s}$$

$$v_\theta = r\dot\theta \Rightarrow \dot\theta = \frac{v_\theta}{r} = \frac{6.88}{7.2} = 0.956\ \text{rad/s}$$



$$a_{\theta z} = \frac{3}{3.6} = 0.833\ \text{m/s}^2$$

$$a_\theta = -0.833\cos 7.55^\circ = -0.826\ \text{m/s}^2$$

$$a_z = -0.833\sin 7.55^\circ = -0.1096\ \text{m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 7.2(0.956)^2 = -6.58\ \text{m/s}^2$$

۲-۱۷۵.

$$\beta = \tan^{-1}\frac{1}{2} = 26.6^\circ$$

$$\phi = \tan^{-1}\frac{3}{5} = 31.0^\circ$$

$$v_R = 400\sin 26.6^\circ \sin 31.0^\circ = 92.0\ \text{km/h} = \dot{R}$$

$$v_\theta = R\dot{\theta}\cos\phi$$

$$\Rightarrow \frac{400}{3.6}\cos 26.6^\circ = 500\dot{\theta} \Rightarrow \dot{\theta} = 0.1988\ \text{rad/s}$$

$$v_\phi = R\dot{\phi}$$

$$\Rightarrow \frac{400}{3.6}\sin 26.6^\circ \cos 31.0^\circ = \frac{500}{\cos 31.0^\circ}\dot{\phi} \Rightarrow \dot{\phi} = 0.0731\ \text{rad/s}$$

۲-۱۷۶. طبق مسئلهٔ ۲-۱۷۵:

$$\phi = \tan^{-1}\frac{3}{5} = 31.0^\circ \quad , \quad \dot{\phi} = 0.0731\ \text{rad/s}$$

$$\dot{R} = \frac{92.0}{3.6} = 25.6\ \text{m/s} \quad , \quad R = \frac{500}{\cos 31.0^\circ} = 583\ \text{m} \quad , \quad \dot{\theta} = 0.1988\ \text{rad/s}$$

$$\mathbf{a} = 0 \Rightarrow a_R = a_\phi = a_\theta = 0$$

$$a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi = 0$$

$$\Rightarrow \ddot{R} = 583(0.0731)^2 + 583(0.1988)^2\cos^2 31.0^\circ = 20.1\ \text{m/s}^2$$

$$a_\theta = R\ddot{\theta}\cos\phi + 2\dot{R}\dot{\theta}\cos\phi - 2R\dot{\theta}\dot{\phi}\sin\phi = 0$$

$$\Rightarrow 583\ddot{\theta}\cos 31.0^\circ + 2(25.6)(0.1988)\cos 31.0^\circ - 2(583)(0.1988)(0.0731)\sin 31.0^\circ = 0$$

$$\Rightarrow \ddot{\theta} = 1.296(10^{-5})\ \text{rad/s}^2$$

$$a_\phi = 2\dot{R}\dot{\phi} + R\ddot{\phi} + R\dot{\theta}^2\sin\phi\cos\phi = 0$$

$$\Rightarrow 2(25.6)(0.0731) + 583\ddot{\phi} + 583(0.1988)^2\sin 31.0^\circ\cos 31.0^\circ = 0$$

$$\Rightarrow \ddot{\phi} = -0.0238\ \text{rad/s}^2$$

۲-۱۷۷.

$$R = 0.75 + 0.5 = 1.25\,\text{m}$$

$$\dot{R} = 0.2\,\text{m/s}$$

$$\ddot{R} = -0.3\,\text{m/s}^2$$

$$\phi = 30^\circ \;,\; \dot{\phi} = 10\left(\frac{\pi}{180}\right)\text{rad/s} \;,\; \ddot{\phi} = 0$$

$$\dot{\theta} = 20\left(\frac{\pi}{180}\right)\text{rad/s} \;,\; \ddot{\theta} = 0$$

$$\begin{cases} v_R = \dot{R} = 0.2\,\text{m/s} \\ v_\theta = R\dot{\theta}\cos\phi = 1.25\left(20\frac{\pi}{180}\right)\cos 30^\circ = 0.378\,\text{m/s} \\ v_\phi = R\dot{\phi} = 1.25\left(10\frac{\pi}{180}\right) = 0.218\,\text{m/s} \end{cases}$$

$$v = \sqrt{v_R^2 + v_\theta^2 + v_\phi^2} = 0.480\,\text{m/s}$$

$$a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$$

$$= -0.3 - 1.25\left(10\frac{\pi}{180}\right)^2 - 1.25\left(20\frac{\pi}{180}\right)^2\cos^2 30^\circ = -0.4523\,\text{m/s}^2$$

$$a_\theta = \cos\phi\,[2\dot{R}\dot{\theta} + R\ddot{\theta}] - 2R\dot{\theta}\dot{\phi}\sin\phi$$

$$= \cos 30^\circ\left[2(0.2)\left(20\frac{\pi}{180}\right) + 1.25(0)\right] - 2(1.25)\left(10\frac{\pi}{180}\right)\left(20\frac{\pi}{180}\right)\sin 30^\circ = 0.0448\,\text{m/s}^2$$

$$a_\phi = 2\dot{R}\dot{\phi} + R\ddot{\phi} + R\dot{\theta}^2\sin\phi\cos\phi$$

$$= 2(0.2)\left(10\frac{\pi}{180}\right) + 1.25(0) + 1.25\left(20\frac{\pi}{180}\right)^2 0.5\,\frac{\sqrt{3}}{2} = 0.1358\,\text{m/s}^2$$

$$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = 0.474\,\text{m/s}^2$$

۲-۱۷۸. مختصات کروی:

$$v_R = \dot{R} = 0.5\,\text{m/s}$$

$$v_\theta = R\dot{\theta}\cos\phi = 15\left(10\frac{\pi}{180}\right)\cos 30^\circ = 2.27\,\text{m/s}$$

$$v_\phi = R\dot{\phi} = 15\left(7\frac{\pi}{180}\right) = 1.833\,\text{m/s}$$

$$v = \sqrt{v_R^2 + v_\theta^2 + v_\phi^2} = 2.96\,\text{m/s}$$

$$a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$$

$$= 0 - 15\left(7\frac{\pi}{180}\right)^2 - 15\left(10\frac{\pi}{180}\right)^2\cos^2 30^\circ = -0.567\,\text{m/s}^2$$

$$a_\theta = \frac{\cos\phi}{R}\,[R^2\ddot{\theta} + 2R\dot{R}\dot{\theta}] - 2R\dot{\theta}\dot{\phi}\sin\phi$$

$$= \frac{\cos 30^\circ}{15}\left[0 + 2(15)(0.5)\left(10\frac{\pi}{180}\right)\right] - 2(15)\left(10\frac{\pi}{180}\right)\left(7\frac{\pi}{180}\right)\sin 30^\circ = -0.1687\ \text{m/s}^2$$

$$a_\phi = \frac{1}{R}\,[R^2\ddot{\phi} + 2R\dot{R}\dot{\phi}] + R\dot{\theta}^2\sin\phi\cos\phi$$

$$= \frac{1}{15}\left[0 + 2(15)(0.5)\left(7\frac{\pi}{180}\right)\right] + 15\left(10\frac{\pi}{180}\right)^2\sin 30^\circ\cos 30^\circ = 0.320\ \text{m/s}^2$$

$$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = 0.672\ \text{m/s}^2$$

۲-۱۷۹.

$R = 24\ \text{m}$ ثابت ، $\dot{\theta} = \omega = \frac{2(2\pi)}{60} = \frac{\pi}{15}\ \text{rad/s}$ ، $\ddot{\theta} = 0$

$\beta = 30^\circ$ ، $\phi = \frac{\pi}{2} - \beta$ ، $\dot{\phi} = -\dot{\beta} = -0.10\ \text{rad/s}$ ، $\ddot{\phi} = -\ddot{\beta} = 0$

$$v_R = \dot{R} = 0\ ,\quad v_\theta = R\dot{\theta}\cos\phi = 24\left(\frac{\pi}{15}\right)\frac{1}{2} = 2.51\ \text{m/s}$$

$$v_\phi = R\dot{\phi} = 24(-0.10) = -2.4\ \text{m/s}$$

$$v = \sqrt{(2.51)^2 + (2.4)^2} = 3.48\ \text{m/s}$$

$$a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi = 0 - 24(-0.10)^2 - 24\left(\frac{\pi}{15}\right)^2\left(\frac{1}{2}\right)^2 = -0.503\ \text{m/s}^2$$

$$a_\theta = \frac{\cos\phi}{R}\frac{d}{dt}(R^2\dot{\theta}) - 2R\dot{\theta}\dot{\phi}\sin\phi = 0 - 2(24)\frac{\pi}{15}(-0.10)\frac{\sqrt{3}}{2} = 0.871\ \text{m/s}^2$$

$$a_\phi = \frac{1}{R}\frac{d}{dt}(R^2\dot{\phi}) + R\dot{\theta}^2\sin\phi\cos\phi = 0 + 24\left(\frac{\pi}{15}\right)\frac{\sqrt{3}}{2}\frac{1}{2} = 0.456\ \text{m/s}^2$$

$$a = \sqrt{(-0.503)^2 + (0.871)^2 + (0.456)^2} = 1.104\ \text{m/s}^2$$

۲-۱۸۰. از معادلهٔ ۲-۱۹ استفاده می‌کنیم که در آن $\dot{\phi} = -\dot{\beta}$ و $L = R$ و $\dot{\theta} = \omega$

$$a_R = 0 - 1.2\left(\frac{3}{2}\right)^2 - 1.2(2)^2\frac{1}{2} = -5.10\ \text{m/s}^2$$

$$a_\theta = \frac{\sin\beta}{L}(2L\dot{L}\omega + 0) + 2L\theta\dot{\beta}\cos\beta = 2\omega(\dot{L}\sin\beta + L\dot{\beta}\cos\beta)$$

$$= 2(2)\left(0.9\frac{1}{\sqrt{2}} + 1.2\left(\frac{3}{2}\right)\frac{1}{\sqrt{2}}\right) = \frac{10.8}{\sqrt{2}} = 7.64\ \text{m/s}^2$$

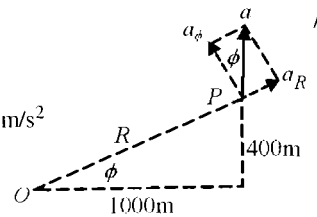
$$a_\phi = -2\dot{L}\dot{\beta} + L\omega^2\cos\beta\sin\beta = -2(0.9)\frac{3}{2} + 1.2(2)^2\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} = -0.3\ \text{m/s}^2$$

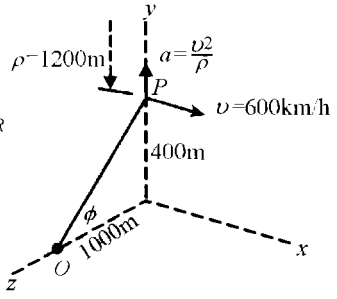
۲-۱۸۱.

$$\phi = \tan^{-1}\frac{400}{1000} = 21.8°$$

$$a = \frac{v^2}{\rho} = \frac{\left(\frac{600}{3.6}\right)^2}{1200} = 23.1\ \text{m/s}^2$$

$$\dot{R} = 0 \quad , \quad \dot{\phi} = 0$$





$$\dot{\theta} = \frac{v_\theta}{R\cos\phi} = \frac{\frac{600}{3.6}}{1000} = 0.1667\ \text{rad/s}$$

معادله ۲-۱۹ : $a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$

$$\Rightarrow 23.1\sin 21.8° = \ddot{R} - 0 - \frac{1000}{\cos 21.8°}(0.1667)^2\cos^2 21.8°$$

$$\Rightarrow \ddot{R} = 34.4\ \text{m/s}^2$$

$$a_\phi = \frac{1}{R}\frac{d}{dt}(R^2\dot{\phi}) + R\dot{\theta}^2\sin\phi\cos\phi = 2\dot{R}\dot{\phi} + R\ddot{\phi} + R\dot{\theta}^2\sin\phi\cos\phi$$

$$\Rightarrow 23.1\cos 21.8° = 0 + \frac{1000}{\cos 21.8°}\ddot{\phi} + \frac{1000}{\cos 21.8°}(0.1667)^2\sin 21.8°\cos 21.8°$$

$$\Rightarrow \ddot{\phi} = 0.01038\ \text{rad/s}^2$$

۲-۱۸۲.

$$R = 200 + 50\sin 4\pi t \qquad \dot{\theta} = 120\left(\frac{2\pi}{60}\right) = 4\pi\ \text{rad/s}$$

$$\dot{R} = 200\pi\cos 4\pi t \qquad \ddot{\theta} = 0$$

$$\ddot{R} = -800\pi^2\sin 4\pi t \qquad \phi = \frac{\pi}{4} - \beta = 60° \quad , \quad \dot{\phi} = \ddot{\phi} = 0$$

برای حداکثر شدن $\dot{R}$ باید $\cos 4\pi t = 1$ و $\sin 4\pi t = 0$ باشد.

معادله ۲-۱۹ : $a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$

$$= 0 - (200 + 0)0 - (200 + 0)(4\pi)^2\cos^2 60° = -800\pi^2\ \text{mm/s}^2$$

$$a_\theta = \frac{\cos\phi}{R}\frac{d}{dt}(R^2\dot{\theta}) - 2R\dot{\theta}\dot{\phi}\sin\phi$$

$$= 2\dot{R}\dot{\theta}\cos\phi - 2R\dot{\theta}\dot{\phi}\sin\phi$$

$$= 2(200\pi\times 1)(4\pi)\cos 60° - 2(200 + 0)(4\pi)(0)\sin 60° = 800\pi^2\ \text{mm/s}^2$$

$$a_\phi = \frac{1}{R}\frac{d}{dt}(R^2\dot{\phi}) + R\dot{\theta}^2\sin\phi\cos\phi$$

$$= 0 + (200 + 0)(4\pi)^2\sin 60°\cos 60° = 800\sqrt{3}\,\pi^2\ \text{mm/s}^2$$

$$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = 17660\ \text{mm/s}^2 \quad \text{یا} \quad 17.66\ \text{m/s}^2$$

◀ ۲-۱۸۳. جملات معادلهٔ ۲-۱۹ عبارتند از:

$$R = 50 + 200\left(\frac{1}{2}\right)^2 = 100\,\text{mm}$$

$$\dot{R} = 400\,t = 400\left(\frac{1}{2}\right) = 200\,\text{mm/s}$$

$$\ddot{R} = 400\,\text{mm/s}^2$$

$$\theta = \omega\,t = \frac{\pi}{3}\left(\frac{1}{2}\right) = \frac{\pi}{6}\,\text{rad}$$

$$\dot{\theta} = \frac{\pi}{3}\,\text{rad/s}$$

$$\ddot{\theta} = 0$$

$$\phi = \dot{\phi}\,t = \frac{2\pi}{3}\left(\frac{1}{2}\right) = \frac{\pi}{3}\,\text{rad}$$

$$\dot{\phi} = \frac{2\pi}{3}\,\text{rad/s}$$

$$\ddot{\phi} = 0$$

$$\sin\theta = \frac{1}{2}\ ,\ \cos\theta = \frac{\sqrt{3}}{2}\ ,\ \sin\phi = \frac{\sqrt{3}}{2}\ ,\ \cos\phi = \frac{1}{2}$$

$$\frac{d}{dt}(R^2\dot{\theta}) = 2R\dot{R}\dot{\theta} + R^2\ddot{\theta} = 2(0.1)(0.2)\frac{\pi}{3} + 0 = \frac{0.04\pi}{3}\,(\text{m/s})^2$$

$$\frac{d}{dt}(R^2\dot{\phi}) = 2R\dot{R}\dot{\phi} + R^2\ddot{\phi} = 2(0.1)(0.2)\frac{2\pi}{3} + 0 = \frac{0.08\pi}{3}\,(\text{m/s})^2$$

بنابراین مؤلفه‌های a از معادله ۲-۱۹ چنین خواهند بود:

$$a_R = 0.40 - 0.10\left(\frac{2\pi}{3}\right)^2 - 0.10\left(\frac{\pi}{3}\right)^2\left(\frac{1}{2}\right)^2 = -0.0661\,\text{m/s}^2$$

$$a_\theta = \frac{\frac{1}{2}}{0.10}\,\frac{0.04\pi}{3} - 2(0.10)\left(\frac{\pi}{3}\right)\left(\frac{2\pi}{3}\right)\left(\frac{\sqrt{3}}{2}\right) = -0.1704\,\text{m/s}^2$$

$$a_\phi = \frac{1}{0.10}\,\frac{0.08\pi}{3} + 0.10\left(\frac{\pi}{3}\right)^2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = 0.885\,\text{m/s}^2$$

$$a = \sqrt{(-0.0661)^2 + (-0.1704)^2 + (0.885)^2} = 0.904\,\text{m/s}^2$$

◀ ۲-۱۸۴.

$$R = \text{ثابت}\ ,\ \theta = \omega\,t\ ,\ \sin\phi = \frac{z}{R}$$

$$z = \frac{h}{2}(1 - \cos 2\theta) \Rightarrow \dot{z} = \omega\,h\sin 2\theta \quad (\dot{\theta} = \omega \text{ که در آن})$$

$$(\cos\phi)\,\dot{\phi} = \frac{1}{R}\,\dot{z} \Rightarrow \dot{\phi} = \frac{\omega\,h\sin 2\theta}{R\cos\phi}$$

$$v_R = \dot{R} = 0$$

$$v_\theta = R\,\dot{\theta}\cos\phi = R\,\omega\sqrt{1 - \sin^2\phi} = R\,\omega\sqrt{1 - \left(\frac{h}{2R}[1 - \cos 2\theta]\right)^2}$$

$$v_\phi = R\,\dot{\phi} = \frac{\omega\,h\sin 2\theta}{\cos\phi} = h\,\omega\,\frac{\sin 2\theta}{\sqrt{1 - \left(\frac{h}{2R}[1 - \cos 2\theta]\right)^2}}$$

وقتی $\theta = \omega t = \frac{\pi}{4}$ باشد $\cos 2\theta = 1 - 1$ خواهد بود بنابراین:

$$v_\theta = R\omega\sqrt{1 - \left(\frac{h}{2R}\right)^2} \quad , \quad v_\phi = \frac{h\omega}{\sqrt{1 - \left(\frac{h}{2R}\right)^2}} \quad , \quad v_R = 0$$

۲-۱۸۵.

$$\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A = 40\mathbf{i} - (-80\mathbf{i}) = 120\mathbf{i} \text{ km/h}$$

$$\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A = 0 - 2\mathbf{i} = -2\mathbf{i} \text{ m/s}^2$$

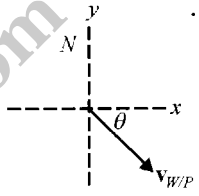
۲-۱۸۶.

الف) $\mathbf{v}_{W/P} = \mathbf{v}_W - \mathbf{v}_P$

$$= 4\left(\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}\right) - (-6\mathbf{i}) = 8.83\mathbf{i} - 2.83\mathbf{j} \text{ km/h}$$

یا $v_{W/P} = (8.83^2 + 2.83^2)^{\frac{1}{2}} = 9.27 \text{ km/h}$

$\theta = \tan^{-1}\frac{2.83}{8.83} = 17.76°$ جنوب شرقی

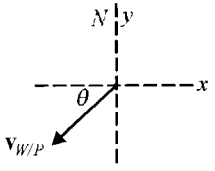


ب) $\mathbf{v}_{W/P} = \mathbf{v}_W - \mathbf{v}_P$

$$= 4\left(\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}\right) - 6\mathbf{i} = -3.17\mathbf{i} - 2.83\mathbf{j} \text{ km/h}$$

یا $v_{W/P} = (3.17^2 + 2.83^2)^{\frac{1}{2}} = 4.25 \text{ km/h}$

$\theta = \tan^{-1}\frac{2.83}{3.17} = 41.7°$ جنوب غربی

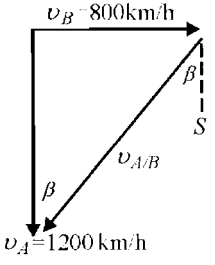


۲-۱۸۷.

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$v_{A/B} = \sqrt{(1200)^2 + (800)^2} = 1442 \text{ km/h}$$

$\beta = \tan^{-1}\frac{800}{1200} = 33.7°$ جنوب غربی

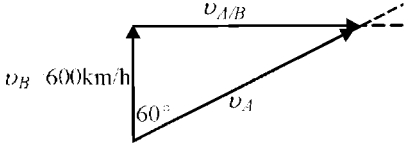


۲-۱۸۸.

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$v_A = \frac{600}{\cos 60°} = 1200 \text{ km/h}$$

$$v_{A/B} = 600 \tan 60° = 1039 \text{ km/h}$$



۲-۱۸۹.

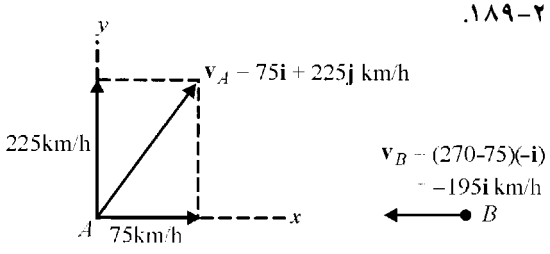
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

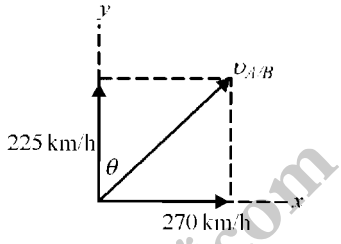
$$75\,\mathbf{i} + 225\,\mathbf{j} = -195\,\mathbf{i} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = 270\,\mathbf{i} + 225\,\mathbf{j}$$

$$\mathbf{v}_{A/B} = \sqrt{270^2 + 225^2} = 351 \text{ km/h}$$

$$\theta = \tan^{-1}\frac{270}{225} = 50.2^\circ \text{ شمال شرقی}$$





۲-۱۹۰.

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$= 120\,[\cos 15^\circ\,\mathbf{i} + \sin 15^\circ\,\mathbf{j}] - 90\,[\cos 60^\circ\,\mathbf{i} + \sin 60^\circ\,\mathbf{j}] = 70.9\,\mathbf{i} - 46.9\,\mathbf{j} \text{ km/h}$$

$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B = 0 - 3(-\cos 60^\circ\,\mathbf{i} - \sin 60^\circ\,\mathbf{j}) = 1.5\,\mathbf{i} + 2.60\,\mathbf{j} \text{ m/s}^2$$

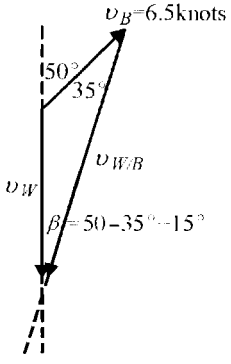
۲-۱۹۱.

W = باد $\quad \mathbf{v}_W = \mathbf{v}_B - \mathbf{v}_{W/B}$

B = قایق

قانون سینوسها : $\dfrac{v_W}{\sin 35^\circ} = \dfrac{6.5}{\sin 15^\circ} \Rightarrow$

$$\Rightarrow v_W = \frac{6.5(0.5736)}{6.2588} = 14.40 \text{ knots}$$



۲-۱۹۲.

$$\mathbf{a}_A = \mathbf{a}_B = 0 \quad \Rightarrow \quad \mathbf{a}_{A/B} = 0$$

در مختصات قطبی نسبت به نقطهٔ B:

$$(a_{A/B})_r = \ddot{r} - r\dot{\theta}^2 = 0 \quad \Rightarrow \quad \ddot{r} = r\dot{\theta}^2$$

$$(a_{A/B})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \quad \Rightarrow \quad \ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r}$$

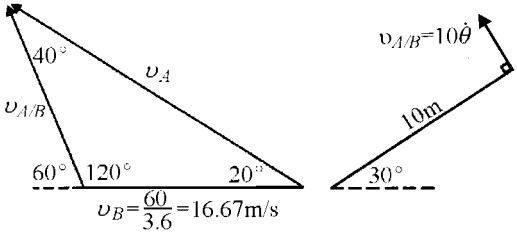
۲-۱۹۳.

قانون سینوسها : $\frac{16.67}{\sin 40^\circ} = \frac{v_A}{\sin 120^\circ}$

$\Rightarrow v_A = 22.5\,\text{m/s}$ یا $v_A = 80.8\,\text{km/h}$

$v_{A/B} = 16.67\,\frac{\sin 20^\circ}{\sin 40^\circ} = 10\,\dot{\theta}$

$\Rightarrow \dot{\theta} = 0.887\,\text{rad/s}$



۲-۱۹۴.

$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B \quad , \quad \Omega = 3\left(\frac{2\pi}{60}\right) = 0.314\,\text{rad/s}$

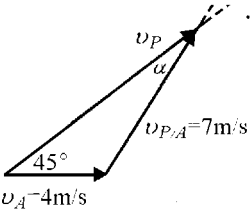
$= \frac{18}{3.6}\mathbf{i} - 9(0.314)(\cos 45^\circ\mathbf{i} - \sin 45^\circ\mathbf{j}) = 3.00\mathbf{i} + 2.00\mathbf{j}\,\text{m/s}$

$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B = 3\mathbf{i} - 9(0.314)^2(\cos 45^\circ\mathbf{i} - \sin 45^\circ\mathbf{j}) = 3.63\mathbf{i} + 0.628\mathbf{j}\,\text{m/s}^2$

۲-۱۹۵.

$\mathbf{v}_P = \mathbf{v}_A + \mathbf{v}_{P/A}$

قانون سینوسها : $\frac{\sin 45^\circ}{7} = \frac{\sin\alpha}{4} \Rightarrow \alpha = 23.8^\circ$



۲-۱۹۶.

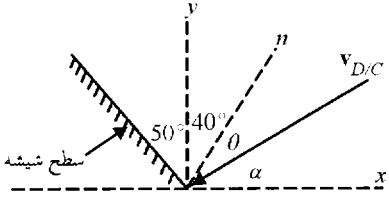
قطره آب : $v_D = \sqrt{2gh} = \sqrt{2(9.81)(6)} = 10.85\,\text{m/s}$

اتومبیل : $v_C = \frac{100}{3.6} = 27.8\,\text{m/s}$

$\mathbf{v}_{D/C} = \mathbf{v}_D - \mathbf{v}_C = -10.85\mathbf{j} - 27.8\mathbf{i}\,\text{m/s}$

$\alpha = \tan^{-1}\frac{10.85}{27.8} = 21.3^\circ$

$40^\circ + \theta + \alpha = 90^\circ \Rightarrow \theta = 28.7^\circ$ زیر امتداد عمود



۲-۱۹۷.

$g = g_0\left(\frac{R}{R+h}\right)^2$

برای A : $g_A = 9.823\left(\frac{6371}{6371+320}\right)^2 = 8.91\,\text{m/s}^2$

برای B : $g_B = 9.823\left(\frac{6371}{6371+36000}\right)^2 = 0.222\,\text{m/s}^2$

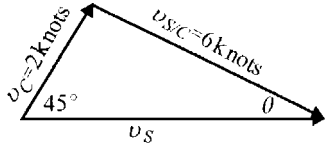
$\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A = 0.222\mathbf{i} - (-8.91)\mathbf{j} = 0.222\mathbf{i} + 8.91\mathbf{j}\,\text{m/s}$

۲-۱۹۸. $\mathbf{v}_S$: سرعت واقعی کشتی (A تا B)

$\mathbf{v}_C$: سرعت واقعی باد (2 گره به سمت شمال شرقی)

$\mathbf{v}_{S/C}$: سرعت کشتی نسبت به باد (6 گره)

$\mathbf{v}_S = \mathbf{v}_C + \mathbf{v}_{S/C}$



قانون سینوسها : $\dfrac{2}{\sin\theta} = \dfrac{6}{\sin 45°} \Rightarrow \theta = 13.63°$

زاویهٔ سمت‌گیری $H = 90 + 13.63° \approx 104°$

$v_S = 2\cos 45° + 6\cos 13.63° = 7.25$ knots

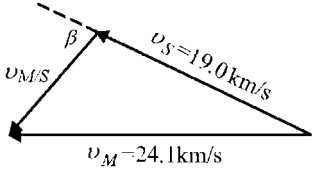
زمان $t = \dfrac{10}{7.25} = 1.380$ hr یا $t = 1$ hr 23 min

۲-۱۹۹.

$\mathbf{v}_M = \mathbf{v}_S + \mathbf{v}_{M/S}$

$(v_{M/S})^2 = (19.0)^2 + (24.1)^2 - 2(19.0)(24.1)\cos 15°$

$= 57.2\,(\text{km/s})^2 \Rightarrow v_{M/S} = 7.56$ km/s

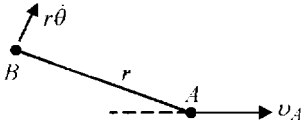


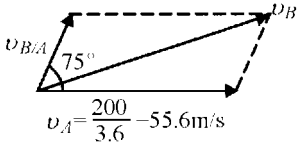
$\dfrac{24.1}{\sin(\pi-\beta)} = \dfrac{7.56}{\sin 15°} \Rightarrow \sin(\pi-\beta) = \sin\beta = 0.8246$

$\Rightarrow \beta = 55.6°$

۲-۲۰۰.

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$, $v_{B/A} = r\dot{\theta} = 60\left(\dfrac{5\pi}{180}\right) = 5.24$ m/s





$v_B^2 = (5.24)^2 + (55.6)^2 + 2(5.24)(55.6)\cos 75° = 3264\,(\text{m/s})^2$

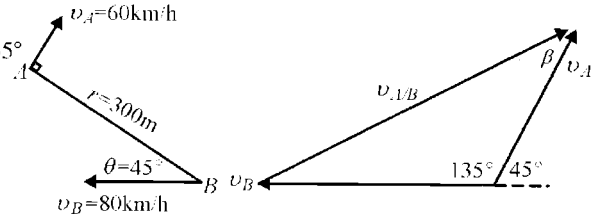
$\Rightarrow v_B = 57.1$ m/s یا $v_B = 57.1(3.6) = 206$ km/h

$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$, $a_A = 0$, $a_{B/A} = r\dot{\theta}^2 = 60\left(\dfrac{5\pi}{180}\right)^2 = 0.457$ m/s^2

$a_B = a_{B/A} = 0.457$m/s^2 از B تا A

۲-۲۰۱.

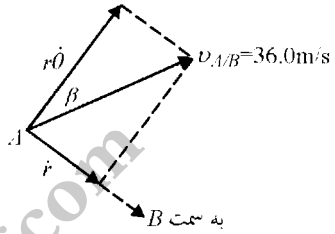
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$



$$v_{A/B}^2 = 60^2 + 80^2 - 2(60)(80)\cos 135°$$

$$v_{A/B} = 129.6\ \text{km/h} \quad \text{یا} \quad 36.0\ \text{m/s}$$

$$\frac{36.0}{\sin 135°} = \frac{\frac{80}{3.6}}{\sin\beta} \Rightarrow \beta = 25.9°$$



$$r\dot{\theta} = v_{A/B}\cos\beta \Rightarrow 30\dot{\theta} = 36.0\cos 25.9°$$

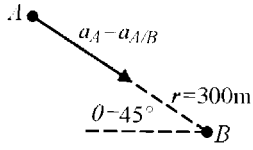
$$\Rightarrow \dot{\theta} = 0.1079\ \text{rad/s}$$

$$\dot{r} = -v_{A/B}\sin\beta \Rightarrow \dot{r} = -36.0\sin 25.9° = -15.71\ \text{m/s}$$

۲-۲۰۲. طبق مسئله ۲-۲۰۱:

$$\dot{r} = -15.71\ \text{m/s}\ ,\quad \dot{\theta} = 0.1079\ \text{rad/s}$$

$$\mathbf{a}_A = \overset{0}{\mathbf{a}_B} + \mathbf{a}_{A/B}$$



$$a_A = \frac{v_A^2}{\rho} = \frac{\left(\frac{60}{3.6}\right)^2}{300} = 0.926\text{m/s}^2 = a_{A/B}$$

$$(a_{A/B})_r = \ddot{r} - r\dot{\theta}^2 = -0.926 = \ddot{r} - 300(0.1079)^2 \Rightarrow \ddot{r} = 2.57\ \text{m/s}^2$$

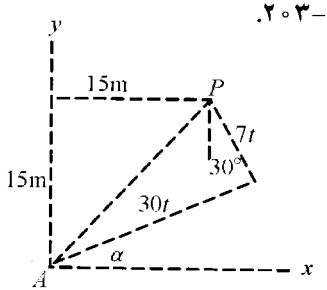
$$(a_{A/B})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \Rightarrow 0 = 300\ddot{\theta} + 2(-15.71)(0.1079) \Rightarrow \ddot{\theta} = 0.01130\ \text{rad/s}^2$$

۲-۲۰۳.

$$\begin{cases} 30t\cos\alpha = 15 + 7t\sin 30° \\ 30t\sin\alpha = 15 - 7t\cos 30° \end{cases}$$

یا

$$\begin{cases} 30\cos\alpha = \frac{15}{t} + 7\sin 30° \\ 30\sin\alpha = \frac{15}{t} - 7\cos 30° \end{cases}$$



کم کردن معادله دوم از اول:

$$30(\cos\alpha - \sin\alpha) = 7\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) \Rightarrow \cos\alpha - \sin\alpha = 0.319$$

$$\cos\alpha = \sqrt{1 - \sin^2\alpha} = 0.319 + \sin\alpha \Rightarrow 2\sin^2\alpha + 0.638\sin\alpha - 0.9 \Rightarrow \alpha = 32°$$

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B = 30[\cos\alpha\,\mathbf{i} + \sin\alpha\,\mathbf{j}] - 7[\sin 30\,\mathbf{i} - \cos 30\,\mathbf{j}] = 21.9\,\mathbf{i} + 21.9\,\mathbf{j}\ \text{m/s}$$

۲-۲۰۴ ◀

$$y = y_0 + v_{y_0}\,t - \frac{1}{2}\,g\,t^2 \Rightarrow 2.1 = 0.9 + 30\sin 30°\,t - \frac{1}{2}(9.81)\,t^2$$

$$\Rightarrow t = 2.97\ \text{s}$$

$$R = x_0 + v_{x_0}\,t = 0 + 30\cos 30°\,(2.97) = 77.2$$

توپ گیر باید مسافت $12.2\,\text{m} = 65 - 77.2$ را در زمان $(2.97 - 0.25)\text{s}$ بدود بنابراین

$$v_B = \frac{12.2}{2.72} = 4.48\ \text{m/s}$$

مولفه‌های سرعت توپ وقتی به دست توپ گیر می‌رسد:

$$v_x = v_{x_0} = 30\cos 30° = 25.98\ \text{m/s}$$

$$v_y = v_{y_0} - gt = 30\sin 30° - 9.81(2.97) = -14.19\ \text{m/s}$$

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B = (25.98\,\mathbf{i} - 14.19\,\mathbf{j}) - 4.48\,\mathbf{i} = 21.5\,\mathbf{i} - 14.19\,\mathbf{j}$$

۲-۲۰۵ ◀

$$v_{B/A} = v_B - v_A = \frac{1500 - 1000}{3.6} = 138.9\ \text{m/s}$$

$$(v_{B/A})_r = \dot{r} = 138.9\cos 30° = 120.3\ \text{m/s}$$

$$(v_{B/A})_\theta = r\dot{\theta} \Rightarrow -138.9\sin 30° = \frac{6000}{\sin 30°}\,\dot{\theta}$$

$$\Rightarrow \dot{\theta} = -0.00579\ \text{rad/s}$$

$$a_{B/A} = a_B - a_A = 0 - 1.2 = -1.2\ \text{m/s}$$

$$(a_{B/A})_r = \ddot{r} - r\dot{\theta}^2$$

$$\Rightarrow -1.2\cos 30° = \ddot{r} - 12000(-0.00579)^2$$

$$\Rightarrow \ddot{r} = -0.637\ \text{m/s}^2$$

$$(a_{B/A})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$\Rightarrow 1.2\sin 30° = 12000\,\ddot{\theta} + 2(120.3)(-0.00579)$$

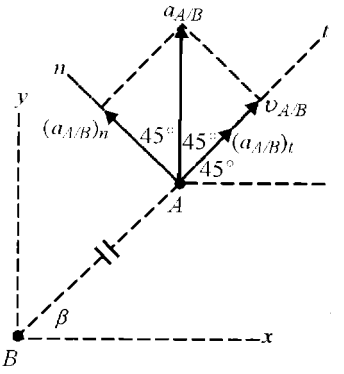
$$\Rightarrow \ddot{\theta} = 0.1660(10^{-3})\ \text{rad/s}^2$$

۲-۲۰۶ ◀

الف) $\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B = 50\,\mathbf{i} - (-50\,\mathbf{j}) = 50\,\mathbf{i} + 50\,\mathbf{j}\ \text{m/s}$

$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B = \frac{{v_A}^2}{\rho_A}\,\mathbf{j} - 0 = \frac{50^2}{2000^2}\,\mathbf{j} + 1.250\,\mathbf{j}\ \text{m/s}^2$$

ب) از نتایج قسمت الف برای یک تحلیل عمودی - مماسی استفاده کنید.



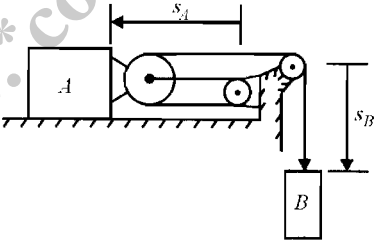
$$v_{A/B} = \sqrt{50^2 + 50^2} = 70.7 \text{ m/s}$$

$$\dot{v}_r = (a_{B/A})_t = a_{A/B}\cos 45° = 1.250\frac{\sqrt{2}}{2} = 0.884 \text{ m/s}^2$$

$$(a_{B/A})_n = a_{B/A}\sin 45° = \frac{v_{A/B}^2}{\rho_r} \Rightarrow 1.250\frac{\sqrt{2}}{2} = \frac{70.7^2}{\rho_r}$$

$$\Rightarrow \rho_r = 5660 \text{ m}$$ در جهت عمود از نقطهٔ A

۲-۲۰۷.

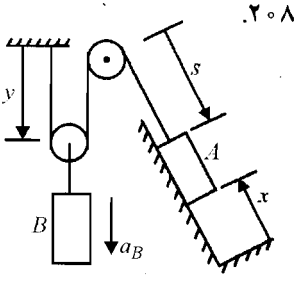


طول کابل $L = 3s_A + s_B + \text{const.}$

مشتق‌گیری : $0 = 3v_A + v_B$

$\Rightarrow v_B = -3v_A = -3(-0.6) = 1.8 \text{ m/s}$ به طرف پائین

۲-۲۰۸.



$$L = 2y + s + \text{const.}$$

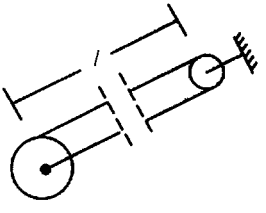
$$0 = 2\dot{y} + \dot{s}$$

$$0 = 2\ddot{y} + \ddot{s}$$

$$a_B = \ddot{y} = -\frac{\ddot{s}}{2} = +\frac{\ddot{s}}{2}$$

$$a_B = \frac{0.044}{2} = 0.022 \text{ m/s}^2$$

۲-۲۰۹.



طول کابل $L = 2l$

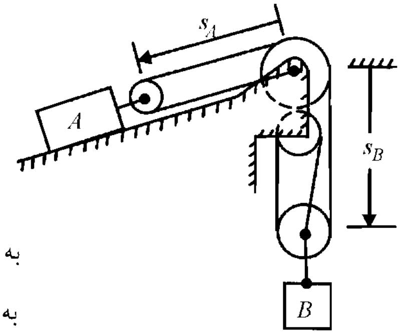
$$\dot{L} = 2\dot{l}$$

بنابراین سرعت کامیون برابر است با:

$$-\dot{l} = \frac{1}{2}(-\dot{L}) = \frac{1}{2}(40) = 20 \text{ mm/s}$$

$$\text{زمان } t = \frac{\text{مسافت}}{\text{سرعت}} = \frac{4(10^3)}{20} = 200 \text{ s} \quad \text{یا} \quad 3 \text{ min } 20 \text{ s}$$

۲-۲۱۰.



طول کابل $L = 2s_A + 3s_B + \text{const.}$

$$0 = 2v_B + 3v_B$$

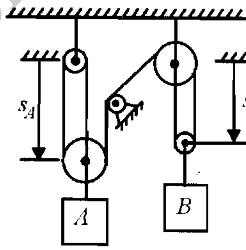
$$0 = 2a_A + 3a_B$$

$$v_A = -\frac{3}{2}v_B = -\frac{3}{2}(0.6) = -0.9\ \text{m/s}$$

$$a_A = -\frac{3}{2}a_B = -\frac{3}{2}(-0.15) = 0.225\ \text{m/s}^2$$

یا $\begin{cases} v_A = 0.9\text{m/s} & \text{به سمت بالای سطح شیبدار} \\ a_A = 0.225\ \text{m/s}^2 & \text{به سمت پائین سطح شیبدار} \end{cases}$

۲-۲۱۱.



طول کابل اصلی $L = 3s_A + 2s_B + \text{const.}$

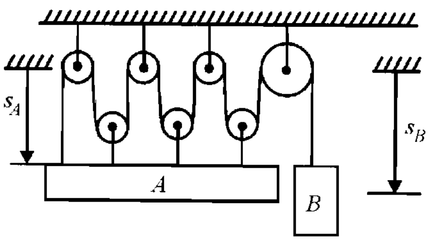
$$0 = 3v_A + 2v_B$$

$$0 = 3a_A + 2a_B$$

$v_A = -\frac{3}{2}v_A = -\frac{3}{2}(0.8) = -1.2\ \text{m/s}$ (به طرف بالا)

$a_B = -\frac{3}{2}a_A = -\frac{3}{2}(-2) = 3\ \text{m/s}^2$ (به طرف پائین)

۲-۲۱۲.

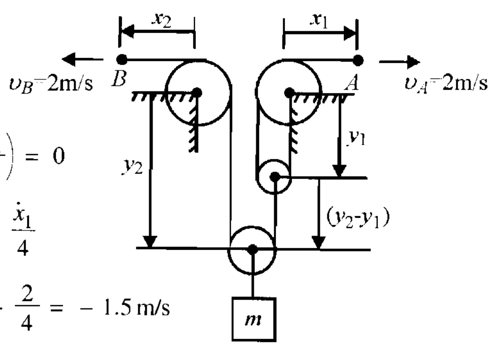


$$L = 7s_A + s_B + \text{const.}$$

$$0 = 7a_A + a_B$$

(برای مختصات نشان داده شده)

۲-۲۱۳.



$$x_1 + 2y_1 = \text{const.}$$

$$\dot{x}_1 + 2\dot{y}_1 = 0 \Rightarrow \dot{y}_1 = -\frac{\dot{x}_1}{2}$$

$$x_2 + y_2 + (y_2 - y_1) = \text{cons.}$$

$$\dot{x}_2 + 2\dot{y}_2 - \dot{y}_1 = 0 \Rightarrow \dot{x}_2 + 2\dot{y}_2 - \left(-\frac{\dot{x}_1}{2}\right) = 0$$

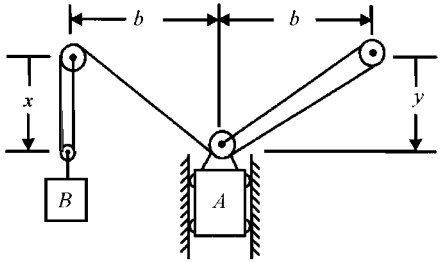
$$\Rightarrow \dot{x}_2 + 2\dot{y}_2 + \frac{\dot{x}_1}{2} = 0 \Rightarrow \dot{y}_2 = -\frac{\dot{x}_2}{2} - \frac{\dot{x}_1}{4}$$

$$= \frac{-2}{2} - \frac{2}{4} = -1.5\ \text{m/s}$$

۲-۲۱۴. طول کل کابل $L = 2x + 3\sqrt{y^2 + b^2} + \text{const.}$

$$\dot{L} = 0 = 2\dot{x} + 3\frac{y\dot{y}}{\sqrt{y^2 + b^2}}$$
$$\dot{x} = v_B \quad , \quad \dot{y} = v_A$$

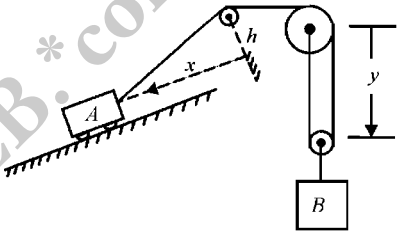
$$\Rightarrow v_B = -\frac{3y}{2\sqrt{y^2 + b^2}}\, v_A$$



۲-۲۱۵. طول کل کابل $L = \sqrt{x^2 + h^2} + 2y + \text{const.}$

$$\dot{L} = 0 = \frac{1}{2}\frac{2x\dot{x}}{\sqrt{x^2 + h^2}} + 2\dot{y}$$
$$v_A = \dot{x} \quad , \quad v_B = -\dot{y}$$

$$\Rightarrow v_A = +\frac{2\sqrt{x^2 + h^2}}{x}\, v_B$$



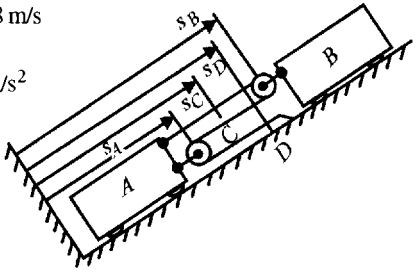
۲-۲۱۶. طول کابل $L = 2(s_B - s_A) + s_D - s_A + \text{const.}$

$$0 = 2v_B - 3v_A \Rightarrow v_A = \frac{2}{3}v_B = \frac{2}{3}(1.2) = 0.8\ \text{m/s}$$

$$0 = 2a_B - 3a_A \Rightarrow a_A = \frac{2}{3}a_B = \frac{2}{3}(2) = \frac{4}{3}\ \text{m/s}^2$$

$$v_{B/A} = v_B - v_A = 1.2 - 0.8 = 0.4\ \text{m/s}$$

$$a_{B/A} = a_B - a_A = 2 - \frac{4}{3} = \frac{2}{3}\ \text{m/s}^2$$



طول کابل بین A و C : $L' = (s_B - s_A) + (s_B - s_C) = 2s_B - s_A - s_C + \text{const.}$

$$\Rightarrow 0 = 2v_B - v_A - v_C \Rightarrow v_C = 2v_B - v_A = 2(1.2) - 0.8 = 1.6\ \text{m/s}$$

۲-۲۱۷. فرض کنید A نقطه‌ای روی کابل 1 و B قرقرهٔ نشان داده شده در شکل باشد.

$L_1 = s_A + 3s_B + \text{cons.}$

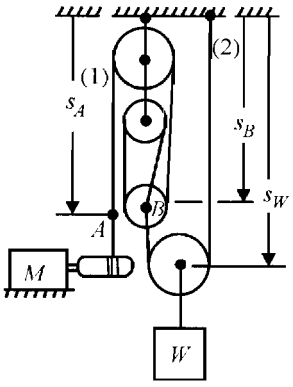
$$0 = v_A + 3v_B \qquad (1)$$

$$L_2 = s_W - s_B + s_W = 2s_W - s_B$$

$$0 = 2v_W - v_B \qquad (2)$$

$$(1)\ ,\ (2)\ :\ v_A + 6v_W = 0 \quad \Rightarrow \quad v_W = -\frac{1}{6}v_A$$

بنابراین وزنهٔ W به اندازهٔ $h = \frac{1}{6}(180)(10) = 300\ \text{mm}$ بالا می‌آید.



۲-۲۱۸.

$$L_1 = l_1 + 2(l_1 - l_2) + \text{const.}$$

$$\dot{L}_1 = -r\omega = 3\dot{l}_1 - 2\dot{l}_2$$

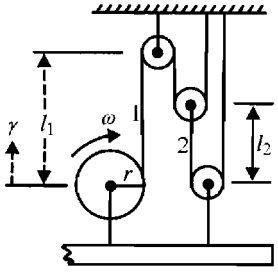
$$L_2 = l_2 + l_1 + \text{cons.}$$

$$\dot{L}_2 = 0 = \dot{l}_2 + \dot{l}_1 \quad \Rightarrow \quad \dot{l}_2 = -\dot{l}_1$$

$$v = -\dot{l}_1$$

$$-r\omega = 3(-v) - 2v \quad \Rightarrow \quad r\omega = 5v$$

$$\Rightarrow \quad v = \frac{r\omega}{5} = \frac{0.1\,(40)\left(\frac{2\pi}{60}\right)}{5} = 0.0838\ \text{m/s} \quad \text{یا} \quad v = 83.8\ \text{mm/s}$$



۲-۲۱۹.

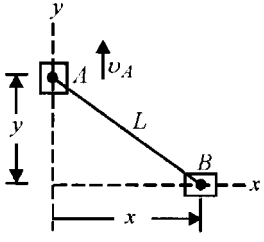
$$x^2 + y^2 = L^2$$

$$x\dot{x} + y\dot{y} = 0$$

$$\dot{x}^2 + x\ddot{x} + \dot{y}^2 + y\ddot{y} = 0$$

$$\dot{y} = v_A\ ,\ \ddot{y} = 0 \quad \Rightarrow \quad a_x = \ddot{x} = \frac{-\dot{x}^2 + \dot{y}^2}{x} = \frac{-\dot{y}^2\frac{y^2}{x^2} + \dot{y}^2}{x}$$

$$a_x = -\frac{L^2}{x^3}\dot{y}^2 = -\frac{L^2 v_A{}^2}{(L^2 - y^2)^{\frac{3}{2}}}$$



۲-۲۲۰.

$$s^2 = 2b^2 + 2b^2\cos\theta = 2b^2(1 + \cos\theta)$$

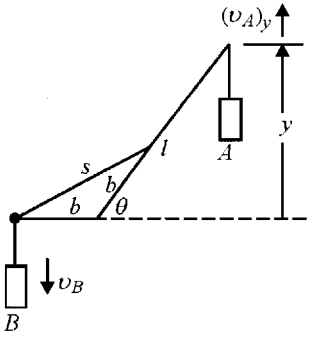
$$2s\dot{s} = 2b^2\,(-\dot{\theta}\sin\theta)$$

$$v_B = -\dot{s} = \frac{b^2}{s}\dot{\theta}\sin\theta$$

$$y = l\sin\theta$$

$$(v_A)_y = \dot{y} = l\dot{\theta}\cos\theta$$

حذف $\dot{\theta}$: $$(v_A)_y = l\cos\theta\frac{s v_B}{b^2\sin\theta} = \frac{l\sqrt{2(1+\cos\theta)}}{b\tan\theta}v_B$$



۲-۲۲۱. ◀

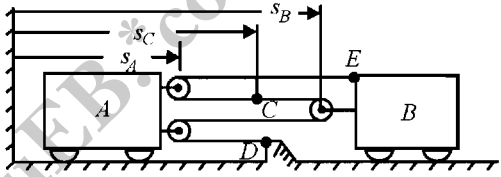
طول کابل $L = 3(s_B - s_A) + (s_D - s_A)$

$$0 = 3v_B - 4v_A$$

$$0 = 3a_B - 4a_A$$

$$v_A = \frac{3}{4}v_B = \frac{3}{4}(2) = 1.5\ \text{m/s}$$

$$a_A = \frac{3}{4}a_B = \frac{3}{4}(2) = 2.25\ \text{m/s}$$



$$v_{B/A} = v_B - v_A = 2 - 1.5 = 0.5\ \text{m/s}$$

$$a_{B/A} = a_B - a_A = 3 - 2.25 = 0.75\ \text{m/s}^2$$

طول کابل بین نقاط E و C : $L' = (s_B - s_A) + (s_C - s_A) + \text{const.}$

$$0 = v_B - 2v_A + v_C \Rightarrow v_C = 2v_A - v_B = 2(1.5) - 2 = 1\ \text{m/s}$$

۲-۲۲۲. ◀

$$l^2 = x^2 + h^2$$

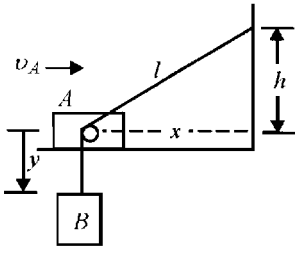
$$l\dot{l} = x\dot{x}$$

$$\dot{y} = -\dot{l} = -\frac{x}{l}\dot{x} \quad , \quad v_A = -\dot{x}$$

$$(v_B)_y = \dot{y} = \frac{x}{l}v_A$$

$$(v_B)_x = \dot{x} = -v_A$$

$$v_B = \sqrt{(v_B)^2_x + (v_B)^2_y} = v_A\sqrt{1 + \frac{x^2}{l^2}} = v_A\sqrt{\frac{2x^2 + h^2}{x^2 + h^2}}$$



۲-۲۲۳.

$$v^2_y = v^2_{y_0} - 2g(y - y_0) \quad +\uparrow$$

$$0^2 = v^2_0 - 2(9.81)(1) \Rightarrow v_0 = 4.43\ \text{m/s}$$

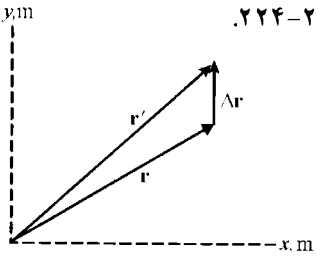
۲-۲۲۴.

$$\Delta \mathbf{r} = \mathbf{r}' - \mathbf{r} = (1.060\,\mathbf{i} + 0.740\,\mathbf{j}) - (1.040\,\mathbf{i} + 0.600\,\mathbf{j})$$

$$= 0.020\,\mathbf{i} + 0.140\,\mathbf{j}\ \text{m}$$

$$\mathbf{v} = \frac{\Delta \mathbf{r}}{\Delta t} = \frac{0.020\,\mathbf{i} + 0.140\,\mathbf{j}}{2.05 - 2} = 0.4\,\mathbf{i} + 2.8\,\mathbf{j}\ \text{m/s}$$

$$v = 2.83\ \text{m/s} \quad @ \quad \theta = 81.9°$$



۲-۲۲۵.

$$\dot{x} = u\cos\theta = 60\cos 60° = 30\ \text{m/s}$$

$$\dot{y} = u\sin\theta - gt = 60\sin 60° - 9.81\,t$$

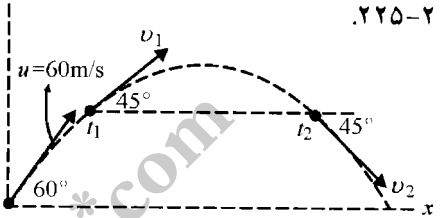
$$= 51.96 - 9.81\,t$$

در زمان t_1 : $\dot{x} = \dot{y} \Rightarrow 30 = 51.96 - 9.81\,t_1$

$$\Rightarrow t_1 = 2.24\ \text{s}$$

در زمان t_2 : $\dot{x} = -\dot{y} \Rightarrow 30 = -51.96 + 9.81\,t_2$

$$\Rightarrow t_2 = 8.35\ \text{s}$$



۲-۲۲۶.

$$\dot{x} = 20\ \text{mm/s} \quad , \quad \ddot{x} = 0$$

$$y = \frac{x^2}{160} \Rightarrow \dot{y} = \frac{x\dot{x}}{80} \quad , \quad \ddot{y} = \frac{(\dot{x}^2 + x\ddot{x})}{80}$$

برای $x = 60\ \text{mm}$: $v = 20\sqrt{1 + \left(\frac{60}{80}\right)^2} = 25\ \text{mm/s}$

$$a = \ddot{y} = \frac{\dot{x}^2}{80} \quad (\text{زیرا } \ddot{x} = 0) \Rightarrow a = \frac{20^2}{80} = 5\ \text{mm/s}^2$$

۲-۲۲۷.

در جهت y : $v^2 = v_1^2 + 2as \Rightarrow 0 = u_y^2 - 2g(12)$ از A تا B

$$\Rightarrow u_y = \sqrt{2(9.81)(12)} = 15.34\ \text{m/s}$$

$$s = v_1 t + \frac{1}{2}at^2 \Rightarrow -30 = 15.34\,t - \frac{9.81}{2}t^2$$

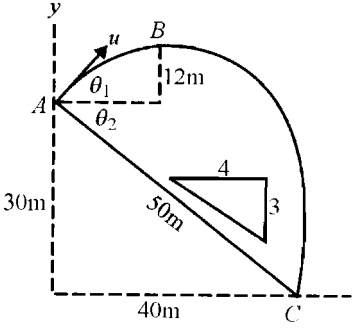
$$\Rightarrow t^2 - 3.128\,t - 6.116 = 0 \Rightarrow t = 4.49\ \text{s}$$

در جهت x : $40 = u_x(4.49) \Rightarrow u_x = 8.91\ \text{m/s}$

$$u = \sqrt{(8.91)^2 + (15.34)^2} = 17.74\ \text{m/s}$$

$$\theta_1 = \tan^{-1}\frac{15.34}{8.91} = 59.86° \quad , \quad \theta_2 = \tan^{-1}\frac{3}{4} = 36.87°$$

$$\Rightarrow \theta = \theta_1 + \theta_2 = 59.86° + 36.87° = 96.7°$$



.۲۲۸-۲

$$\theta = 4[t + 30e^{-0.03t} - 30] \text{ (rad)}$$

$$\dot{\theta} = 4[1 - 0.9e^{-0.03t}] \text{ (rad/s)}$$

$$\ddot{\theta} = 0.1080\, e^{-0.03t} \text{ (rad/s}^2\text{)}$$

$$r\dot{\theta}^2 = 10[4(1 - 0.9e^{-0.03t})]^2 = 9.81(10)$$

$$\Rightarrow (1 - 0.9e^{-0.03t})^2 = 0.613 \Rightarrow 1 - 0.9e^{-0.03t} = \pm 0.783$$

علامت منفی نتیجهٔ $t < 0$ خواهد داد بنابراین علامت + را در نظر می‌گیریم:

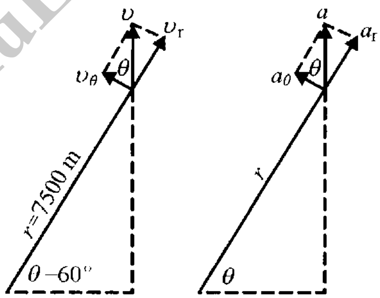
$$1 - 0.9e^{-0.03t} = 0.783 \Rightarrow t = 47.4 \text{ s}$$

$$\ddot{\theta} = 0.1080\, e^{-0.03(47.4)} = 0.026 \text{ rad/s}^2$$

$$r\ddot{\theta} = 10(0.026) = 0.260 \text{ m/s}^2 = 0.026\, g$$

بنابراین a_t قابل صرف‌نظر کردن است.

.۲۲۹-۲



$$r = 7500 \text{ m} \quad \theta = 60°$$

$$\dot{\theta} = 0.03 \text{ rad/s} \quad a = 20 \text{ m/s}^2$$

$$v_\theta = r\dot{\theta} = 7500(0.03) = 225 \text{ m/s}$$

$$v = \frac{225}{\cos 60°} = 450 \text{ m/s}$$

$$v_r = \dot{r} = 45 \cos 30° = 389.7 \text{ m/s}$$

$$a_\theta = a \cos 60° = 20(0.5) = 10 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \Rightarrow r\ddot{\theta} = 10 - 2(389.7)(0.03) = -13.38 \text{ m/s}^2$$

$$\ddot{\theta} = \frac{-13.38}{7500} = -1.784(10^{-3}) \text{ rad/s}^2$$

$$a_r = a \sin 60° = 20\left(\frac{\sqrt{3}}{2}\right) = 17.32 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \Rightarrow \ddot{r} = 17.32 + 7500(0.03)^2 = 24.1 \text{ m/s}^2$$

.۲۳۰-۲

$$\mathbf{v}_A = \mathbf{v}_W + \mathbf{v}_{A/W} = -48\mathbf{i} + 220\mathbf{i} = 172\mathbf{i} \text{ km/h}$$

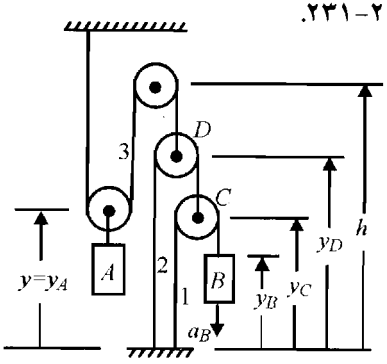
موقع فرود $\mathbf{v}_A = 172(\cos 10\mathbf{i} - \sin 10\mathbf{j}) \text{ km/h}$

$$\mathbf{v}_{A/C} = \mathbf{v}_A - \mathbf{v}_C = 172(\cos 10°\mathbf{i} - \sin 10°\mathbf{j}) - 30\mathbf{i}$$

$$= 139.4\mathbf{i} - 29.9\mathbf{j} \text{ km/h}$$

$$\beta = \tan^{-1}\left(\frac{29.9}{139.4}\right) = 12.09°$$

۲-۲۳۱.



$$y = y_A = \frac{t^2}{4}\ \text{m}$$

$$\dot{y}_A = \frac{t}{2}\ \text{m/s}$$

$$a_A = \ddot{y}_A = \frac{1}{2}\ \text{m/s}^2$$

یک درجه آزادی

طول کابل‌ها $L_1 = y_C + (y_C - y_B) + C_1 \Rightarrow 0 = 2\ddot{y}_C - \ddot{y}_B$ (که در آن $a_B = -\ddot{y}_B$)

$$L_2 = y_D + (y_D - y_C) + C_2 \Rightarrow 0 = 2\ddot{y}_D - \ddot{y}_C$$

$$L_3 = 2(h - y_A) + h - y_D + C_3 \Rightarrow 0 = -2\ddot{y}_A - \ddot{y}_D$$

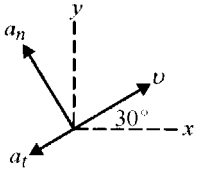
حذف $\ddot{y}_C$ و $\ddot{y}_D$ $\Rightarrow \ddot{y}_B = -8\ddot{y}_A \Rightarrow a_B = 8\ddot{y}_A = 4\ \text{m/s}^2$

با بررسی جابجائی‌های قرقره نیز خواهیم داشت:

$$-dy_B = 8\,dy_A \Rightarrow a_B = 8a_A = 4\ \text{m/s}^2$$

۲-۲۳۲.

$$v = \frac{1000}{3.6} = 278\ \text{m/s}\ ,\ a_t = \frac{15}{3.6} = 4.17\ \text{m/s}^2$$



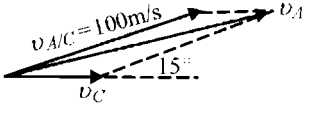
$$a_n = \frac{v^2}{\rho} = \frac{(278)^2}{1500} = 51.4\ \text{m/s}^2$$

$$\ddot{x} = -51.4\sin 30° - 4.17\cos 30° = -29.3\ \text{m/s}^2$$

$$\ddot{y} = 51.4\cos 30° - 4.17\sin 30° = 42.5\ \text{m/s}^2$$

۲-۲۳۳. ناو با سرعت ثابت حرکت می‌کند، بنابراین به عنوان مبنای دستگاه مختصات مورد استفاده قرار گیرد. سرعت جنگنده نسبت به ناو برابر است با:

$$v^2_{A/C} = 2as = 2(50)(100) = 10000(\text{m/s}^2) \Rightarrow v_{A/C} = 100\ \text{m/s}$$



$$\mathbf{v}_A = \mathbf{v}_C + \mathbf{v}_{A/C}$$

$$v_C = \frac{30(1.852)}{3.6} = 15.43\ \text{m/s}$$

$$v_A{}^2 = (100)^2 + (15.43)^2 + 2(100)(15.43)\cos 15° = 13220\,(\text{m/s})^2$$

$$\Rightarrow v_A = 115.0\ \text{m/s} \quad \text{یا} \quad v_A = v = 115.0(3.6) = 414\ \text{km/h}$$

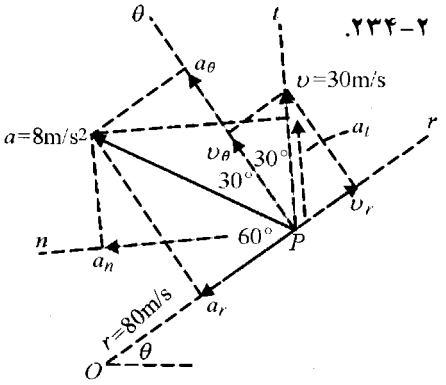
$a_r = -8\cos 60° = -4\ \text{m/s}^2$

$a_\theta = 8\cos 30° = 6.93\ \text{m/s}^2$

$v_r = 30\cos 60° = 15\ \text{m/s}$

$v_\theta = 30\cos 30° = 26.0\ \text{m/s}$

۲-۲۳۴.



$[r-\theta]$ $\quad v_r = \dot{r} = 15\ \text{m/s}$

$v_\theta = r\dot{\theta} \Rightarrow 26.0 = 80\dot{\theta} \Rightarrow \dot{\theta} = 0.325\ \text{rad/s}$

$a_r = \ddot{r} - r\dot{\theta}^2 \Rightarrow -4 = \ddot{r} - 80(0.325)^2 \Rightarrow \ddot{r} = 4.44\ \text{m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \Rightarrow 6.93 = 80\ddot{\theta} + 2(15)(0.325) \Rightarrow \ddot{\theta} = -0.0352\ \text{rad/s}^2$

$[n-t]$ $\quad a_n = 8\cos 30° = 6.93\ \text{m/s}^2$

$a_t = 8\sin 30° = 4\ \text{m/s}^2$

$a_n = \dfrac{v^2}{\rho} \Rightarrow \rho = \dfrac{v^2}{a_n} = \dfrac{30^2}{6.93} = 129.9\ \text{m}$

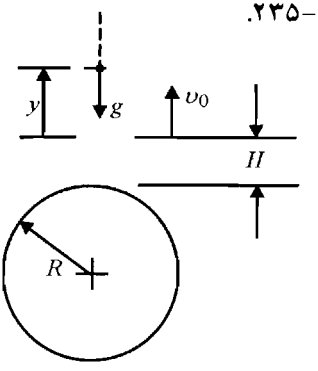
۲-۲۳۵.

$v_0 = \dfrac{27000}{3.6} = 7500\ \text{m/s}$

$H = 35(10^4)\ \text{m}$

$g_0 = 9.832\ \text{m/s}^2$ (شکل ۱-۱)

$R = 6371\ \text{km}$ یا $6.371(10^6)\ \text{m}$



$g = g_0\left(\dfrac{R}{R+H+y}\right)^2$

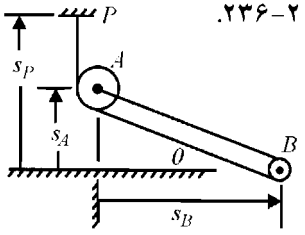
$a = v\dfrac{dv}{dy} \Rightarrow \int_{v_0}^{0} v\,dv = \int_0^h -g_0\left(\dfrac{R}{R+H+y}\right)^2 dy$

$\Rightarrow -\dfrac{1}{2}v_0^2 = g_0R^2\dfrac{1}{R+H+y}\Big|_0^h \Rightarrow v_0^2 = 2g_0R\dfrac{h}{(R+H)(R+H+h)}$

$\Rightarrow h = \dfrac{(R+H)^2 v_0^2}{2g_0R^2 - (R+H)v_0^2}$

جای‌گذاری مقادیر عددی : $h = 6048(10^3)\ \text{m}$ یا $h = 6048\ \text{km}$

۲-۲۳۶.



$$\theta = 45^\circ : \frac{s_B}{s_A} = 1$$

$$\theta = 30^\circ : \frac{s_B}{s_A} = \sqrt{3}$$

$$\theta = 15^\circ : \frac{s_B}{s_A} = 3.73$$

$$L = (s_P - s_A) + 2\sqrt{{s_A}^2 + {s_B}^2}$$

$$0 = -v_A + 2(s_A v_A + s_B v_B)/\sqrt{{s_A}^2 + {s_B}^2} \Rightarrow v_B = \left[\sqrt{1 + \left(\frac{s_B}{s_A}\right)^2} - 2\right] \frac{v_A}{2\left(\frac{s_B}{s_A}\right)}$$

$$\theta = 45^\circ : v_B = -0.293\, v_A = -0.293(-1) = 0.293 \text{ m/s}$$

$$\theta = 30^\circ : v_B = 0$$

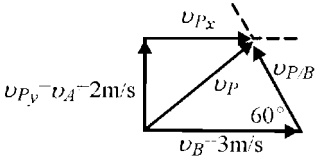
$$\theta = 15^\circ : v_B = 0.250\, v_A = 0.250(-1) = -0.250 \text{ m/s}$$

۲-۲۳۷. فرض کنید B نقطه‌ی روی شیار B منطبق بر P باشد.

$$\mathbf{v}_P = \mathbf{v}_B + \mathbf{v}_{P/B} \quad , \quad \mathbf{v}_P = \mathbf{v}_{Px} + \mathbf{v}_{Py} = \mathbf{v}_{Px} + \mathbf{v}_A$$

$$\Rightarrow \mathbf{v}_{Px} + \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{P/B}$$

$$\Rightarrow v_P = \sqrt{2^2 + \left(3 - \frac{2}{\sqrt{3}}\right)^2} = 2.72 \text{ m/s}$$



۲-۲۳۸. روش ترسیمی : $a = 7.1 \text{ m/s}^2$, $a_n = 6.8 \text{ m/s}^2$, $a_r = -1.83 \text{ m/s}^2$

$$\rho = \frac{v^2}{a_n} = \frac{3.2^2}{6.8} = 1.51 \text{ m}$$

روش تحلیلی : $\overline{1-2} = 1.83 + 5\cos 60^\circ = 4.33 \text{ m/s}^2$

$$\overline{1-3} = \frac{\overline{1-2}}{\cos 30^\circ} = \frac{4.33}{0.866} = 5 \text{ m/s}^2$$

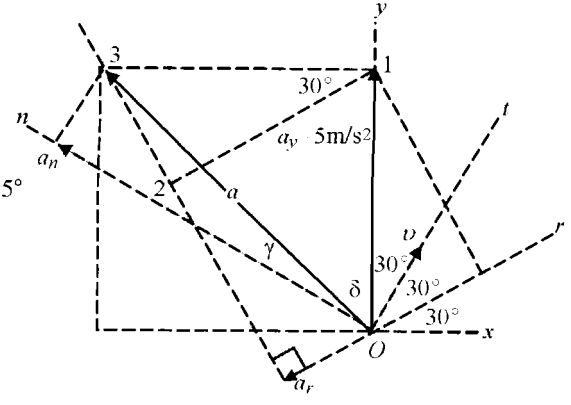
$$a = \overline{0-3} = \sqrt{5^2 + 5^2} = 7.07 \text{ m/s}^2$$

$$\delta = 45^\circ \ , \ \gamma = 90^\circ - 30^\circ - 45^\circ = 15^\circ$$

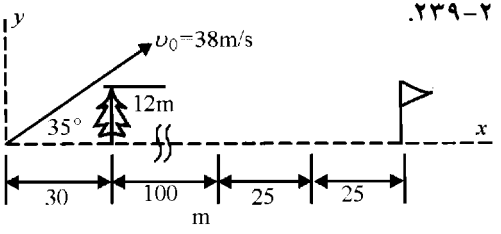
$$a_n = 7.07\cos 15^\circ = 6.83 \text{ m/s}^2$$

$$\rho = \frac{v^2}{a_n} = \frac{3.2^2}{6.83} = 1.499 \text{ m}$$

$$a_x = -a\cos 45^\circ = -5 \text{ m/s}^2$$



۲-۲۳۹.



زمان تا درخت : $x = x_0 + v_{x_0} t$

$\Rightarrow 30 = 0 + 38 \cos 35° t$

$\Rightarrow t = 0.964\,\text{s}$

ارتفاع : $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$

$\Rightarrow y = 0 + 38 \sin 35° (0.964) - \frac{9.81}{2} (0.964)^2 = 16.45\,\text{m}$

بنابراین توپ از درخت عبور می‌کند.

زمان کل حرکت توپ : $y = y_0 + v_{y_0} - \frac{1}{2} g t^2$

$\Rightarrow 0 = 0 + 38 \sin 35° t_f - \frac{9.81}{2} t_f^2 \Rightarrow \begin{cases} t_f = 0 & \text{زمان ضربه خوردن} \\ t_f = 4.44 & \text{زمان برخورد به زمین} \end{cases}$

$x = x_0 + v_{x_0} t \Rightarrow R = 0 + 38 \cos 35° (4.44) = 138.3\,\text{m}$

توپ در محدودهٔ آب به زمین برخورد می‌کند.

۲-۲۴۰.

$x = 50\,\text{m}$, $\dot{x} = -10\,\text{m/s}$, $\ddot{x} = -10\,\text{m/s}^2$

$y = 20\,\text{m}$, $\dot{y} = 10\,\text{m/s}$, $\ddot{y} = 5\,\text{m/s}^2$

$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(-10)^2 + 10^2} = 10\sqrt{2}\ \text{m/s}$

$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(-10)^2 + 5^2} = 11.18\ \text{m/s}^2$

$\mathbf{e}_t = \frac{\mathbf{v}}{v} = \frac{(-10\mathbf{i} + 10\mathbf{j})}{10\sqrt{2}} = \frac{\sqrt{2}}{2}(-\mathbf{i} + \mathbf{j})$

$a_t = \mathbf{a} \cdot \mathbf{e}_t = (-10\mathbf{i} + 5\mathbf{j}) \cdot \frac{\sqrt{2}}{2}(-\mathbf{i} + \mathbf{j}) = 10.61\ \text{m/s}^2$

$\mathbf{a}_t = a_t \cdot \mathbf{e}_t = 10.61 \frac{\sqrt{2}}{2}(-\mathbf{i} + \mathbf{j}) = -7.5\mathbf{i} + 7.5\mathbf{j}\ \text{m/s}^2$

$\mathbf{a}_n = \mathbf{a} - \mathbf{a}_t = (-10\mathbf{i} + 5\mathbf{j}) - (-7.5\mathbf{i} + 7.5\mathbf{j}) = -2.5(\mathbf{i} + \mathbf{j})\ \text{m/s}^2$

$a_n = \sqrt{2.5^2 + 2.5^2} = 3.54\ \text{m/s}^2$

$\rho = \frac{v^2}{a_n} = \frac{(10\sqrt{2})^2}{3.54} = 56.6\,\text{m}$

$\mathbf{e}_n = \frac{\mathbf{a}_n}{a_n} = \frac{-2.5(\mathbf{i} + \mathbf{j})}{3.54} = -\frac{\sqrt{2}}{2}(\mathbf{i} + \mathbf{j})$

$$\mathbf{e}_r = \frac{\mathbf{r}}{r} = \frac{50\,\mathbf{i} + 25\,\mathbf{j}}{\sqrt{50^2 + 25^2}} = 0.894\,\mathbf{i} + 0.447\,\mathbf{j}$$

$\mathbf{e}_\theta = \mathbf{e}_r$ (در خلاف ساعت بچرخد $90°$) $= -0.447\,\mathbf{i} + 0.894\,\mathbf{j}$

$v_r = \mathbf{v} \cdot \mathbf{e}_r = (-10\,\mathbf{i} + 10\,\mathbf{j}) \cdot (0.894\,\mathbf{i} + 0.447\,\mathbf{j}) = -4.47$ m/s

$\mathbf{v}_r = v_r\,\mathbf{e}_r = -4.47\,(0.894\,\mathbf{i} + 0.447\,\mathbf{j}) = -4\,\mathbf{i} - 2\,\mathbf{j}$ m/s

$v_\theta = \mathbf{v} \cdot \mathbf{e}_\theta = (-10\,\mathbf{i} + 10\,\mathbf{j}) \cdot (-0.447\,\mathbf{i} + 0.894\,\mathbf{j}) = 13.42$ m/s

$\mathbf{v}_\theta = v_\theta\,\mathbf{e}_\theta = 13.42\,(-0.447\,\mathbf{i} + 0.894\,\mathbf{j}) = -6\,\mathbf{i} + 12\,\mathbf{j}$ m/s

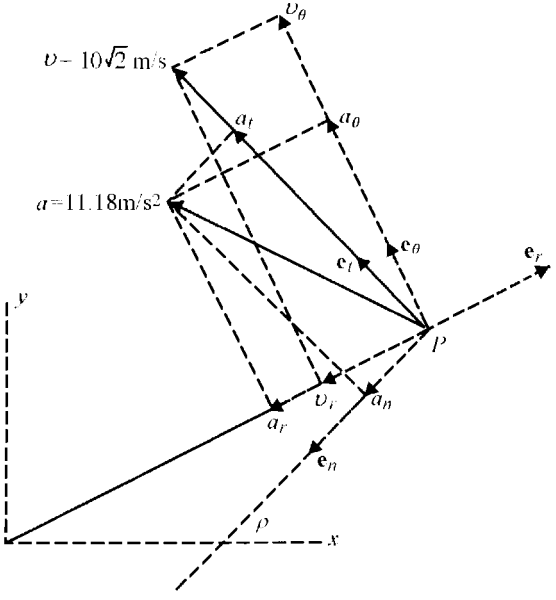
$a_r = \mathbf{a} \cdot \mathbf{e}_r = (-10\,\mathbf{i} + 5\,\mathbf{j}) \cdot (0.894\,\mathbf{i} + 0.447\,\mathbf{j}) = -6.71$ m/s^2

$\mathbf{a}_r = a_r\,\mathbf{e}_r = -6.71\,(0.894\,\mathbf{i} + 0.447\,\mathbf{j}) = -6\,\mathbf{i} - 3\,\mathbf{j}$ m/s^2

$a_\theta = \mathbf{a} \cdot \mathbf{e}_\theta = (-10\,\mathbf{i} + 5\,\mathbf{j}) \cdot (-0.447\,\mathbf{i} + 0.894\,\mathbf{j}) = 8.94$ m/s^2

$\mathbf{a}_\theta = a_\theta\,\mathbf{e}_\theta = 8.94\,(-0.447\,\mathbf{i} + 0.894\,\mathbf{j}) = -4\,\mathbf{i} + 8\,\mathbf{j}$ m/s^2

$r = \sqrt{x^2 + y^2} = \sqrt{50^2 + 25^2} = 55.9$ m



(توجه: مرکز منحنی در نقطه $(10, -15)$ قرار دارد)

$\dot{r} = v_r = -4.47\ \text{m/s}$

$v_\theta = r\dot{\theta} \Rightarrow \dot{\theta} = \frac{v_\theta}{r} = \frac{13.42}{55.9} = 0.240\ \text{rad/s}$

$a_r = \ddot{r} - r\dot{\theta}^2 \Rightarrow \ddot{r} = a_r + r\dot{\theta}^2 = -6.71 + 55.9\,(0.240)^2 = -3.49\ \text{m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \Rightarrow \ddot{\theta} = \frac{1}{r}(a_\theta - 2\dot{r}\dot{\theta}) = \frac{1}{55.9}[8.94 - 2(-4.47)(0.240)] = 0.1984\ \text{rad/s}^2$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{25}{50}\right) = 26.6°$

◀ ۲-۲۴۱.

$\theta = \theta_0 \cos\omega t$

$\dot{\theta} = -\theta_0 \omega \sin\omega t$

$\ddot{\theta} = -\theta_0 \omega^2 \cos\omega t$

$\dot{\phi} = k \ , \ \ddot{\phi} = 0 \ , \ R = b \ , \ \dot{R} = \ddot{R} = 0$

طبق معادله ۲-۱۹ : $a_R = 0 - bk^2 - b\theta_0^2\,\omega^2 \sin^2\omega t \cos^2\phi$

$a_\theta = b\cos\phi\,(-\theta_0\omega^2\cos\omega t) - 2b(-\theta_0\omega\sin\omega t)\,k\sin\theta$

$a_\phi = 0 + b(\theta_0\omega\sin\omega t)^2 \sin\phi\cos\phi$

الف) $\cos\omega t = -1 \ , \ \sin\omega t = 0$

$a_R = -bk^2 \ , \ a_\theta = b\omega^2\theta_0\cos\phi \ , \ a_\phi = 0$

$a = b\sqrt{k^4 + \omega^4\theta_0^2\cos^2\phi}$

ب) $\cos\omega t = 0 \ , \ \sin\omega t = 1 \ , \ \phi = \frac{\pi}{2}$

$a = bk\sqrt{k^2 + 4\omega^2\theta_0^2}$

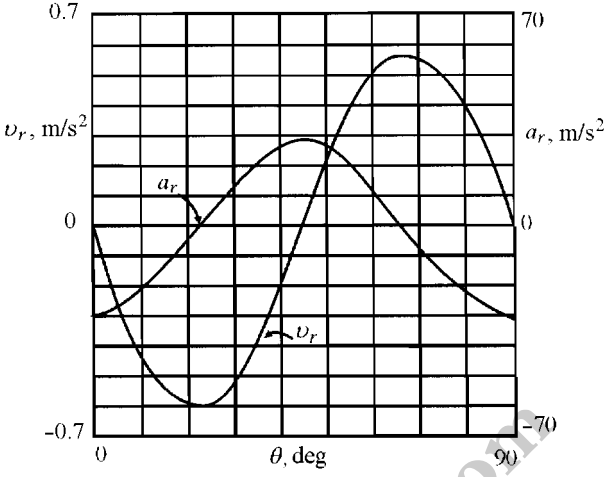
* ۲-۲۴۲. حرکت در جهت r می‌باشد بنابراین $a = a_r = \ddot{r}$

$r = 80 + 12\cos 4\theta \ , \ \dot{r} = -48\dot{\theta}\sin 4\theta \ , \ \ddot{r} = -192\dot{\theta}^2\cos 4\theta$

الف) $\theta = 0 \Rightarrow a = -192(4\pi)^2(1) = -30.3(10^3)\ \text{mm/s}^2$ یا $-30.3\ \text{m/s}^2$

ب) $\theta = \frac{\pi}{8} \Rightarrow a = -192(4\pi)^2(0) = 0$

ج) $\theta = \frac{\pi}{4} \Rightarrow a = -192(4\pi)^2\cos\pi = 30.3(10^3)\ \text{mm/s}^2$ یا $30.3\ \text{m/s}^2$



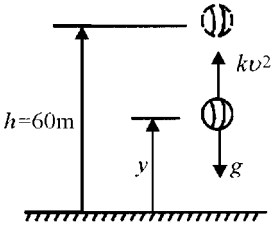
۲-۲۴۳ *

$$a = v\frac{dv}{dy} = -g + kv^2 \quad , \quad h = 60\,\text{m}$$

$$\int_0^v \frac{v\,dv}{-g+kv^2} = \int_h^y dy \Rightarrow \frac{1}{2k}\ln\left[-g+kv^2\right]_0^v = y\Big|_h^y$$

$$\Rightarrow -\frac{1}{2k}\ln\left[\frac{-g+kv^2}{-g}\right] = y-h$$

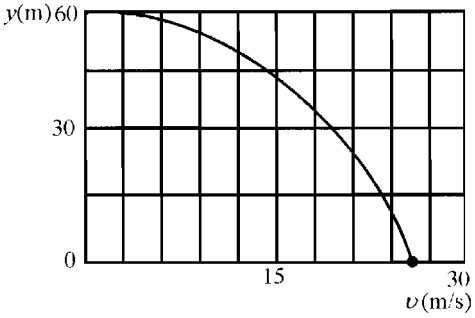
$$\Rightarrow v = \sqrt{\frac{g}{k}\left[1-e^{2k(y-h)}\right]}$$



جای‌گذاری اعداد : $26 = \sqrt{\frac{9.81}{k}\left[1-e^{2k(0-60)}\right]}$ ⇒ حل عددی $k = 0.01029\,\text{m}^{-1}$

سرعت حد : $g = kv^2 \Rightarrow 9.81 = 0.01029\,v_t^2 \Rightarrow v_t = 30.9\,\text{m/s}$

بدون مقاومت هوا : $v' = \sqrt{2gh} = \sqrt{2(9.81)(60)} = 34.3\,\text{m/s}$



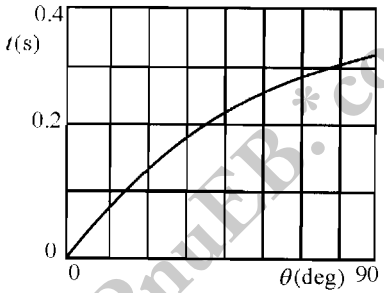
* ۲-۲۴۴.

$$\ddot{\theta} = \dot{\theta}\frac{d\dot{\theta}}{d\theta} = \frac{g}{l}\cos\theta$$

$$\int_{\dot{\theta}_0}^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = \frac{g}{l}\int_0^{\theta}\cos\theta\,d\theta \quad \Rightarrow \quad \dot{\theta} = \left[\dot{\theta}_0^2 + \frac{2g}{l}\sin\theta\right]^{\frac{1}{2}}$$

$$\Rightarrow \dot{\theta} = \frac{d\theta}{dt} = \left[\dot{\theta}_0^2 + \frac{2g}{l}\sin\theta\right]^{\frac{1}{2}} \quad \Rightarrow \quad t = \int_0^{\theta}\frac{d\theta}{\sqrt{\dot{\theta}_0^2 + \frac{2g}{l}\sin\theta}}$$

$\dot{\theta}_0 = 2\,\text{rad/s}$, $l = 0.6\,\text{m}$, $g = 9.81$, $\theta = \frac{\pi}{2}$ $\Rightarrow$ انتگرال‌گیری عددی $t' = 0.349\,\text{s}$



* ۲-۲۴۵.

$$a = \frac{dv}{dt} = C_1 - C_2 v^2$$

$$\int_0^t dt = \int_0^v \frac{dv}{C_1 - C_2 v^2} = \frac{1}{\sqrt{C_1 C_2}}\tanh^{-1}\sqrt{\frac{C_2}{C_1}}\,v\Big|_0^v \quad \Rightarrow \quad t = \frac{1}{\sqrt{C_1 C_2}}\tanh^{-1}\sqrt{\frac{C_2}{C_1}}\,v$$

$$\Rightarrow v = \frac{ds}{dt} = \sqrt{\frac{C_1}{C_2}}\tanh\sqrt{C_1 C_2}\,t \quad \Rightarrow \quad \int_0^s ds = \sqrt{\frac{C_1}{C_2}}\int_0^t \tanh\sqrt{C_1 C_2}\;t\,dt$$

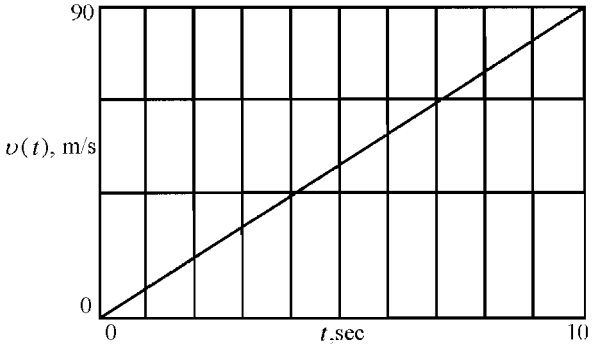
$$\Rightarrow s = \sqrt{\frac{C_1}{C_2}}\cdot\frac{1}{\sqrt{C_1 C_2}}\ln\left(\cosh\sqrt{C_1 C_2}\,t\right)\Big|_0^t = \frac{1}{C_2}\ln\left(\cosh\sqrt{C_1 C_2}\,t\right)$$

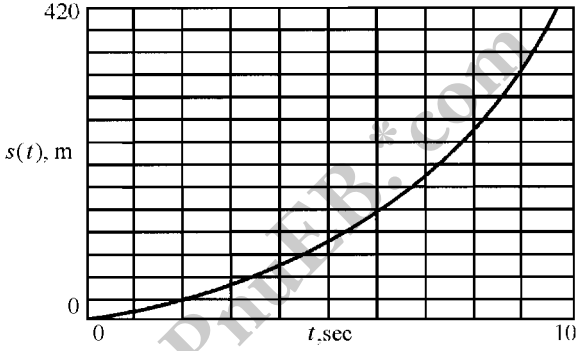
$s = 402\,\text{m}$, $C_1 = 9.14\,\text{m/s}^2$, $t = 9.40\,\text{s}$

$$\Rightarrow \quad 402 = \frac{1}{C_2}\ln\left(\cosh\sqrt{9.14\,C_2}\;9.40\right) = \frac{1}{C_2}\ln\left(\cosh\;28.4\sqrt{C_2}\right)$$

حل عددی : $C_2 = 33.2(10^{-6})\,\text{m}^{-1}$

در نمودارهای زیر توجه کنید که نمودار سرعت - زمان خطی است در حالیکه این طور نیست!)





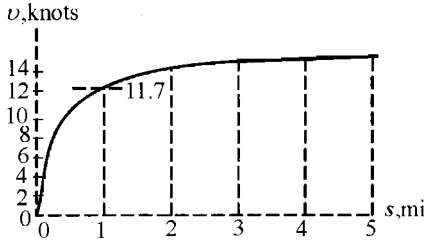
۲-۲۴۶. *

$$v\,dv = a\,ds \Rightarrow a = \frac{T - 4.50v^2}{m}$$

$$\int_0^v \frac{m\,v\,dv}{T - 4.50v^2} = \int_0^s ds \Rightarrow s = \frac{m}{9.00}\ln\frac{T}{T - 4.50v^2}$$

$$\Rightarrow e^{9.00\,s/m} = \frac{T}{T - 4.50\,v^2} \Rightarrow v = \sqrt{\frac{T}{4.50}(1 - e^{-9.00\,s/m})}$$

که در آن $T = 250\,\text{kN}$ و s = مسافت بر حسب متر و $m = 16000\,\text{tons}$ و v = سرعت برحسب m/s است. اگر v = سرعت برحسب گره دریایی (knots) باشد، s = مسافت برحسب مایل دریایی می‌باشد.



$$v = \sqrt{\frac{250}{4.50}\left(1 - e^{-9.00 \times 1852s/16000}\right)}\ \frac{3.6}{1.852} = 14.49\sqrt{1 - e^{-1.042s}}$$

$$v_{min} = 11.7 \text{ knots}$$

$$v_{max} = 14.49 \text{ knots}$$

* ۲۴۷-۲.

$$\omega_n = \sqrt{\frac{g}{l}} \text{ : فرض} \Rightarrow \begin{cases} \theta = \theta_0 \sin \omega_n t \\ \dot{\theta} = \theta_0 \omega_n \cos \omega_n t \\ \ddot{\theta} = -\theta_0 {\omega_n}^2 \sin \omega_n t \end{cases}$$

$$a_t = l\ddot{\theta} = -l\theta_0 {\omega_n}^2 \sin \omega_n t = -g\theta_0 \sin \omega_n t$$

$$a_n = l\dot{\theta}^2 = l\theta_0^2 {\omega_n}^2 \cos^2 \omega_n t = g\theta_0^2 \cos^2 \omega_n t$$

$$a = \sqrt{{a_t}^2 + {a_n}^2} = g\theta_0\sqrt{\sin^2 \omega_n t + \theta_0^2 \cos^4 \omega_n t}$$

$$\frac{da^2}{d\theta} = 0 = g^2\theta_0^2\left[2 \sin \omega_n t(\cos \omega_n t) + \theta_0^2 4 \cos^3 \omega_n t\,(-\sin \omega_n t)\right]$$

$$\Rightarrow \left[1 - 2\theta_0^2 \cos^2 \omega_n t\right] = 0$$

$$\left[1 - 2\left(\frac{\pi}{3}\right)^2 \cos^2 \omega_n t\right] = 0 \Rightarrow \omega_n t = 0.830 \text{ rad}$$

$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.81}{0.8}} = 3.50 \text{ s}^{-1} \Rightarrow t = 0.237 \text{ s}$$

$$\theta = \frac{\pi}{3} \sin (0.830) = 0.772 \text{ rad } (44.3°)$$

همانطور که از نمودارهای زیر مشاهده می‌شود، اعداد فوق شتاب کمینه را مشخص می‌کند:

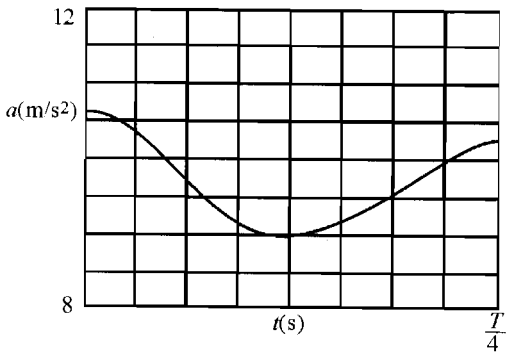
$$a_{min} = 9.03 \text{ m/s}^2 \quad @ \quad \theta = 44.3° \ , \ t = 0.237 \text{ s}$$

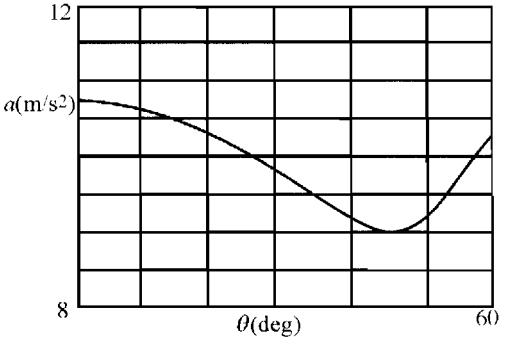
$$a_{max} = 10.76 \text{ m/s}^2 \quad @ \quad \theta = 0 \ , \ t = 0$$

توجه:

$$T = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.50} = 1.794 \text{ s}$$

$$\frac{T}{4} = 0.449 \text{ s}$$





۲۴۸-۲ *

$$a = v\frac{dv}{dy} = -g - kv^2$$

$$\int_{v_0}^{0} \frac{v\,dv}{-g-kv^2} = \int_0^h dy$$

فرض : $u = g + kv^2 \Rightarrow du = 2kv\,dv$

$$\int_{v=v_0}^{v=0} -\frac{1}{2k}\frac{du}{u} = h \Rightarrow h = -\frac{1}{2k}\ln(g+kv^2)\Big|_{v_0}^{0} = \frac{-1}{2k}\ln\left(\frac{g}{g+kv_0^2}\right)$$

$$\Rightarrow 2kh = \ln\left(1+\frac{kv_0^2}{g}\right)$$

$$\Rightarrow 2k(1600) = \ln\left(1+\frac{k(600)^2}{9.81}\right) \xrightarrow{\text{حل عددی}} k = 11.89(10^{-4})\ \text{m}^{-1}$$

فصل سوم

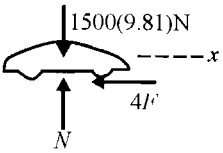
سینتیک ذرات

۳-۱.

$$v^2_2 - v^2_1 = 2a(x_2 - x_1)$$

$$0^2 - \left(\frac{100}{3.6}\right)^2 = 2a_x(50) \quad \Rightarrow \quad a_x = -7.72\ \text{m/s}^2$$

$$\Sigma F_x = m a_x \ : \ -4F = 1500(-7.72) \quad \Rightarrow \quad F = 2890\ \text{N}$$



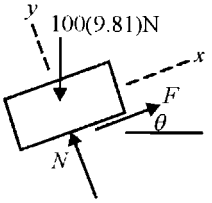
۳-۲.

$$\theta_{\max} = \tan^{-1}\mu_s = \tan^{-1}(0.30) = 16.70°$$

الف) $\theta = 15° \quad \Rightarrow$ بدون حرکت $\Rightarrow \quad a = 0$

ب) $\Sigma F_y = 0 \ : \ N = 100(9.81)\cos 20° = 921.8\ \text{N}$

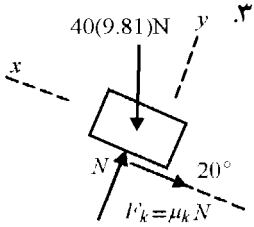
$$F = \mu_k N = 0.25(921.8) = 230.5\ \text{N}$$



$$\Sigma F_x = m a_x \ : \ -100(9.81)\sin 20° + 230.5 = 100a \quad \Rightarrow \quad a = -10.5\ \text{m/s}^2$$

(صندوق به طرف پائین شتاب می‌گیرد)

۳-۳.



$\Sigma F_y = 0 \quad : \quad N - 40(9.81)\cos 20° = 0 \quad \Rightarrow \quad N = 368.7\,\text{N}$

$\Sigma F_x = m a_x \quad : \quad -0.25(368.7) - 40(9.81)\sin 20° = 40\,a$

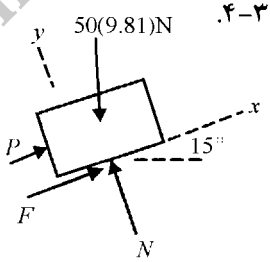
$\Rightarrow \quad a = -5.659\,\text{m/s}^2$

$v = v_0 + a t \quad : \quad 0 = +10 - 5.659\,t \quad \Rightarrow \quad t = 1.767\,\text{s}$

$v^2 = v_0^2 + 2a(s - s_0) \quad : \quad 0^2 = 10^2 + 2(-5.659)\,d \quad \Rightarrow \quad d = 8.83\,\text{m}$

$v^2 = v_0^2 + 2a(s - s_0) \quad : \quad 5^2 = 10^2 + 2(-5.659)\,d' \quad \Rightarrow \quad d' = 6.63\,\text{m}$

۴-۳.



$\Sigma F_y = 0 \quad : \quad N - 50(9.81)\cos 15° = 0 \quad \Rightarrow \quad N = 474\,\text{N}$

الف) $P = 0$:

بررسی تعادل : $\Sigma F_x = 0 \quad : \quad F - 50(9.81)\sin 15° = 0$

$\Rightarrow \quad F = 127.0\,\text{N}$

$F_{\max} = \mu_s N = 0.2(474) = 94.8\,\text{N} < F \Rightarrow$ حرکت ↙

$\Sigma F_x = m a_x \quad : \quad 0.15(474) - 50(9.81)\sin 15° = 50\,a_x \quad \Rightarrow \quad a_x = -1.118\,\text{m/s}^2$

ب) $P = 150\,\text{N}$:

بررسی تعادل : $\Sigma F_x = 0 \quad : \quad 150 + F - 50(9.81)\sin 15° = 0$

$\Rightarrow \quad F = -23.0\,\text{N} \quad , \quad |F| < F_{\max} \quad \Rightarrow \quad$ بدون حرکت , $a = 0$

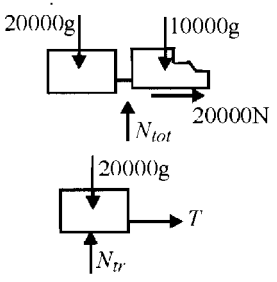
ج) $P = 300\,\text{N}$:

بررسی تعادل : $\Sigma F_x = 0 \quad : \quad 300 - F - 50(9.81)\sin 15° = 0$

$\Rightarrow \quad F = -173.0\,\text{N} \quad , \quad |F| > F_{\max} \quad \Rightarrow \quad$ حرکت ↗ $F = F_k$ ↙

$\Sigma F_x = m a_x \quad : \quad 300 - 0.15(474) - 50(9.81)\sin 15° = 50\,a_x \quad \Rightarrow \quad a_x = 2.04\,\text{m/s}^2$

۵-۳.



برای کل مجموعه : $\Sigma F = ma \quad : \quad 20000 = 30000\,a$

$\Rightarrow \quad a = 0.667\,\text{m/s}^2$

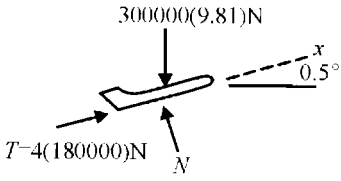
برای تریلی : $T = 20000(0.667) = 13330\,\text{N}$

یا $T = 13.33\,\text{kN}$

۳-۶.

$$\Sigma F_x = m a_x \; : \; 4(180000) - 300000\,(9.81)\,\sin\frac{1}{2}^\circ = 300000\,a_x$$

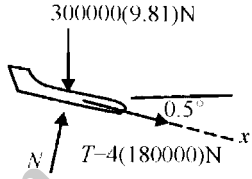
$$\Rightarrow \; a_x = 2.31\ \text{m/s}^2$$



$$v^2 = 2\,a_x s \; : \left(\frac{220}{3.6}\right)^2 = 2(2.31)\,s \quad \Rightarrow \quad s_u = 807\ \text{m}$$

$$\Sigma F_x = m a_x \; : 4(180000) + 300000(9.81)\,\sin\frac{1}{2}^\circ = 300000\,a_x$$

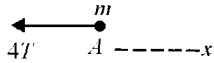
$$\Rightarrow \; a_x = 2.49\ \text{m/s}^2$$



$$v^2 = 2a_x s \; : \left(\frac{220}{3.6}\right)^2 = 2(2.49)\,s \quad \Rightarrow \quad s_d = 751\ \text{m}$$

۳-۷.

$$\overset{+}{\leftarrow} \Sigma F = ma \; : \; 4(200000) = 340000\,a \quad \Rightarrow \quad a = 2.35\ \text{m/s}^2$$



$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B = -2.35\,\mathbf{i} - 0 = -2.35\,\mathbf{i}\ \text{m/s}^2$$

$$v_A = (v_A)_0 + at = 0 + 2.35(10) = 23.5\ \text{m/s}$$

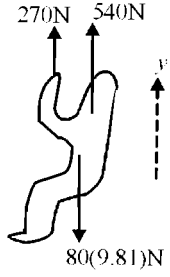
$$v_B = \frac{25}{3.6} = 6.94\ \text{m/s}$$

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B = -23.5\,\mathbf{i} - 6.94(\cos 30^\circ\,\mathbf{i} + \sin 30^\circ\,\mathbf{j}) = -29.5\,\mathbf{i} - 3.47\,\mathbf{j}\ \text{m/s}$$

۳-۸.

$$\Sigma F_y = ma_y \; : \; 270 + 540 - 80(9.81) = 80a$$

$$\Rightarrow \; a = 0.315\ \text{m/s}^2$$

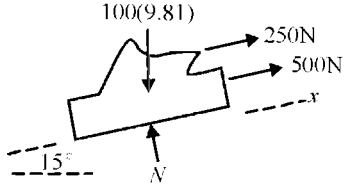


۳-۹.

$$\Sigma F_x = ma_x$$

$$250 + 500 - 100(9.81)\,\sin 15^\circ = 100\,a$$

$$\Rightarrow \; a = 4.96\ \text{m/s}^2$$

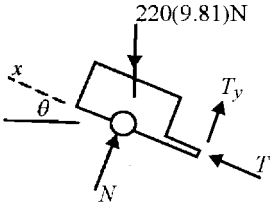


۳-۱۰.

$\nwarrow + \quad v^2 - v_0^2 = 2a(s - s_0)$

$0^2 - \left(\frac{8}{3.6}\right)^2 = 2a(-1.2) \Rightarrow a = 2.06\ \text{m/s}^2$

دیاگرام آزاد گاری (به عنوان یک ذره رفتار می‌کند) (استاتیک)



$(\theta = \tan^{-1}\frac{4}{12} = 18.43°)$

$\Sigma F_x = 0 \quad : \quad T_s - 220(9.81)\sin 18.43° = 0$

$\Rightarrow T_s = 682.3\ \text{N}$

$\Sigma F_x = m a_x \quad : \quad T - 220(9.81)\sin 18.43° = 220(2.06)$

$\Rightarrow T = 1135.5\ \text{N}$

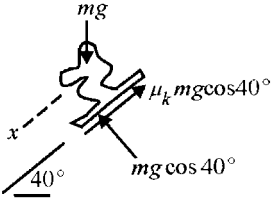
درصد افزایش : $n = \frac{1135.5 - 682.3}{682.3}(100\%) = 66\%$

۳-۱۱.

$\Sigma F_x = m a_x \quad : \quad mg\sin 40° - \mu_k mg\cos 40° = ma$

$\Rightarrow a = 9.81(\sin 40° - \mu_k \cos 40°)$

$= 6.31 - 7.51\mu_k$



برای شتاب ثابت : $s = v_0 t + \frac{1}{2} a t^2$

$20 = 0 + \frac{1}{2}(6.31 - 7.51\mu_k)\ 2.58^2$

$\mu_k = 0.0395$

۳-۱۲.

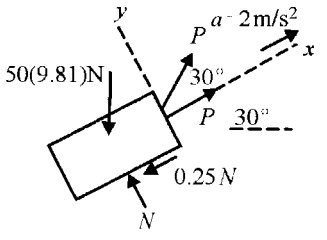
$\Sigma F_x = m a_x$

$P(1 + \cos 30°) - 0.25\ \text{N} - 50(9.81)\sin 30° = 50(2)$

$\Sigma F_y = 0$

$N + P\sin 30° - 50(9.81)\cos 30° = 0$

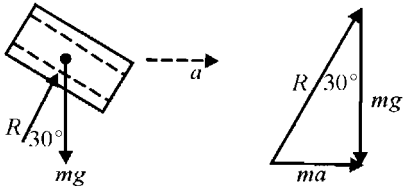
$\Rightarrow \begin{cases} 1.866P - 0.25N = 345.25 \\ 0.5P + N = 424.78 \end{cases} \Rightarrow \begin{matrix} P = 227\ \text{N} \\ N = 311.3\ \text{N} \end{matrix}$



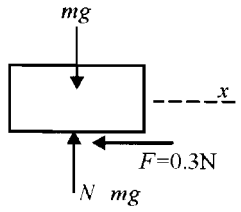
۳-۱۳.

$\Sigma \mathbf{F} = m\mathbf{a} \quad :$

$ma = mg\tan 30° \Rightarrow a = g\tan 30° = 5.66\ \text{m/s}^2$



۳-۱۴. فرض کنید m جرم صندوق باشد.

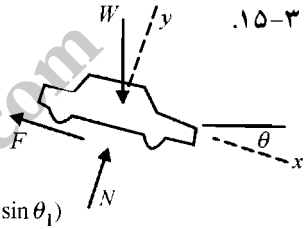


$$\Sigma F_x = m a_x \quad : \quad -0.3\,mg = m a_x$$

$$\Rightarrow \quad a_x = -0.3\,g = -0.3\,(9.81) = -2.94\ \text{m/s}^2$$

$$\int_v^0 v\,dv = \int_0^s a_x\,dx \quad \Rightarrow \quad -\frac{v^2}{2} = a_x s \quad \Rightarrow \quad s = \frac{-\dfrac{\left(-\dfrac{70}{3.6}\right)^2}{2}}{-2.94} = 64.3\ \text{m}$$

۳-۱۵.



وقتی $\theta = \theta_1$, $a = 0$

$$\Sigma F_x = 0 = -F + W\sin\theta_1 \quad \Rightarrow \quad F = W\sin\theta_1$$

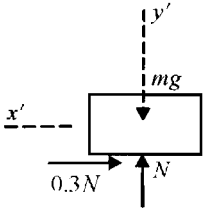
وقتی $\theta = \theta_2$

$$\Sigma F_x = ma \quad : W\sin\theta_2 - W\sin\theta_1 = \frac{W}{g}a \quad \Rightarrow \quad a = g(\sin\theta_2 - \sin\theta_1)$$

$$\left.\begin{array}{l}\theta_1 = 6^\circ \\ \theta_2 = 2^\circ\end{array}\right\} \quad a = g(\sin 2^\circ - \sin 6^\circ) = -0.0696\,g$$

۳-۱۶.

B تا A :

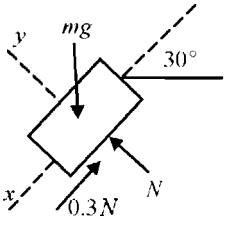


$$\Sigma F_y = 0 \quad \Rightarrow \quad N = 0.866\,mg$$

$$\Sigma F_x = m a_x \ :$$

$$mg\sin 30^\circ - 0.3(0.866\,mg) = ma$$

$$\Rightarrow \quad a_x = 2.36\ \text{m/s}^2$$



$$v_B^2 = v_A^2 + 2a_x d \ : \ v_B^2 = 0.8^2 + 2(2.36)(2) \quad \Rightarrow \quad v_B = 3.17\ \text{m/s}$$

C تا B :

$$\Sigma F_{y'} = 0 \quad \Rightarrow \quad N = mg$$

$$\Sigma F_{x'} = m a_{x'} \ : \ -0.3\,(mg) = m a_{x'} \quad \Rightarrow \quad a_{x'} = -2.94\ \text{m/s}^2$$

$$v_C^2 = v_B^2 + 2a_{x'}s \ : \ 0 = 3.17^2 - 2(2.94)s \quad \Rightarrow \quad s = 1.710\ \text{m}$$

۳-۱۷.

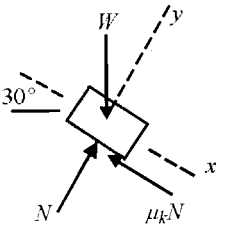
$$\Sigma F_y = 0 \;:\; N = W\cos 30°$$

$$\Sigma F_x = m a_x \;:\; W\sin 30° - \mu_k W\cos 30° = \frac{W}{g} a_x$$

$$\Rightarrow\; a_x = 9.81\left(\frac{1}{2} - \mu_k \frac{\sqrt{3}}{2}\right)$$

$$v_2^2 - v_1^2 = 2a(x_2 - x_1) \;:\; 0.9^2 - 0.4^2 = 2(9.81)\left(\frac{1}{2} - \mu_k \frac{\sqrt{3}}{2}\right)2$$

$$\Rightarrow\; \mu_k = 0.558$$

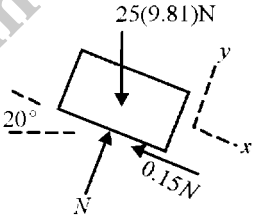


۳-۱۸. قاب و گوی بعنوان یک مجموعه:

$$\Sigma F_y = 0 \;:\; N - 25(9.81)\cos 20° = 0 \;\Rightarrow\; N = 230\ \text{N}$$

$$\Sigma F_x = m a_x \;:\; 25(9.81)\sin 20° - 0.15(230) = 25a$$

$$\Rightarrow\; a = 1.973\ \text{m/s}^2$$



گوی به تنهایی:

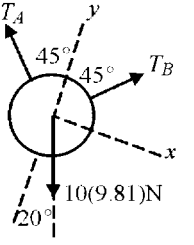
$$\Sigma F_y = 0 \;:\; (T_A + T_B)\cos 45° - 10(9.81)\cos 20° = 0$$

$$\Rightarrow\; T_A + T_B = 130.4\ \text{N} \qquad (1)$$

$$\Sigma F_x = m a_x \;:\; (T_B - T_A)\sin 45° + 98.1\sin 20° = 10(1.973)$$

$$\Rightarrow\; T_B - T_A = 19.56\ \text{N} \qquad (2)$$

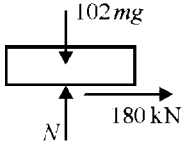
حل (1) و (2): $T_A = 75.0\ \text{N}$, $T_B = 55.4\ \text{N}$



۳-۱۹. فرض کنید m جرم هر واگن و $2m$ جرم لوکوموتیو باشد.

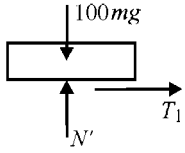
$$\xrightarrow{+} \Sigma F = ma \;:\; 180000 = 102(90000)\,a$$

$$\Rightarrow\; a = 0.0196\ \text{m/s}^2$$



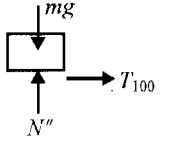
$$\xrightarrow{+} \Sigma F = ma \;:\; T_1 = 100(90000)(0.0196)$$

$$\Rightarrow\; T_1 = 176470\ \text{N} \quad \text{یا} \quad 176.5\ \text{kN}$$

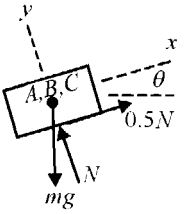


$$\xrightarrow{+} \Sigma F = ma \;:\; T_{100} = 1(90000)(0.0196)$$

$$\Rightarrow\; T_{100} = 1765\ \text{N}$$



۳-۲۰.

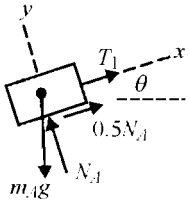


مجموعهٔ واگن :
$$\begin{cases} \Sigma F_y = 0 \Rightarrow N = mg\cos\theta \\ \Sigma F_x = m a_x : 0.5 mg\cos\theta - mg\sin\theta = ma \end{cases}$$

$$\Rightarrow a = g(0.5\cos\theta - \sin\theta) = 4.41\ \text{m/s}^2$$

$$\theta = \tan^{-1}\left(\frac{5}{100}\right) = 2.86°$$

واگن A : $\Sigma F_y = 0 : N_A = m_A g\cos\theta$



$$\Sigma F_x = m a_x : T_1 + 0.5 m_A g\cos\theta - m_A g\sin\theta = m_A(4.41)$$

$$\Rightarrow T_1 = 0$$

طبق دیاگرام آزاد واگن C : $T_2 = 0$

با تحلیل مشابه :

	ب	ج	د
a	2.78 m/s^2	2.78 m/s^2	2.78 m/s^2
T_1	32700 N	16330 N	16330 N
T_2	16330 N	16330 N	32700 N

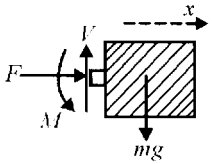
۳-۲۱.

$$x = X\sin\omega t$$

$$\dot{x} = X\omega\cos\omega t$$

$$\ddot{x} = -X\omega^2\sin\omega t \quad , \quad \ddot{x}_{max} = X\omega^2$$

دیاگرام آزاد مدار چاپی:



$$\Sigma F_x = m a_x$$

$$F = m(-X\omega^2\sin\omega t)$$

$$F_{max} = mX\omega^2$$

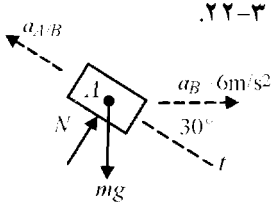
۳-۲۲.

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

$$(a_A)_t = 6\cos 30° - a_{A/B}$$

$$\Sigma F_t = m a_t \Rightarrow mg\sin 30° = m(6\cos 30° - a_{A/B})$$

$$\Rightarrow a_{A/B} = 0.291\ \text{m/s}^2$$

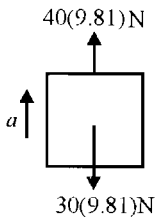


۳-۲۳.

الف) $\Sigma F = ma$

$40(9.81) - 30(9.81) = 30\,a$

$a = 3.27\ \text{m/s}^2$

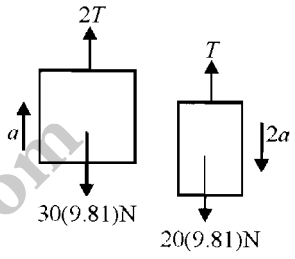


ب) $\Sigma F = ma$

30 kg وزنه : $2T - 30(9.81) = 30\,a$

20 kg وزنه : $20(9.81) - T = 20(2\,a)$

$\Rightarrow$ $T = 160.5\,\text{N}$, $a = 0.892\ \text{m/s}^2$

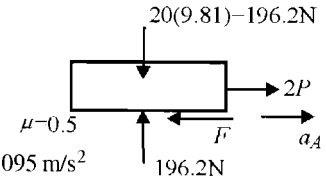


۳-۲۴.

الف) $2P = 120\,\text{N}$

$F_{\max} = 0.5(196.2) = 98.1\,\text{N} < 2P$

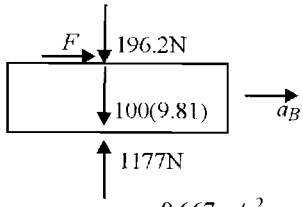
فرض می‌کنیم لغزش اتفاق بیفتد و $F = 98.1\,\text{N}$



قطعه A : $\Sigma F = m\,a$: $120 - 98.1 = 20\,a_A$ $\Rightarrow$ $a_A = 1.095\ \text{m/s}^2$

گاری B : $\Sigma F = m\,a$: $98.1 = 100\,a_B$ $\Rightarrow$ $a_B = 0.981\ \text{m/s}^2$

$a_A > a_B$ $\Rightarrow$ فرض صحیح است.



ب) $2P = 80\,\text{N} < F_{\max}$ $\Rightarrow$ هیچ لغزشی رخ نمی‌دهد

برای مجموعهٔ گاری و قطعه : $\Sigma F = ma$ $\Rightarrow$ $80 = 120\,a$ $\Rightarrow$ $a_A = a_B = a = 0.667\ \text{m/s}^2$

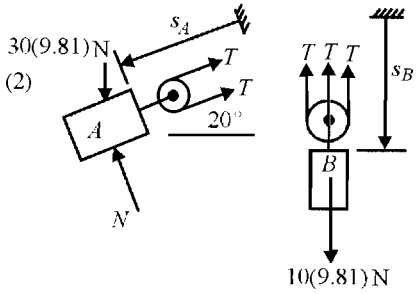
۳-۲۵.

قیود سینماتیک : $L = 2s_A + 3s_B$

$\Rightarrow 0 = 2a_A + 3a_B$ (1)

$\nearrow \Sigma F = m_A a_A$: $30(9.81)\sin 20° - 2T = 30\,a_A$ (2)

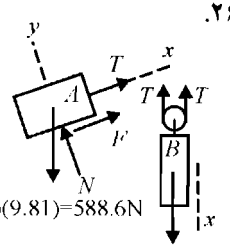
$+\downarrow \Sigma F = m_B a_B$: $10(9.81) - 3T = 10\,a_B$ (3)



حل معادلات (1) تا (3) :

$$\begin{cases} a_A = 1.024\ \text{m/s}^2 \\ a_B = -0.682\ \text{m/s}^2 \\ T = 35.0\,\text{N} \end{cases}$$

۳-۲۶. بررسی امکان حرکت با فرض تعادل استاتیکی :



$B : 2T = 196.2 \Rightarrow T = 98.1\ \text{N}$

$A : \Sigma F_x = 0 : 9.81 - 588.6 \sin 30 + F = 0 \Rightarrow F = 196.2\ \text{N}$

$F_{\max} = \mu_s N = (0.25)(588.6)\cos 30° = 127.4\ \text{N}$

$F > F_{\max} \Rightarrow$ حرکت ↙

از سینماتیک : $a_A = 2a_B = 2a$

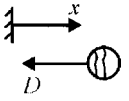
$A : \Sigma F_x = m a_x : T + 0.2(588.6\cos 30°) - 588.6 \sin 30° = 60(2a)$

$B : \Sigma F_x = m a_x : -2T + 196.2 = 20a$

$\Rightarrow a = -0.725\ \text{m/s}^2 \quad , \quad T = 105.4\ \text{N}$

۳-۲۷. فعلاً از وزن صرفنظر کنید.

$$\Sigma F_x = m a_x : -D = -C_D \frac{1}{2}\rho v^2 s = m v \frac{dv}{dx}$$



$$\Rightarrow \int_0^x \left(-C_D \frac{1}{2}\rho s\right)dx = m\int_{v_0}^{v}\frac{dv}{v} \Rightarrow v = v_0 e^{\left(-\frac{1}{2}(C_D \rho s x/m)\right)}$$

$$\Rightarrow v = v_0 e^{\left(-\frac{1}{2}(0.3)(1.2063)\pi\left(\frac{232}{2\pi}\right)^2 x/146\right)} = v_0 e^{-5.31(10^{-3})x}$$

$v_0 = 150\ \text{km/h} \quad , \quad x = 18\ \text{m} \Rightarrow v = 136.3\ \text{km/h}$

توضیح حرکت در جهت y: فرض کنید $v = 150\ \text{k m/h}$ ثابت باشد. زمان رسیدن توپ به هوم پلیت:

$$t = \frac{18}{\frac{150}{3.6}} = 0.432\ \text{s}$$

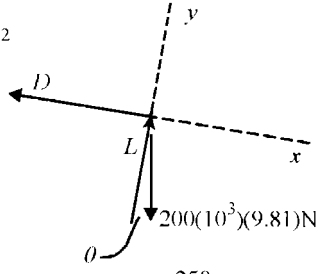
$$v_y = v_{y_0} - gt = -9.81(0.432) = -4.24\ \text{m/s}$$

که نمی‌تواند تغییر قابل ملاحظه‌ای روی $v = \sqrt{v_x^2 + v_y^2}$ داشته باشد.

۳-۲۸.

$$v_B^2 - v_A^2 = 2a_x(s_B - s_A)$$

$$\left(\frac{200}{3.6}\right)^2 - \left(\frac{300}{3.6}\right)^2 = 2a_x\left(\frac{2000}{\cos 7.13°}\right) \Rightarrow a_x = -0.957\ \text{m/s}^2$$



$$\theta = \tan^{-1}\frac{250}{2000} = 7.13°$$

$$\Sigma F_x = m a_x \;:\; -D + 200(10^3)(9.81)\sin 7.13^\circ = 200(10^3)(-0.957)$$

$$\Rightarrow D = 435\,\text{kN}$$

$$\Sigma F_y = 0 \;:\; L - 200(10^3)(9.81)\cos 7.13^\circ = 0 \;\Rightarrow\; L = 1.947\,\text{MN}$$

نیروی ایرودینامیک خالص : $R = \sqrt{L^2 + D^2} = \sqrt{1.947^2 + 0.435^2} = 1.995\,\text{MN}$

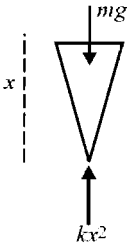
۲۹-۳. دیاگرام آزاد مخروط در طی نفوذ

$$\Sigma F_x = m a_x \;:\; mg - kx^2 = mv\frac{dv}{dx}$$

$$\int_0^d \left(g - \frac{k}{m}x^2\right)dx = \int_{v_0}^0 v\,dv \;\Rightarrow\; gd - \frac{k}{3m}d^3 = -\frac{v_0^2}{2}$$

که در آن $v_0 = \sqrt{2gh}$

$$\Rightarrow k = \frac{3mg}{d^3}(h + d)$$



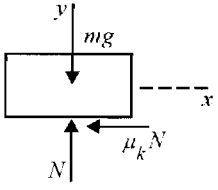
۳۰-۳.

به طرف پائین : $\Sigma F_y = m a_y \;:\; -mg + N = -ma$

$$\Rightarrow N = m(g - a)$$

$$\Sigma F_x = m a_x \;:\; -\mu_k m(g - a) = m a_x$$

$$\Rightarrow a_x = -\mu_k(g - a)$$



به طرف بالا : $\Sigma F_y = m a_y \;:\; N - mg = ma \;\Rightarrow\; N = m(g + a)$

$$\Sigma F_x = m a_x \;:\; -\mu_K m(g + a) = m a_x \;\Rightarrow\; a_x = -\mu_K(g + a)$$

پائین : $v^2 = v_0^2 + 2as \;:\; 0 + v^2 - 2\mu_k(g - a)s_1$

بالا : $0 = v^2 - 2\mu_k(g + a)s_2$

$$\Rightarrow (g - a)s_1 = (g + a)s_2 \;\Rightarrow\; a = g\frac{s_1 - s_2}{s_1 + s_2}$$

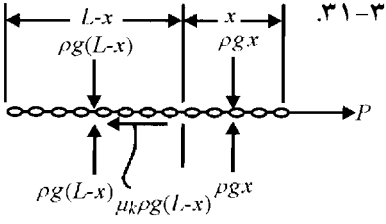
۳۱-۳.

$$\Sigma F_x = m a_x \;:\; P - \mu_k \rho g(L - x) = \rho L \ddot{x}$$

$$\int v\,dv = \int \ddot{x}\,dx$$

$$\Rightarrow \int_0^v v\,dv = \int_0^l \left(\frac{P}{\rho L} - \mu_k g + \mu_k g\frac{x}{L}\right)dx$$

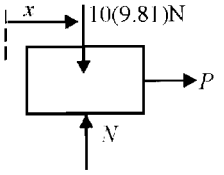
$$\Rightarrow \frac{v^2}{2} = \left(\frac{P}{\rho L} - \mu_k g\right)L + \frac{\mu_k g}{2L}L^2 \;\Rightarrow\; v = \sqrt{\frac{2P}{\rho} - \mu_k gL}$$



۳-۳۲.

$$\Sigma F_x = m a_x \;:\; P = 10 a_x \;\Rightarrow\; \frac{P}{10} = \frac{dv}{dt}$$

$$\Rightarrow\; v = \int_0^t \frac{P}{10}\, dt$$



$$P_1 = 10t \quad \text{برای} \;:\; v = \frac{t^2}{2} \;,\; s = \frac{t^3}{6}$$

$$t = 5\,\text{s} \quad \text{در} \;:\; v = 12.5\,\text{m/s} \;,\; s = 20.8\,\text{m}$$

$$P = k t^2 \quad \text{برای} \;:\; 50 = k(5)^2 \;\Rightarrow\; k = 2\,\text{N/s}^2 \;\Rightarrow\; P = 2t^2$$

$$v = \int_0^t \frac{2t^2}{10}\, dt = \frac{t^3}{15} \;,\; s = \frac{t^4}{60}$$

$$t = 5\,\text{s} \quad \text{در} \;:\; v = 8.33\,\text{m/s} \;,\; s = 10.42\,\text{m}$$

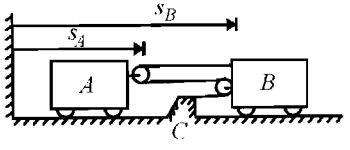
۳-۳۳.

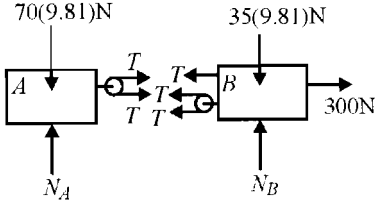
$$L = 2(s_B - s_A) + (s_B - s_C) + \text{const.}$$

$$0 = 3a_B - 2a_A \qquad (1)$$

$$\xrightarrow{+} \Sigma F = ma \;:\; (A)\quad 2T = 70\, a_A \qquad (2)$$

$$(B)\quad 300 - 3T = 35\, a_B \qquad (3)$$





حل معادلات (1) تا (3) :

$$\begin{cases} a_A = 2.34\,\text{m/s}^2 \\ a_B = 1.558\,\text{m/s}^2 \\ T = 81.8\,\text{N} \end{cases}$$

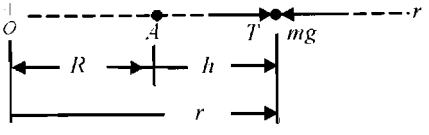
۳-۳۴.

$$g = g_0 \frac{R^2}{r^2}$$

$$\Sigma F_r = m a_r \;:\; T - m g_0 \frac{R^2}{r^2} = m v \frac{dv}{dr}$$

$$\Rightarrow \int_R^{2R} \left(\frac{T}{m} - g_0 \frac{R^2}{r^2}\right) dr = \int_0^v v\, dv$$

$$\Rightarrow v = \sqrt{\frac{2TR}{m} - g_0 R} = \sqrt{R\left(\frac{2T}{m} - g_0\right)}$$

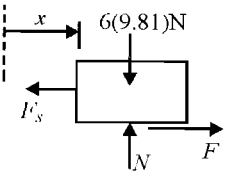


جای‌گذاری اعداد :

$$v = \sqrt{\frac{3476(1000)}{2}\left(\frac{2(2500)}{1200} - 1.62\right)} = 2100\,\text{m/s}$$

۳-۳۵.

$F_s = 150x + 400x^2$ (N)



$+\uparrow \Sigma F = 0 \Rightarrow N = 58.9\,\text{N}$

$F_{\max} = \mu_s N = 0.30(58.9) = 17.66\,\text{N}$

الف) $x = 50\,\text{mm} : F_s = 150(0.050) + 400(0.050)^2 = 8.5\,\text{N} < F_{\max} \Rightarrow a = 0$

ب) $x = 100\,\text{mm} : F_s = 150(0.1) + 400(0.1)^2 = 19\,\text{N} > F_{\max}$

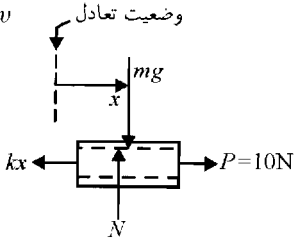
$\Sigma F_x = m a_x : -19 + 0.25(58.9) = 6a \Rightarrow a = -0.714\,\text{m/s}^2$

۳-۳۶.

$\Sigma F_x = m\ddot{x} : P - kx = m\ddot{x}$

$10 - 200x = 2\ddot{x} \Rightarrow \ddot{x} = 5 - 100x$

$\Rightarrow 5 - 100x = v\frac{dv}{dx} \Rightarrow \int_0^{0.040} (5 - 100x)\,dx = \int_0^v v\,dv$



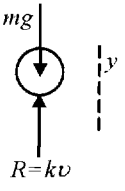
$\Rightarrow 5x - 50x^2 \Big|_0^{0.040} = \frac{v^2}{2}\Big|_0^v \Rightarrow v = 0.490\,\text{m/s}$

$5x - 50x^2 = 0 \Rightarrow \begin{cases} x = 0 \\ x = 0.10\,\text{m} \quad \text{یا} \quad x = 100\,\text{mm} \end{cases}$ (شرایط اولیه)

۳-۳۷.

$\Sigma F_y = m a_y : mg - kv = ma \Rightarrow a = g - \frac{k}{m}v$

$v\,dv = a\,dy \Rightarrow \int_0^v \frac{v\,dv}{g - \frac{k}{m}v} = \int_0^h dy$



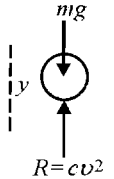
$\Rightarrow \frac{m^2}{k^2}\left[\left(g - \frac{k}{m}v\right) - g\ln\left(g - \frac{k}{m}v\right)\right]_0^v = h$

$\Rightarrow h = \frac{m^2}{k^2}\left[-\frac{k}{m}v - g\ln\left(1 - \frac{kv}{mg}\right)\right]$

$\Rightarrow h = \frac{m^2}{k^2} g \ln\left(\frac{1}{1 - \frac{kv}{mg}}\right) - \frac{mv}{k}$

۳-۳۸.

$\Sigma F_y = m a_y : mg - cv^2 = ma \Rightarrow a = g - \frac{c}{m}v^2$



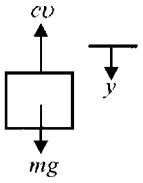
$v\,dv = a\,dy \Rightarrow \int_0^v \frac{v\,dv}{g - \frac{c}{m}v^2} = \int_0^h dy$

$\Rightarrow -\frac{m}{2c}\ln\left(g - \frac{c}{m}v^2\right)\Big|_0^v = h \Rightarrow h = \frac{m}{2c}\ln\left(\frac{mg}{mg - cv^2}\right)$

۳-۳۹.

$$\Sigma F_y = m a_y \;:\; mg - cv = m a_y$$

الف) $a_y = 0 \Rightarrow mg - cv = 0 \Rightarrow v_s = \dfrac{mg}{c} = \dfrac{100(9.81)}{3000} = 0.327\ \text{m/s}$



ب) $mg - cv = m\dfrac{dv}{dt} \Rightarrow \int_0^t dt = \int_0^v \dfrac{dv}{g - \frac{c}{m}v} = -\dfrac{m}{c}\int_0^v \dfrac{-\frac{c}{m}dv}{g - \frac{c}{m}v}$

$$\Rightarrow t = -\frac{m}{c}\ln\left(g - \frac{c}{m}v\right)_0^v = -\frac{m}{c}\ln\left(\frac{g - \frac{c}{m}v}{g}\right)$$

$$\Rightarrow v = \frac{mg}{c}\left[1 - e^{-ct/m}\right] = 0.327\left[1 - e^{-30t}\right]$$

$$\left[1 - e^{-30t}\right] = 0.9 \Rightarrow t = 0.0768\ \text{s}$$

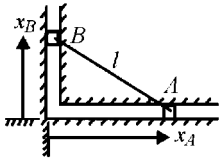
$$v = \frac{dy}{dt} = 0.327\left[1 - e^{-30t}\right] \Rightarrow \int_0^y dy = \int_0^t 0.327\left[1 - e^{-30t}\right]dt$$

$$\Rightarrow y = 0.327\left[t + \frac{1}{30}(e^{-30t} - 1)\right]$$

$$t = 0.0768\ \text{s} \Rightarrow y = 0.01529\ \text{m}$$

۳-۴۰.

$$x_A^2 + x_B^2 = l^2$$



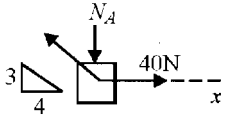
$$2x_A\dot{x}_A + 2x_B\dot{x}_B = 0$$

$$x_A\ddot{x}_A + \dot{x}_A^2 + x_B\ddot{x}_B + \dot{x}_B^2 = 0$$

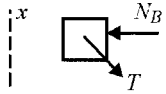
$$\Rightarrow \dot{x}_B = -\frac{x_A\dot{x}_A}{x_B} = \frac{-(0.4)(0.9)}{0.3} = -1.2\ \text{m/s}$$

$$\ddot{x}_B = \frac{-\dot{x}_B^2 - \dot{x}_A^2 - x_A\ddot{x}_A}{x_B} = \frac{-1.2^2 - 0.9^2 - 0.4\ddot{x}_A}{0.3} = -7.5 - \frac{4}{3}\ddot{x}_A$$

یا $a_B = -7.5 - \dfrac{4}{3}a_A \quad (1)$

A: 

$$\Sigma F_x = m a_x \;:\; 40 - \frac{4}{5}T = 2a_A \qquad (2)$$

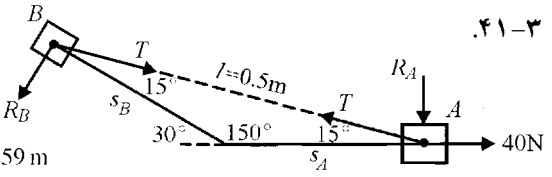
B: 

$$\Sigma F_x = m a_x \;:\; -\frac{3}{5}T = 3a_B \qquad (3)$$

حل معادلات (1) تا (3) :

$$\begin{cases} a_A = 1.364\ \text{m/s}^2 \\ a_B = -9.32\ \text{m/s}^2 \\ T = 46.6\ \text{N} \end{cases}$$

۳-۴۱.



$$\sin\frac{150^\circ}{l} = \sin\frac{15^\circ}{s_B} \quad , \quad s_B = s_A = 0.259\,\text{m}$$

قانون کسینوسها : $l^2 = s_A^2 + s_B^2 - 2 s_A s_B \cos 150^\circ$

$$2l\dot{l} = 0 = 2 s_A v_A + 2 s_B v_B - 2\left(-\frac{\sqrt{3}}{2}\right)(s_A v_B + s_B v_A)$$

$$s_A v_A + s_B v_B + \frac{\sqrt{3}}{2}(s_A v_B + s_A v_B) = 0$$

با جای‌گذاری $s_A = s_B = 0.259\,\text{m}$ و $v_A = 0.4\,\text{m/s}$ و $v_B = -0.4\,\text{m/s}$ و انتگرال‌گیری از رابطهٔ فوق خواهیم داشت:

$$v_A^2 + s_A a_A + v_B^2 + s_B a_B + \frac{\sqrt{3}}{2}(s_A a_B + v_A v_B + a_A s_B + v_A v_B) = 0$$

$$0.483\,a_A + 0.483\,a_B + 0.0429 = 0 \qquad (1)$$

سینتیک : $\nwarrow^{+} \Sigma F = m a_B : -T\cos 15^\circ = 3 a_B \qquad (2)$

$\rightarrow^{+} \Sigma F = m a_A : 40 - T\cos 15^\circ = 2 a_A \qquad (3)$

حل معادلات (1) تا (3) : $T = 25.0\,\text{N}$, $a_A = 7.95\,\text{m/s}^2$, $a_B = -8.04\,\text{m/s}^2$

۳-۴۲.

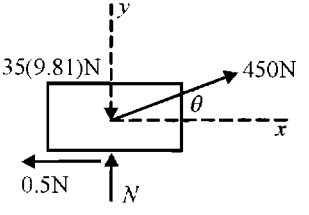
$\Sigma F_x = m a_x$: $450\cos\theta - 0.5\,\text{N} = 35(a)$

$\Sigma F_y = 0$: $N - 35(9.81) + 450\sin\theta = 0$

حذف N : $2\cos\theta + \sin\theta = 2.163$

با قرار دادن $\sin\theta = \sqrt{1 - \cos^2\theta}$ خواهیم داشت:

$$5\cos^2\theta - 8.652\cos\theta + 3.678 = 0 \Rightarrow \theta = 11.88^\circ\,,\,41.3^\circ$$



حرکت قطعه را در دو حالت بررسی می‌کنیم:

$\theta = 11.88^\circ$:

$N = 35(9.81) - 450\sin 11.88^\circ = 250.7\,\text{N}$

$F_{\max} = 0.6(250.7) = 150.4\,\text{N} < 450\cos 11.88 = 440.4\,\text{N}$ ✓

$\theta = 41.3^\circ$:

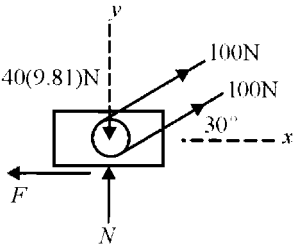
$N = 46.35\,\text{N}$

$F_{\max} = 0.6(46.35) = 27.81\,\text{N} < 450\cos 41.3^\circ = 338\,\text{N}$ ✓

۳-۴۳.

$$\Sigma F_y = 0 \;:\; N + 200\sin 30° - 40(9.81) = 0$$

$$N = 292\,\text{N}$$



با فرض تعادل استاتیکی : $\Sigma F_x = 0 \;:\; -F + 200\cos 30° = 0$

$$F = 173.2\,\text{N}$$

$$F_{\max} = \mu_s N = 0.5(292) = 146.2\,\text{N} < F$$

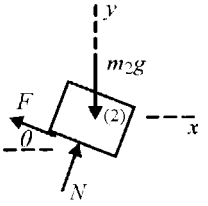
بنابراین فرض غلط است و قطعه حرکت می‌کند.

$$F = \mu_k N = 0.4(292) = 117.0\,\text{N}$$

$$\Sigma F_x = m\,a_x \;:\; -117 + 200\cos 30° = 40\,a_x \;\Rightarrow\; a_x = a = 1.406\,\text{m/s}_2$$

۳-۴۴.

$$\begin{cases} \Sigma F_x = m\,a_x \;:\; -F\cos\theta + N\sin\theta = m_2 a \\ \Sigma F_y = 0 \;:\; F\sin\theta + N\cos\theta - m_2 g = 0 \end{cases}$$



$$\Rightarrow \begin{cases} F = m_2(g\sin\theta - a\cos\theta) \\ N = m_2(a\sin\theta + g\cos\theta) \end{cases}$$

(فرض: شروع لغزش به سمت پائین)

در آستانهٔ لغزش : $F = \mu_s N$

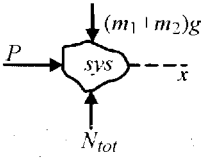
$$\Rightarrow\; m^2(g\sin\theta - a\cos\theta) = \mu_s m_2(a\sin\theta + g\cos\theta)$$

$$\Rightarrow\; a = g\,\frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}$$

جای‌گذاری اعداد : $a = 0.0577\,g$ $\qquad (\theta = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.70°$:نکته)

فرض کنید لغزش رو به بالای سطح شیبدار شروع شود (نیروی F در دیاگرام آزاد فوق برعکس باشد):

$$a = g\,\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = 0.745\,g$$



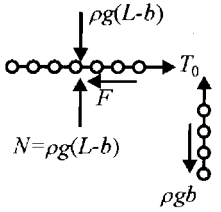
$$\Sigma F_x = m\,a_x \;:\; P = (m_1 + m_2)a$$

$$\Rightarrow\; 0.0577(m_1 + m_2)\,g \le P \le 0.745(m_1 + m_2)\,g$$

◀ ۳-۴۵. فرض کنید $\rho = \frac{\text{جرم}}{\text{طول}}$

$$F = \mu N = \mu g \rho (L - b)$$

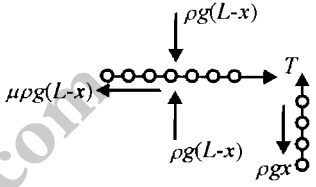
$$\left.\begin{aligned} \Sigma F = 0 \;:\; T_0 - \mu g \rho (L - b) = 0 \\ T_0 = \rho g b \end{aligned}\right\} \Rightarrow b = \frac{\mu L}{1 + \mu}$$



$$\Sigma F = ma \;:\; T - \mu \rho g (L - x) = \rho (L - x) a$$

$$\rho g x - T = \rho x a$$

حذف T $\Rightarrow$ $a = \ddot{x} = \frac{g}{L}\left[x(1 + \mu) - \mu L\right]$



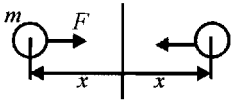
$$v\,dv = \ddot{x}\,dx : \int_0^v v\,dv = \int_b^L \frac{g}{L}\left[x(1 + \mu) - \mu L\right] dx$$

$$\Rightarrow \frac{1}{2} v^2 = \frac{g}{L}\left[\frac{x^2}{2}(1 + \mu) - \mu L x\right]_b^L$$

با جای‌گذاری $b = \frac{\mu L}{1 + \mu}$ و ساده‌سازی : $v = \sqrt{\frac{gL}{1 + \mu}}$

◀ ۳-۴۶.

$$F = \frac{Gm^2}{x^2}$$



$$m = \rho V = 7210\left(\frac{4}{3}\pi\, 0.05^3\right) = 3.775\,\text{kg}$$

$$\Sigma F_x = m a_x : -\frac{Gm^2}{(2x)^2} = m v \frac{dv}{dx} \Rightarrow -\frac{Gm}{4}\int_{x_0 = 0.5}^{x} \frac{dx}{x^2} = \int_{v_0 = 0}^{v} v\,dv$$

$$\Rightarrow v = \sqrt{Gm}\sqrt{\frac{1}{2x} - 1} = \sqrt{(6.673 \times 10^{-11})(3.775)}\sqrt{\frac{1}{2(0.05)} - 1} = 4.76 \times 10^{-5}\ \text{m/s}$$

$$\frac{dx}{dt} = -\sqrt{Gm}\sqrt{\frac{\frac{1}{2} - x}{x}} \Rightarrow \int_{x_0 = 0.5}^{x = 0.05} \frac{\sqrt{x}\,dx}{\sqrt{\frac{1}{2} - x}} = -\sqrt{Gm}\int_0^t dt$$

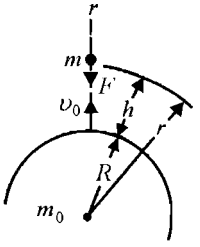
$$\Rightarrow \left[-\sqrt{x}\sqrt{\frac{1}{2} - x} + \frac{1}{2}\sin^{-1}\sqrt{2x}\right]_{x_0 = 0.5}^{x = 0.05} = -\sqrt{Gm}\,t \Rightarrow t = 48800\,\text{s} \quad \text{یا} \quad t = 13\,\text{hr}\ 33\,\text{min}$$

◀ ۳-۴۷.

$$\left.\begin{aligned} F = G\frac{mm_0}{r^2} \\ r = R \quad \text{وقتی} \;:\; F = mg \end{aligned}\right\} \Rightarrow Gm_0 = gR^2$$

$$\Rightarrow F = mg\frac{R^2}{r^2}$$

(که در آن g شتاب جاذبه زمین در سطح زمین است)



$$\Sigma F_r = m a_r \;:\; -mg\frac{R^2}{r^2} = ma_r \Rightarrow a_r = -g\frac{R^2}{r^2}$$

$$v\,dv = a_r\,dr \;:\; \int_{v_0}^{0} v\,dv = \int_{R}^{\infty} -g\frac{R^2}{r^2}\,dr$$

$$\Rightarrow -\frac{v_0^2}{2} = gR^2\frac{1}{r}\Big|_R^\infty = 0 - gR$$

طبق جدول ۱-۵: $\begin{cases} R = 6.371(10^6)\,\text{m} \\ g = 9.825\,\text{m/s}^2 \end{cases}$

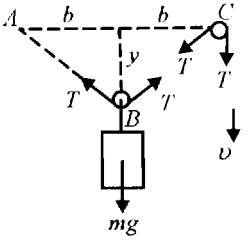
$$\Rightarrow v_0^2 = 2gR \Rightarrow v_0 = \sqrt{2gR} = \sqrt{2(8.25)(6.371(10^6))} = 11190\,\text{m/s} \quad \text{یا} \quad v_0 = 11.19\,\text{km/s}$$

(در جدول د-۲ مقدار 11.18 km/s است.)

◀ ۳-۴۸.

$$+\uparrow \Sigma F = ma \;:\; 2T\frac{y}{\sqrt{b^2+y^2}} - mg = ma \quad (a = -\ddot{y})$$

$$\Rightarrow T = \frac{m(a+g)\sqrt{b^2+y^2}}{2y} \qquad a = f(v, y) \text{ که در آن}$$



فرض کنید L طول کابل ABC یعنی $L = 2\sqrt{b^2+y^2}$ باشد:

$$-v = \dot{L} = 2\frac{y\dot{y}}{\sqrt{b^2+y^2}}, \quad \ddot{L} = 2\frac{\sqrt{b^2+y^2}\,(\dot{y}^2 + y\ddot{y})}{b^2+y^2} - 2\frac{y\dot{y}(y\dot{y})}{(b^2+y^2)\sqrt{b^2+y^2}} = 0$$

$$\Rightarrow \sqrt{b^2+y^2}\left(\frac{v^2(b^2+y^2)}{4y^2} + y\ddot{y}\right) = \frac{y^2}{\sqrt{b^2+y^2}}\frac{v^2(b^2+y^2)}{4y^2}$$

ساده سازی : $\ddot{y} - \frac{b^2v^2}{4y^3} = -a$

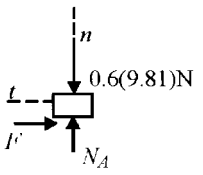
$$\Rightarrow T = \frac{m\left(g+\frac{b^2v^2}{4y^3}\right)\sqrt{b^2+y^2}}{2y} \Rightarrow T = \frac{m}{2y}\sqrt{b^2+y^2}\left(g+\frac{b^2v^2}{4y^3}\right)$$

۳-۴۹.

$$\Sigma F_n = m\frac{v^2}{\rho}$$

$$N_A - 0.6(9.81) = 0.6\frac{5^2}{3}$$

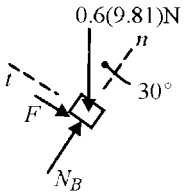
$$N_A = 10.89\,\text{N}$$



$$\Sigma F_n = m\frac{v^2}{\rho}$$

$$N_B - 0.6(9.81)\cos 30^\circ = 0.6\frac{4^2}{3}$$

$$N_B = 8.30\,\text{N}$$



: B

(نکته: اصطکاک در راستای محور t بوده و در محاسبات فوق اثری ندارد)

۳-۵۰

$$\mathbf{r} = 8t^2\mathbf{i} + 3t^3\mathbf{j} - \frac{1}{2}t^4\mathbf{k}\ \text{m}$$

$$\dot{r} = 16t\mathbf{i} + 9t^2\mathbf{j} - 2t^3\mathbf{k}\ \text{m/s}$$

$$\ddot{r} = 16\mathbf{i} + 18t\mathbf{j} - 6t^2\mathbf{k}\ \text{m/s}^2$$

$$\ddot{r}_{t=2\text{s}} = 16\mathbf{i} + 36\mathbf{j} - 24\mathbf{k}\ \text{m/s}^2$$

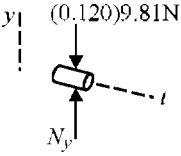
$$a = |\ddot{\mathbf{r}}| = \sqrt{16^2 + 36^2 + 24^2} = 46.1\ \text{m/s}^2$$

$$\Sigma F = ma \quad \Rightarrow \quad F = 1.5(46.1) = 69.2\,\text{N}$$

۳-۵۱

الف) $\Sigma F_y = 0 \quad : \quad N_y - (0.120)(9.81) = 0$

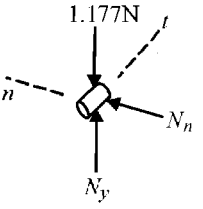
$$N_y = R = 1.177\,\text{N}$$



ب) $N_y = 1.177\,\text{N}$ (مشابه قسمت الف)

$$\Sigma F_n = m\frac{v^2}{\rho} \quad : \quad N_n = 0.120\frac{1.4^2}{0.200} = 1.176\,\text{N}$$

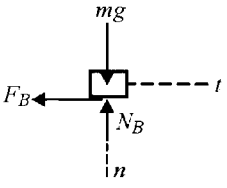
$$R = \sqrt{N_y^2 + N_n^2} = 1.664\,\text{N}$$



۳-۵۲

$$\Sigma F_n = m\,a_n = m\frac{v^2}{\rho}$$

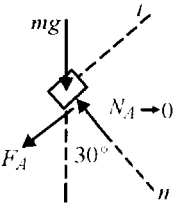
$$2(9.81) - N = 2\frac{3.5^2}{2.4} \quad \Rightarrow \quad N_B = 9.41\,\text{N}$$



$N_A \to 0$: قطع تماس در نقطه A

$$\Sigma F_n = m a_n = m \frac{v^2}{\rho}$$

$$mg \cos 30° = m \frac{v^2}{2.4} \Rightarrow v = 4.52 \text{ m/s}$$



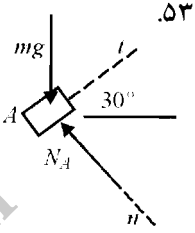
۳-۵۳.

$$\Sigma F_n = m a_n : -N_A + mg \cos 30° = m \frac{v_A^2}{\rho}$$

$$N_A = m \left(g \cos 30° - \frac{v_A^2}{\rho} \right) = 2 \left(9.81 \cos 30° - \frac{4.5^2}{2.4} \right)$$

$$N_A = 0.1164 \text{ N}$$

$$\Sigma F_t = m a_t : -mg \sin 30° = m a_t \Rightarrow a_t = -\frac{g}{2} = -4.90 \text{ m/s}^2$$

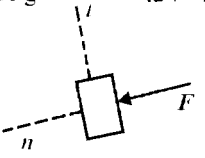


۳-۵۴.

$$a_n = \frac{v^2}{\rho} = \frac{\left[\frac{55}{3.6}\right]^2}{30} = 7.78 \text{ m/s}^2 \qquad \left(\frac{1 \text{ g}}{9.81 \text{ m/s}^2} \right) = 0.793 \text{ g}$$

$$\Sigma F_n = m a_n : F = 1400(7.78) = 10892 \text{ N} \text{ یا } 10.89 \text{ kN}$$

(به طور متوسط 2.72 N بر هر چرخ)

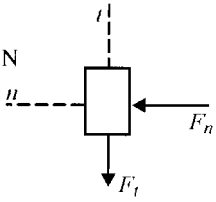


۳-۵۵.

$$\Sigma F_n = m a_n : F_n = 1400 \frac{\left[\frac{40}{3.6}\right]^2}{30} = 5761.3 \text{ N}$$

$$\sqrt{F_n^2 + F_t^2} = F_{tot} \Rightarrow 5761.3^2 + F_t^2 = 10600^2 \Rightarrow F_t = 8897.6 \text{ N}$$

$$\Sigma F_t = m a_t : -8897.6 = 1400 a_t \Rightarrow a_t = -6.36 \text{ m/s}^2$$



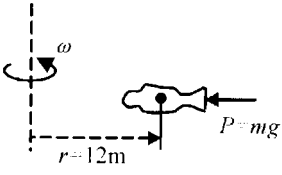
۳-۵۶.

g = شتاب جاذبه زمین روی سطح زمین

$$\Sigma F_n = m a_n \Rightarrow mg = m r \omega^2 \Rightarrow \omega = \sqrt{\frac{g}{r}}$$

$$\omega = \sqrt{\frac{9.81}{12}} = 0.904 \text{ rad/s}$$

$$N = 0.904 \left(\frac{60}{2\pi} \right) = 8.63 \text{ rpm}$$



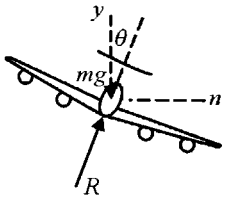
۳-۵۷.

$$\Sigma F_y = 0 \quad : \quad R\cos\theta - mg = 0 \Rightarrow R\cos\theta = mg$$

$$\Sigma F_n = m a_n \quad : \quad R\sin\theta = m\frac{v^2}{\rho}$$

از ترکیب دو رابطه فوق : $\tan\theta = \dfrac{v^2}{\rho g}$

$$\Rightarrow \quad \theta = \tan^{-1}\frac{v^2}{\rho g} = \tan^{-1}\frac{\left(\frac{600}{3.6}\right)^2}{3000\times 9.81} = 43.3^\circ$$



۳-۵۸.

$$\Sigma F_n = m a_n \quad :$$

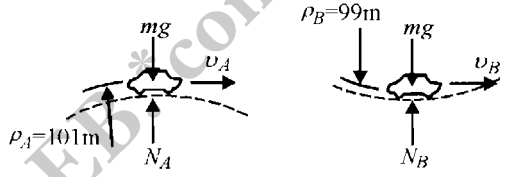
$$A: \quad mg - N_A = m\frac{{v_A}^2}{\rho_A}$$

$$B: \quad N_B - mg = m\frac{{v_B}^2}{{\rho_B}^2}$$

$$N_B = 2N_A \quad \Rightarrow \quad m\left(\frac{{v_B}^2}{{\rho_B}^2} + g\right) = 2m\left(g - \frac{{v_A}^2}{\rho_A}\right)$$

$$\Rightarrow \quad {v_B}^2 = \rho_B g - 2{v_A}^2\frac{\rho_B}{\rho_B} = 99(9.81) - 2\left(\frac{60}{3.6}\right)^2\frac{99}{101} = 427\ \frac{\text{m}^2}{\text{s}^2}$$

$$\Rightarrow \quad v_B = 20.7\ \text{m/s} \quad \text{یا} \quad v_B = 74.4\ \text{km/h}$$



۳-۵۹.

$$a_n = g \quad , \quad a_n = \frac{v^2}{\rho} = v\dot{\theta} \Rightarrow \dot{\theta} = \frac{a_n}{v} = \frac{g}{v}$$

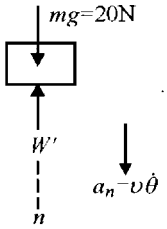
$$\dot{\theta} = \frac{9.81}{\frac{600}{3.6}} = 5.89(10^{-2})\ \text{rad/s} \quad \text{یا} \quad \dot{\theta} = 3.37\ \text{deg/s}$$

۳-۶۰.

$$W = mg \quad : \quad m = \frac{20}{9.81} = 2.04\ \text{kg}$$

$$\Sigma F_n = ma_n = 20 - W' = 2.04\left(\frac{800}{3.6}\cdot\frac{(1)\pi}{180}\right) = 7.91\ \text{N}$$

$$\Rightarrow \quad W' = 20 - 7.91 = 12.09\ \text{N}$$

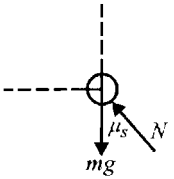


۶۱-۳.

$$\Sigma F_y = 0 \;:\; N\frac{\sqrt{2}}{2} - mg = 0 \quad \Rightarrow \quad N = \frac{2}{\sqrt{2}}mg$$

$$\Sigma F_n = m a_n \;:\; N\frac{\sqrt{2}}{2} = m\left(3R + R\frac{\sqrt{2}}{2}\right)\Omega^2$$

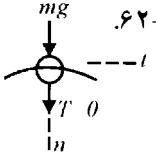
$$\Rightarrow \frac{2}{\sqrt{2}}mg\left(\frac{\sqrt{2}}{2}\right) = mR\left(3 + \frac{\sqrt{2}}{2}\right)\Omega^2$$



$$R = 0.200\,\text{m} \quad \Rightarrow \quad \Omega = 3.64\,\text{rad/s}$$

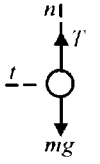
۶۲-۳.

(1) $$\Sigma F_n = m\frac{v^2}{\rho} \;:\; mg = m\frac{v^2}{1} \quad \Rightarrow \quad v = \sqrt{g} = 3.13\,\text{m/s}$$



(2) $$\Sigma F_n = m\frac{v^2}{\rho} \;:\; T - mg = m\frac{g}{1} \quad \Rightarrow \quad T = 2mg = 2(0.050)(9.81)$$

$$\Rightarrow \quad T = 0.981\,\text{N}$$

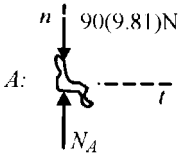


۶۳-۳.

$$\Sigma F_n = m a_n$$

$$N_A - 90(9.81) = 90\frac{\left(\frac{600}{3.6}\right)^2}{1000}$$

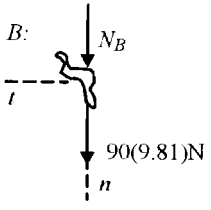
$$N_A = 3380\,\text{N}$$



$$\Sigma F_n = m a_n \;:$$

$$N_B + 90(9.81) = 90\frac{\left(\frac{600}{3.6}\right)^2}{1000}$$

$$N_B = 1617\,\text{N}$$

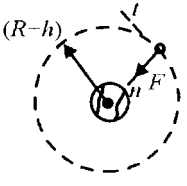


(توجه: در شرایط استاتیکی عادی: $mg = 90(9.81) = 883\,\text{N}$)

۶۴-۳.

$$\Sigma F_n = m a_n \;:\; F = \frac{G m_e m}{(R+h)^2} = m\frac{v^2}{(R+h)}$$

$$v = \frac{s}{t} = \frac{2\pi(R+h)}{(23.9344)(3600)}$$



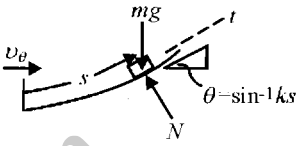
ترکیب دو معادله فوق : $v = \dfrac{2\pi(R+h)}{(23.9344)(3600)} = \sqrt{\dfrac{G m_e}{(R+h)}} \Rightarrow h = 3.580\times 10^7 \text{ m}$

یا $h = 35800 \text{ km}$

۳-۶۵.

$\Sigma F_t = m a_t \; : \; -mg\sin\theta = m a_t$

$\Rightarrow a_t = -g\sin\theta = -gks$



$v\,dv = a_t\,ds \Rightarrow \int_{v_0}^{0} v\,dv = -gk\int_0^s s\,ds$

$\Rightarrow -\dfrac{1}{2}v_0^2 = -gk\dfrac{s^2}{2} \Rightarrow s = \dfrac{v_0}{\sqrt{kg}}$

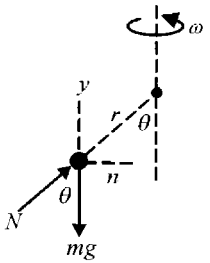
۳-۶۶.

$\Sigma F_y = 0 \; : \; N\cos\theta - mg = 0 \Rightarrow N = \dfrac{mg}{\cos\theta}$

$\Sigma F_n = m a_n \; : \; N\sin\theta = m(r\sin\theta)\omega^2$

$\left(\dfrac{mg}{\cos\theta}\right)\sin\theta = m r\sin\theta\,\omega^2$

$\Rightarrow \omega = \sqrt{\dfrac{g}{r\cos\theta}}$



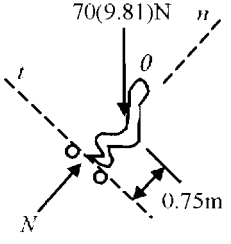
توجه داشته باشید که $\cos\theta = \dfrac{g}{r\omega^2} \le 1$ است بنابراین $\omega^2 \ge \dfrac{g}{r}$ یک محدودیت است.

۳-۶۷.

$\rho = 4.5 - 0.75 = 3.75 \text{ m}$

$\Sigma F_n = m a_n \; : \; N - 70(9.81)\cos\theta = 70\dfrac{v^2}{3.75}$

$\Rightarrow N = 70\left(9.81\cos\theta + \dfrac{v^2}{3.75}\right)$



$\theta = 0 \; : \; N_0 = 70\left(9.81 + \dfrac{8.5^2}{3.75}\right) = 20.40 \text{ N}$

$\theta = 45° \; : \; N_{45°} = 70\left(9.81 + \dfrac{\sqrt{2}}{2} + \dfrac{6^2}{3.75}\right) = 1158 \text{ N}$

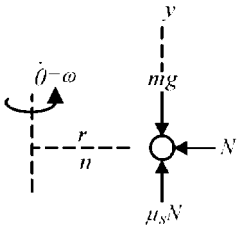
$\theta = 90° \; , \; N_{90°} = 0$

۳-۶۸.

$$\Sigma F_n = m a_n \Rightarrow N = m r\omega^2$$

$$\Sigma F_y = 0 \Rightarrow \mu_s(m r\omega^2) = m g$$

$$\Rightarrow \omega^2 = \frac{g}{\mu_s r} \Rightarrow \omega = \sqrt{\frac{g}{\mu_s r}}$$



۳-۶۹.

$$\Sigma F_t = m a_t :$$

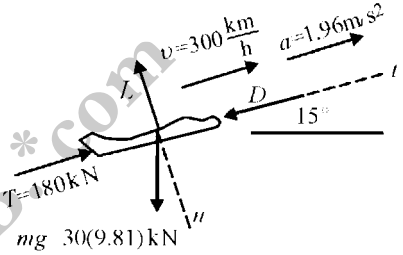
$$180000 - 30000(9.81)\sin 150° - D = 30000(1.96)$$

$$\Rightarrow D = 45000\,\text{N} \quad \text{یا} \quad D = 45.0\,\text{kN}$$

$$\Sigma F_n = m a_n :$$

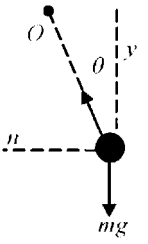
$$30000(9.81)\cos 15° - L = 30000\,\frac{\left(\frac{300}{3.6}\right)^2}{20000}$$

$$\Rightarrow L = 274000\,\text{N} \quad \text{یا} \quad L = 274\,\text{kN}$$



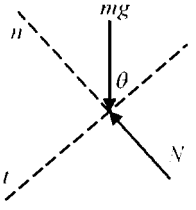
۳-۷۰. آونگ را در وسیله نقلیه طوری نصب کنید که قاب آونگ عمودی بوده و در عرض وسیله نقلیه باشد.

$$\begin{cases} \Sigma F_y = 0 : T\cos\theta - mg = 0 \quad (1) \\ \Sigma F_n = m\dfrac{v^2}{\rho} : T\sin\theta = m\dfrac{v^2}{r} \quad (2) \end{cases} \Rightarrow v = \sqrt{g r \tan\theta}$$



۳-۷۱. کودک را بعنوان یک ذره در نظر بگیرید.

$$\begin{cases} \Sigma F_t = m a_t : mg\cos\theta = m a_t \quad (1) \\ \Sigma F_n = m a_n : N - mg\sin\theta = m\dfrac{v^2}{R} \quad (2) \end{cases}$$



(1) از رابطه : $g\cos\theta = v\dfrac{dv}{ds} = v\dfrac{dv}{R\,d\theta} \Rightarrow \int_{\theta_0=20}^{\theta} R\,g\cos\theta\,d\theta = \int_{v_0=0}^{v} v\,dv$

$$\Rightarrow v = \left[2Rg(\sin\theta - \sin 20^\circ)\right]^{\frac{1}{2}}$$

(2) از رابطه : $N = m\left(g\sin\theta + \dfrac{v^2}{R}\right)$

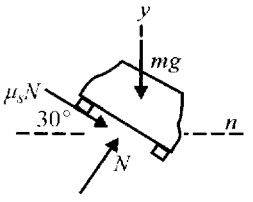
جای‌گذاری اعداد $g = 9.81\ \text{m/s}^2$ و $R = 2.5\ \text{m}$

$\theta = 30^\circ$:
$a_t = 8.50\ \text{m/s}^2$
$v = 2.78\ \text{m/s}$
$N = 280\ \text{N}$

$\theta = 90^\circ$:
$a_t = 0$
$v = 5.65\ \text{m/s}$
$N = 795\ \text{N}$

۳-۷۲. برای عدم تمایل به لغزش باید $F_{\max} = \mu_s N$ را در دیاگرام آزاد معادل صفر در نظر بگیریم.

$$\begin{cases} \Sigma F_y = 0 : N\cos 30^\circ - mg = 0 \\ \Sigma F_n = m\dfrac{v^2}{\rho} : N\sin 30^\circ = m\dfrac{v^2}{350} \end{cases} \Rightarrow \begin{array}{l} N = 1.155\,mg \\ v = 44.5\ \text{m/s} \\ \text{یا}\quad v = 160.3\ \text{km/h} \end{array}$$



$v_{\min} = 0 \quad : \theta_{\max} = \tan^{-1}\mu_s = \tan^{-1}(0.9) = 42.0^\circ > 30^\circ$

برای $v_{\max}$، از $F_{\max}$ نشان داده شده در دیاگرام آزاد استفاده می‌کنیم.

$$\begin{cases} \Sigma F_y = 0 : N\cos 30^\circ - mg - \mu_s N\sin 30^\circ = 0 \\ \Sigma F_n = m\dfrac{v^2}{\rho} : \mu_s N\cos 30^\circ + N\sin 30^\circ = m\dfrac{v_{\max}{}^2}{\rho} \end{cases}$$

$\mu_s = 0.9 \Rightarrow N = 2.40\,mg$ ، $v = 102.78\ \text{m/s}$ یا $370\ \text{km/h}$

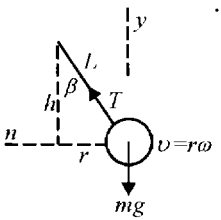
۳-۷۳.

$\Sigma F_y = 0 : T\cos\beta - mg = 0 \Rightarrow T\cos\beta = mg$

$\Sigma F_n = m\,a_n : T\sin\beta = m\dfrac{v^2}{r}$

تقسیم دو رابطه بر هم : $\tan\beta = \dfrac{v^2}{gr} = \dfrac{r\omega^2}{g}$

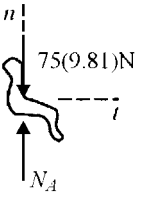
$r = L\sin\beta \Rightarrow \tan\beta = L\omega^2\dfrac{\sin\beta}{g} \Rightarrow L\cos\beta = \dfrac{g}{\omega^2}$



$h = L\cos\beta \Rightarrow h = \frac{g}{\omega^2}$ (فقط به ω و g بستگی دارد)

$$\Rightarrow T = \frac{mg}{\cos\beta} = \frac{mg}{\frac{h}{L}} = \frac{mgL}{\frac{g}{\omega^2}} = mL\omega^2$$

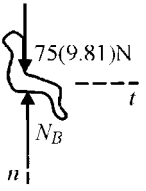
۳-۷۴.



A نقطه : $\Sigma F_n = m a_n$

$$N_A - 75(9.81) = 75\frac{22^2}{40}$$

$$N_A = 1643\,\text{N}$$



B نقطه : $\Sigma F_n = m a_n$

$$75(9.81) - N_B = 75\frac{12^2}{20}$$

$$N_B = 195.8\,\text{N}$$

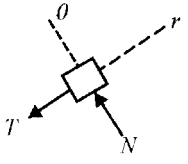
(توجه کنید که مقدار نیروی عمود در حالت استاتیکی $N = mg = 75(9.81) = 736\,\text{N}$ می‌باشد)

۳-۷۵.

$\Sigma F_r = m a_r = m(\ddot{r} - r\dot{\theta}^2)$

$-T = 2(0 - 0.225(6^2))$

$T = 16.20\,\text{N}$



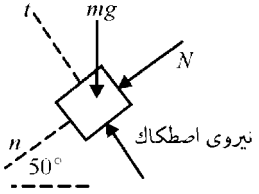
$\Sigma F_\theta = m a_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

$N = 2\left[0.225(-2) + 2(-0.050)(6^2)\right] = -8.1\,\text{N}$ (B تماس در سمت)

۳-۷۶.

$\Sigma F_n = m a_n$: $mg\sin 50° = m r\Omega^2$

$$\Omega = \sqrt{\frac{g\sin 50°}{r}} = \sqrt{\frac{9.81\sin 50°}{0.330}} = 4.77\,\text{rad/s} \quad (45.6\,\text{rpm})$$

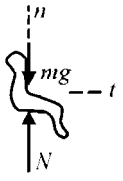


۳-۷۷.

$\theta = \frac{\pi}{3}\sin 0.950t$

$\dot{\theta} = \frac{\pi}{3}(0.950)\cos 0.950t$

$\Rightarrow \dot{\theta}_{max} = \frac{\pi}{3}(0.950) = 0.995\,\text{rad/s}$ (θ=0 وقتی)



$$\Sigma F_n = m a_n \;:\; N - mg = m r\dot{\theta}^2$$

$$N = mg + m(11)(0.995)^2 = 20.7\,m$$ (وقتی m برحسب kg باشد، N برحسب نیوتن است)

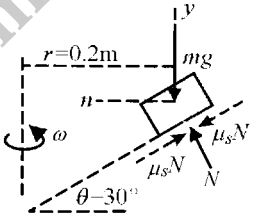
به سرنشینان نزدیک به وسط بیشترین نیروی عمودی وارد می‌شود و به سرنشینان دو انتهای وسیله کمترین نیرو وقتیکه θ در مقدار حداکثر (یا حداقل) باشد.

۷۸-۳. برای $\omega_{\max}$، $\mu_s N$ به سمت پائین است.

برای $\omega_{\min}$، $\mu_s N$ به سمت بالا است.

$$\Sigma F_y = 0 \;:\; N\cos\theta \mp \mu_s N\sin\theta = mg$$

$$\Sigma F_n = m a_n \;:\; N\sin\theta \pm \mu_s N\cos\theta = m r\omega^2$$



علامت بالایی برای $\omega_{\max}$ و علامت پائینی برای $\omega_{\min}$ می‌باشد.

ترکیب دو رابطه : $$\frac{\sin\theta \pm \mu_s\cos\theta}{\cos\theta \mp \mu_s\sin\theta} = \frac{r\omega^2}{g}$$

$$\Rightarrow\ \omega = \sqrt{\frac{g}{r}}\sqrt{\frac{\sin\theta \pm \mu_s\cos\theta}{\cos\theta \mp \mu_s\sin\theta}} = \sqrt{\frac{9.81}{0.2}}\sqrt{\frac{0.5 \pm 0.3(0.866)}{0.866 \mp 0.3(0.5)}}$$

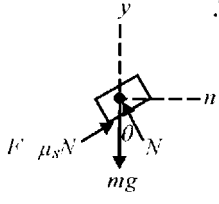
علامت بالایی $\omega_{\max} = 7.21$ rad/s

علامت پائینی $\omega_{\min} = 3.41$ rad/s

۷۹-۳.

$$\begin{cases}\Sigma F_y = 0 \;:\; N\cos\theta - mg + \mu_s N\sin\theta = 0\\ \Sigma F_n = m a_n \;:\; -N\sin\theta + \mu_s N\cos\theta = m r\omega^2\end{cases}$$

$$\Rightarrow\ \omega = \sqrt{\frac{g}{r}\frac{(\mu_s\cos\theta - \sin\theta)}{(\cos\theta + \mu_s\sin\theta)}} = 2.73\ \text{rad/s}$$

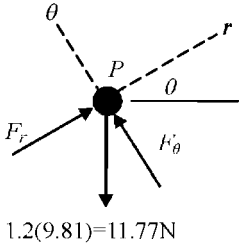


۸۰-۳.

$\theta = 30°$ $\qquad r = 1.25$ m

$\dot{\theta} = 40\left(\frac{\pi}{180}\right) = 0.698$ rad/s $\qquad \dot{r} = 0.4$ m/s

$\ddot{\theta} = 120\left(\frac{\pi}{180}\right) = 2.09$ rad/s^2 $\qquad \ddot{r} = -0.3$ m/s^2



1.2(9.81)=11.77N

$\Sigma F_r = m a_r$: $F_r - 11.77 \sin 30° = 1.2[-0.3 - 1.25(0.698)^2]$ $\Rightarrow$ $F_r = 4.79\,\text{N}$

$\Sigma F_\theta = m a_\theta$: $F_\theta - 11.77 \cos 30° = 1.2[1.25(2.09) + 2(0.4)(0.698)]$ $\Rightarrow$ $F_\theta = 14.00\,\text{N}$

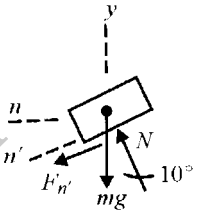
در حالت تعادل استاتیکی $a_r = a_\theta = 0$ خواهد بود بنابراین:

$(F_r)_{st} = 5.89\,\text{N}$

$(F_\theta)_{st} = 10.19\,\text{N}$

۳-۸۱.

صندوق :
$$\begin{cases} \Sigma F_y = 0 : N\cos 10° - F_{n'}\sin 10° - mg = 0 \\ \Sigma F_n = m a_n : F_{n'}\cos 10° + N\sin 10° = m\dfrac{(2t)^2}{30} \\ \Sigma F_t = m a_t : F_t = m(2) \end{cases}$$



(F_t و t به سمت داخل کاغذ)

حل دو معادله اول : $F_{n'} = m\left[\dfrac{4t^2\cos 10°}{30} - g\sin 10°\right]$

$N = m\left[\dfrac{4t^2\sin 10°}{30} + g\cos 10°\right]$

شرایط لغزش : $\sqrt{F_t^2 + F_{n'}^2} = \mu_s N$

$$\sqrt{2^2 + \left(\frac{4t^2\cos 10°}{30} - g\sin 10°\right)^2} = 0.3\left[\frac{4t^2\sin 10°}{30} + g\cos 10°\right] \Rightarrow t = 5.58\,\text{s}$$

۳-۸۲. مسافت طی شده از A تا C برابر است با:

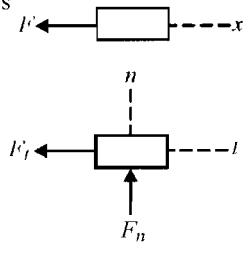
$$(s_C - s_A) = 32 + 80\left(30\frac{\pi}{180}\right) = 73.89\,\text{m}$$

شتاب مماسی یکنواخت : $v_C^2 = v_A^2 + 2a_t(s_C - s_A)$ $\Rightarrow$ $0^2 = \left(\dfrac{100}{3.6}\right)^2 + 2a_t(73.89)$

$\Rightarrow$ $a_t = -5.22\,\text{m/s}^2$

سرعت در نقطه B : $v_B^2 = v_A^2 + 2a_t(s_B - s_A)$ $\Rightarrow$ $v_B^2 = \left(\dfrac{100}{3.6}\right)^2 + 2(-5.22)(32)$

$\Rightarrow$ $v_B = 20.9\,\text{m/s}$



(الف) $\Sigma F_x = m a_x$: $-F = 1500(-5.22)$ $\Rightarrow$ $F = 7830\,\text{N}$

(ب) $\Sigma F_t = m a_t$: $-F_t = 1500(-5.22)$ $\Rightarrow$ $F_t = 7830\,\text{N}$

$\Sigma F_n = m\dfrac{v^2}{\rho}$: $F_n = 1500\dfrac{(20.9)^2}{80} = 8190\,\text{N}$

$F = \sqrt{F_t^2 + F_n^2} = 11331\,\text{N}$

(ج) $v \Rightarrow F = F_t = 7830\,\text{N}$ و در نتیجه F_n به سمت صفر می روند

(در تمام دیاگرام‌های آزاد، یک نیروی وزن عمود بر کاغذ و یک نیروی عمود استاتیکی به طرف خارج صفحه وجود دارد)

۸۳-۳

$a_t = 0 \quad , \quad a_n = r\omega^2$

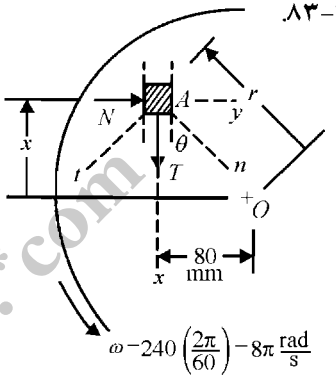
$\Sigma F_x = m a_x \quad : \quad (400)(x - 0.025) = 0.5\, r(8\pi)^2 \cos\theta$

$= 0.5\,(8\pi)^2 x$

$\Rightarrow \quad x = 0.1188\,\text{m} \quad$ یا $\quad x = 118.8\,\text{mm}$

$\Sigma F_y = m a_y \quad : \quad N = 0.5\, r(8\pi)^2 \sin\theta = 0.5\,(8\pi)^2(0.080)$

$= 25.3\,\text{N}$



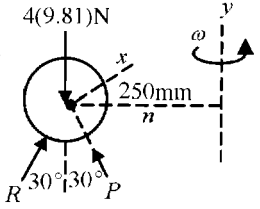
۸۴-۳

$\omega = 30 \times \dfrac{2\pi}{60} = \pi\ \text{rad/s}$

حل ۱ :

$\Sigma F_y = 0 \quad : \quad (R + P)\cos 30° = 4(9.81)$

$\Sigma F_n = m a_n \quad : \quad (R - P)\sin 30° = 4(0.250)\pi^2 \Rightarrow R = 32.5\,\text{N}$



حل ۲ : (یک معادله نیرو)

$\Sigma F_x = m a_x \quad : \quad R\cos 30° - 4(9.81)\sin 30° = 4(0.250)\pi^2 \cos 30° \quad \Rightarrow \quad R = 32.5\,\text{N}$

۸۵-۳

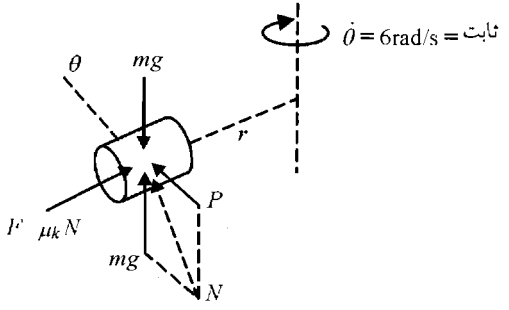
$mg = 0.8(9.81) = 7.85\,\text{N}$

$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

$P = 0.8(0 + 2(0.8)6) = 7.68\,\text{N}$

$N = \sqrt{7.68^2 + 7.85^2} = 10.98\,\text{N}$

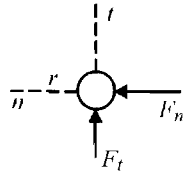
$F = \mu_k N = 0.4(10.98) = 4.39\,\text{N}$



(توجه: نیروی فنرها در دیاگرام آزاد نشان داده نشده است)

۸۶-۳

$$\left.\begin{aligned}\Sigma F_t &= m a_t : \; F_t = m r\alpha \\ \Sigma F_n &= m a_n \; : \; F_n = m r\omega^2 \\ F_{\max} &= \mu_s N = \mu_s mg\end{aligned}\right\} \Rightarrow \; m r \sqrt{\alpha^2 + \omega^2} = \mu_s mg$$



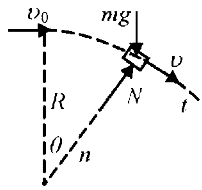
$$\Rightarrow \; \omega^2 = \frac{1}{r}\sqrt{\mu_s^2 g^2 - r^2\alpha^2}$$

برای α ثابت : $\omega^2 = 2\alpha\theta = 2\alpha(2\pi N)$

تعداد دور : $N = \dfrac{\omega^2}{4\pi\alpha} = \dfrac{1}{4\pi}\sqrt{\left(\dfrac{\mu_s g}{r\alpha}\right)^2 - 1}$

۸۷-۳

$$\Sigma F_t = m a_t \; : \; m g\sin\theta = m a \;\; \Rightarrow \; a_t = g\sin\theta$$



$$\int v\, dv = \int a_t\, ds \;\; \Rightarrow \int_{v_0}^{v} v\, dv = \int_0^{\theta} g\sin\theta\,(R\, d\theta)$$

$$\Rightarrow \; v^2 = v_0^2 + 2gR(1 - \cos\theta)$$

$$\Sigma F_n = m a_n \; : \; mg\cos\theta - N = m\frac{v^2}{R}$$

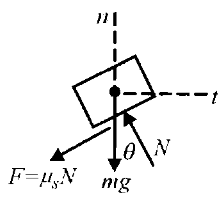
$$N = mg\cos\theta - \frac{m}{R}v_0^2 - 2mg(1 - \cos\theta) = mg\left(3\cos\theta - 2 - \frac{v_0^2}{gR}\right)$$

$N = 0 \;,\; \theta = \beta$ وقتی : $3\cos\beta = 2 + \dfrac{v_0^2}{gR} \;\Rightarrow\; \beta = \cos^{-1}\left(\dfrac{2}{3} + \dfrac{v_0^2}{3gR}\right)$

$v_0 = 0$ برای : $\beta = \cos^{-1}\left(\dfrac{2}{3}\right) = 48.2°$

۸۸-۳

بسته : $\begin{cases}\Sigma F_t = m a_t = -\mu_s N\cos\theta - N\sin\theta = -m\dfrac{g}{2} \\ \Sigma F_n = m a_n = N\cos\theta - \mu_s N\sin\theta - mg = m\left(\dfrac{(70/3.6)^2}{80}\right)\end{cases}$



معادله اول : $N = \dfrac{\dfrac{mg}{2}}{\sin\theta + \mu_s\cos\theta}$

معادله دوم : $\left(\dfrac{\dfrac{mg}{2}}{\sin\theta + \mu_s\cos\theta}\right)(\cos\theta - \mu_s\sin\theta) - mg = m(4.726)$

$$\Rightarrow \; \tan\theta = \left(\frac{1 - 2.9635\mu_s}{\mu_s + 2.9635}\right)$$

$\mu_s = 0.2$ برای $\Rightarrow \theta = 7.34°$

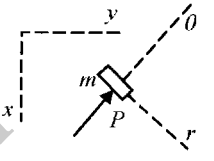
$\mu_s = 0.4$ برای $\Rightarrow \theta = -3.16°$!!

(نکته: برای $\theta = -3.16°$، $N > 0$ می‌باشد)

۳-۸۹. (توجه: نیروهای mg و عمود بر سطح، بر سطح کاغذ عمود هستند)

$$\Sigma F_r = m a_r : \quad 0 = m(\ddot{r} - r\Omega^2) \qquad (1)$$

$$\Sigma F_\theta = m a_\theta : \quad P = m(r\ddot{\theta}^{\,0} + 2\dot{r}\Omega) \qquad (2)$$



$$(1) : \ddot{r} = \dot{r}\frac{d\dot{r}}{dr} = r\Omega^2 \Rightarrow \int_{\dot{r}_0}^{\dot{r}} \dot{r}\,d\dot{r} = \int_{r_0}^{r} \Omega^2 r\,dr \Rightarrow \dot{r}^2 = \dot{r}_0^{\,2} + \Omega^2(r^2 - r_0^{\,2})$$

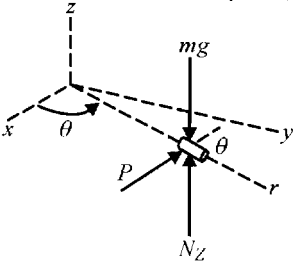
جای‌گذاری اعداد انتهای لوله : $\dot{r} = \left[20^2 + 7^2(1^2 - 0.15^2)\right]^{\frac{1}{2}} = 21.16\ \text{m/s}$

$$(2) : P = m(2\dot{r}\Omega) = 0.180(2)(21.16)(7) = 53.4\ \text{N}$$

۳-۹۰.

$$\Sigma F_z = 0 \Rightarrow N_z = mg$$

طبق جواب مسئله ۳-۸۹ : $\dot{r} = \left[\dot{r}_0^{\,2} + \Omega^2(r^2 - r_0^{\,2})\right]^{\frac{1}{2}}$



$$\int_{r_0}^{r} \frac{dr}{\sqrt{\dot{r}_0^{\,2} + \Omega^2(r^2 - r_0^{\,2})}} = \int_0^t dt$$

$$\Rightarrow \frac{1}{\Omega}\ln\left[r + \sqrt{r^2 + \frac{\dot{r}_0^{\,2}}{\Omega^2} - r_0^{\,2}}\right]_{r_0}^{r} = t$$

$$\Rightarrow \frac{1}{\Omega}\ln\left[\frac{r + \sqrt{r^2 + \frac{\dot{r}_0^{\,2}}{\Omega^2} - r_0^{\,2}}}{r_0 + \frac{\dot{r}_0}{\Omega}}\right] = t$$

با جای‌گذاری اعداد $r = 1\ \text{m}$ ، $r_0 = 0.15\ \text{m}$ ، $\Omega = 7\ \text{rad/s}$ ، $\dot{r}_0 = 20\ \text{m/s}$ خواهیم داشت:

$$t = 0.0416 \Rightarrow \theta = \Omega t = 7(0.0416) = 0.291\ \text{rad}$$

طبق جواب مسئله ۳-۸۹ : $P = 53.4\ \text{N}$

$$P_x = -P\sin\theta = -53.4\sin 0.291^{\text{R}} = -15.31\ \text{N}$$

$$P_y = P\cos\theta = 53.4\cos 0.291^{\text{R}} = 51.1\ \text{N}$$

۳-۹۱.

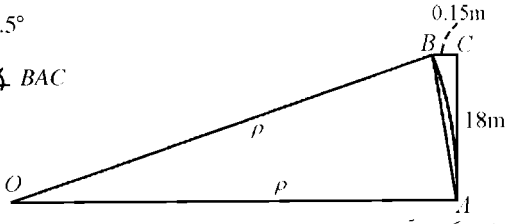
$$\measuredangle BAC = \tan^{-1}\frac{0.15}{18} = 0.477^\circ$$

$$\measuredangle OBA = \measuredangle OAB = (90 - 0.477) = 89.5^\circ$$

$$\measuredangle BOA = 180 - 2(89.5) = 0.955^\circ = 2\measuredangle BAC$$

$$\overline{AB} = \sqrt{18^2 + 0.15^2} = 18.0006\,\text{m}$$

$$\frac{\sin 0.955^\circ}{18.0006} = \frac{\sin 89.5^\circ}{\rho} \Rightarrow \rho = 1080\,\text{m}$$



(دیاگرام آزاد نیروهای افقی)

$$\Sigma F_n = m a_n \;:\; R = 0.146\frac{38^2}{1080} = 0.1952\,\text{N}$$

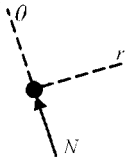
۳-۹۲.

$$\Sigma F_r = m a_r \;:\; 0 = m(\ddot{r} - r\dot{\theta}^2)$$

$$\ddot{r} = r\dot{\theta}^2 = r\omega_0^2$$

$$\dot{r}\frac{d\dot{r}}{dr} = r\omega_0^2 \Rightarrow \int_0^{\dot{r}} \dot{r}\,d\dot{r} = \omega_0^2\int_{r_0}^{r} r\,dr$$

$$\Rightarrow \dot{r} = \omega_0\sqrt{r^2 - r_0^2} = v_r$$



$$\frac{dr}{dt} = \omega_0\sqrt{r^2 - r_0^2} \Rightarrow \int_{r_0}^{r}\frac{dr}{\sqrt{r^2 - r_0^2}} = \omega_0\int_0^t dt \Rightarrow \ln\left[r + \sqrt{r^2 - r_0^2}\right]\Big|_{r_0}^{r} = \omega_0 t$$

$$\Rightarrow r = \frac{r_0}{2}\left[e^{-\omega_0 t} + e^{\omega_0 t}\right]$$

$$v_\theta = r\dot{\theta} = r\omega_0 = \frac{r_0\omega_0}{2}\left[e^{-\omega_0 t} + e^{\omega_0 t}\right]$$

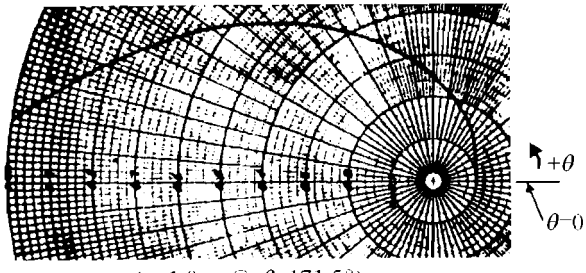
$v_r = \dfrac{r_0\omega_0}{2}\left(e^{\omega_0 t} - e^{-\omega_0 t}\right)$ به صورت تابعی از t

برحسب توابع هیپربولیک:

$$v_r = r_0\omega_0\sinh\omega_0 t$$
$$r = r_0\cosh\omega_0 t$$
$$v_\theta = r_0\omega_0\cosh\omega_0 t$$

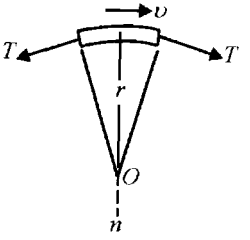
عددگذاری ⇒

$$\begin{cases} v_r = 0.1\sinh t \\ r = 0.1\cosh t \\ v_\theta = 0.1\cosh t \end{cases}$$



(r=1.0m @ θ=171.5°)

۳-۹۳.



$$\Sigma F_n = m a_n \;:\; 2T \sin\frac{d\theta}{2} = \rho r d\theta \times \frac{v^2}{r}$$

$$T d\theta = \rho v^2 d\theta \;\Rightarrow\; T = \rho v^2$$

$$\delta_t = \frac{T}{A} = \frac{\rho v^2}{A}$$

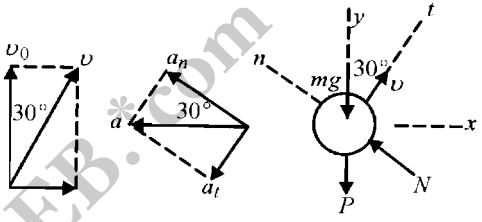
۳-۹۴.

$$mg = 0.5(9.81)\,N$$

$$v = \frac{v_0}{\cos 30^\circ} = \frac{2}{0.866} = 2.31\ \text{m/s}$$

$$a_n = \frac{v^2}{r} = \frac{(2.31)^2}{0.25} = 21.3\ \text{m/s}^2$$

$v_y =$ ثابت $\Rightarrow \dot{v}_y = a_y = 0 \quad , a = a_x$



$$a_t = a_n \tan 30^\circ = 21.3 \tan 30^\circ = 12.32\ \text{m/s}^2$$

$$\Sigma F_t = m a_t \;:\; -0.5(9.81)\cos 30^\circ - P\cos 30^\circ = -0.5(12.32) \;\Rightarrow\; P = 2.21\ \text{N}$$

$$\Sigma F_x = m a_x \;:\; -N\cos 30^\circ = 0.5\left(-\frac{21.3}{\cos 30^\circ}\right) \;\Rightarrow\; N = 14.22\ \text{N}$$

۳-۹۵.

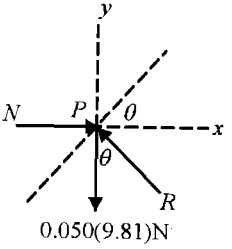
$$y = 0.015x^2 \qquad (\text{mm برحسب } y \text{ و } x)$$

$$\frac{dy}{dx} = 0.030x$$

$$x = 40\ \text{mm} \;\Rightarrow\; \frac{dy}{dx} = \tan\theta = 0.030(40) = 1.2 \Rightarrow \theta = 50.2^\circ$$

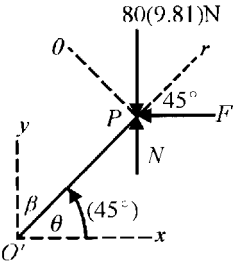
$$\dot{y} = 2(0.015)x\dot{x} = 0.030x\dot{x} = 0.030(40)(100) = 120\ \text{mm/s}$$

$$\ddot{y} = 0.030\dot{x}^2 + \underbrace{0.030x\ddot{x}}_{0} = 0.030(100)^2 = 300\ \text{mm/s}^2$$



$$\begin{cases} \Sigma F_x = m\ddot{x} \;:\; N - R\sin 50.2^\circ = 0 \\ \Sigma F_y = m\ddot{y} \;:\; R\cos 50.2^\circ - 0.050(9.81) = 0.050(0.3) \end{cases} \Rightarrow \begin{cases} R = 0.790\ \text{N} \\ N = 0.607\ \text{N} \end{cases}$$

۳-۹۶. دیاگرام آزاد سرنشین در نقطهٔ P بصورت زیر است که سرنشین بعنوان یک ذره رفتار می‌کند.



$$O'P = 6\,\text{m} \;,\; \theta = 45^\circ \;,\; \dot{\theta} = -0.8\,\text{rad/s} \;,\; \ddot{\theta} = -0.4\,\text{rad/s}^2$$

$$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$-80(9.81)\frac{\sqrt{2}}{2} + N\frac{\sqrt{2}}{2} - F\frac{\sqrt{2}}{2} = 80[0 - 6(-0.8)^2] \quad (1)$$

$$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$ (مرکز دایره‌ای که توسط P طی می‌شود)

$$-80(9.81)\frac{\sqrt{2}}{2} + N\frac{\sqrt{2}}{2} + F\frac{\sqrt{2}}{2} = 80[6(-0.4) + 0] \quad (2)$$

حل (1) و (2) : $\begin{cases} N = 432\,\text{N} \\ F = 81.5\,\text{N} \end{cases}$ در حالت استاتیکی $\begin{cases} N_s = 785\,\text{N} \\ F_s = 0 \end{cases}$

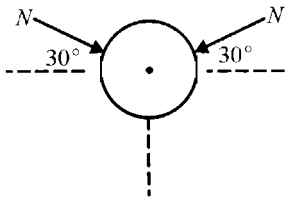
۳-۹۷.

$$\Sigma F_n = m a_n = m r\omega^2$$

$$2N\sin 30^\circ = 2.5(0.150)\left[\frac{600(2\pi)}{60}\right]^2$$

$$N = 1480\,\text{N}$$

$$F = 4N\cos 30^\circ = 5130\,\text{N}$$



۳-۹۸.

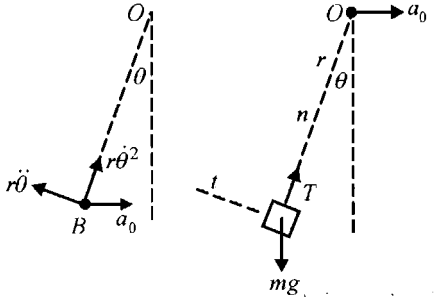
شتاب سطل $\mathbf{a}_B = \mathbf{a}_O + (\mathbf{a}_{B/O})_n + (\mathbf{a}_{B/O})_t$

$$\Sigma F_t = m a_t \;:\; -mg\sin\theta = m(r\ddot{\theta} - a_0\cos\theta)$$

$$\Rightarrow \ddot{\theta} = +\frac{1}{r}(a_0\cos\theta - g\sin\theta)$$

$\dot{\theta}$ وقتی به حداکثر می‌رسد که $\ddot{\theta} = 0$ باشد بنابراین:

$$a_0\cos\theta = g\sin\theta \;\Rightarrow\; \theta = \tan^{-1}\left(\frac{a_0}{g}\right)$$



$$\ddot{\theta} = \dot{\theta}\frac{d\dot{\theta}}{d\theta} : \int_0^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = \int_0^{\theta} \frac{1}{r}(a_0\cos\theta - g\sin\theta)\,d\theta \;\Rightarrow\; \dot{\theta}^2 = \frac{2}{r}(a_0\sin\theta + g\cos\theta - g)$$

$$\Sigma F_n = m a_n \;:\; T - mg\cos\theta = m(r\dot{\theta}^2 + a_0 \sin\theta)$$

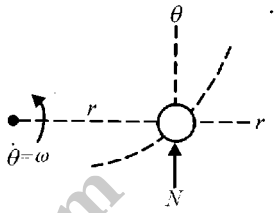
$\dot{\theta}^2$ جای‌گذاری $\longrightarrow$ $T = m(3a_0 \sin\theta + 3g\cos\theta - 2g)$

۳-۹۹.

$$\Sigma F_r = m a_r \;:\; 0 = m(\ddot{r} - r\omega^2)$$

$$\Rightarrow r = r_0 \cosh\omega t$$

$$\dot{r} = r_0 \omega \sinh\omega t$$



$$\Sigma F_\theta = m a_\theta \;:\; N = m(0 + 2\dot{r}\omega) = 2m r_0 \omega^2 \sinh\omega t$$

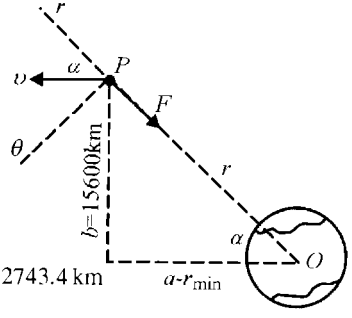
$$\cosh^2\omega t - \sinh^2\omega t = 1 \;\Rightarrow\; \sinh^2\omega t = \cosh^2\omega t - 1 \;\Rightarrow\; \sinh\omega t = \sqrt{\left(\frac{r}{r_0}\right)^2 - 1}$$

$$\Rightarrow N = 2m r_0 \omega^2 \sqrt{\left(\frac{r}{r_0}\right)^2 - 1} = 2m\omega^2\sqrt{r^2 - r_0^2}$$

۳-۱۰۰.

نصف قطر اصلی بیضی $a = \dfrac{r_{max} + r_{min}}{2} = \dfrac{39771 + 6671}{2} = 23221 \text{ km}$

$$\alpha = \tan^{-1}\frac{b}{a - r_{min}} = \tan^{-1}\frac{15600}{23221 - 6671} = 43.3°$$



$$\dot{r} = v_r = v\cos\alpha = 4230\cos 43.3° = 3078.5 \text{ m/s}$$

$$r\dot{\theta} = v_\theta = v\sin\alpha \;\Rightarrow\; \dot{\theta} = \frac{v\sin\alpha}{r}$$

$$r = \sqrt{b^2 + (a - r_{min})^2} = \sqrt{15600^2 + (23221 - 6671)^2} = 22743.4 \text{ km}$$

$$\Rightarrow \dot{\theta} = \frac{(4230)\sin 43.3°}{22743.4(10^3)} = 1.276(10^{-4}) \text{ rad/s}$$

$$\Sigma F_r = m a_r \;:\; -m\frac{gR^2}{r^2} = m(\ddot{r} - r\dot{\theta}^2) \;\Rightarrow\; \ddot{r} = r\dot{\theta}^2 - \frac{gR^2}{r^2}$$

$$\Rightarrow \ddot{r} = 22743.4(10^3)\,.\,[1.276(10^{-4})]^2 - \frac{9.825[6371(10^3)]^2}{[22743.4(10^3)]^2} = -0.401 \text{ m/s}^2$$

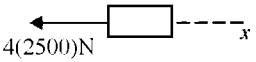
$$\Sigma F_\theta = m a_\theta \;:\; 0 = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$\Rightarrow \ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} = -2\frac{(3078.5)(1.276(10^{-4}))}{22743.4(10^3)} = 3.45(10^{-8}) \text{ rad/s}^2$$

۳-۱۰۱. ◀

حرکت از A تا B :

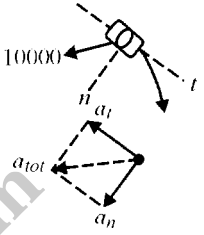
$$\Sigma F_x = m a_x \; : \; -4(2500) = 1350\, a \;\Rightarrow\; a = -7.407\ \text{m/s}^2$$



$$v_B^2 - v_A^2 = 2a(x_B - x_A) \;\Rightarrow\; v_B^2 - 25^2 = 2(-7.407)(10) \;\Rightarrow\; v_B = 21.84\ \text{m/s}$$

بعد از B :

$$F = ma_{tot} \;\Rightarrow\; a_{tot} = \frac{10000}{1350} = 7.407\ \text{m/s}^2$$



$$a_{tot} = \sqrt{a_n^2 + a_t^2} = \sqrt{\frac{v^4}{\rho^2} + a_t^2}$$

$$a_t = -\sqrt{a_{tot}^2 - \frac{v^4}{\rho^2}} = v\frac{dv}{ds}$$

$$\int_{10}^{s} ds = -\rho \int_{v_B}^{0} \frac{v\,dv}{\sqrt{\rho^2 a_{tot}^2 - v^4}}$$

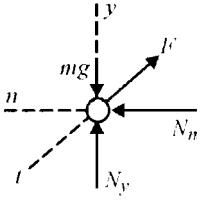
فرض کنید $x = v^2$ $\Rightarrow$

$$s - 10 = -\rho \int_{v_B^2}^{0} \frac{\frac{dx}{2}}{\sqrt{\rho^2 a_{tot}^2 - x^2}} \;\Rightarrow\; s = 10 + \frac{\rho}{2}\sin^{-1}\left(\frac{v_B^2}{\rho\, a_{tot}}\right) = 47.4\ \text{m}$$

۳-۱۰۲. ◀

$$\Sigma F_y = 0 \quad , \quad N_y = mg$$

$$\Sigma F_n = m a_n \; : \; N_n = m\frac{v^2}{r}$$



$$F = \mu_k N_{tot} = \mu_k \sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)^2}$$

$$= \frac{\mu_k m}{r}\sqrt{r^2 g^2 + v^4}$$

$$\Sigma F_t = ma_t \; : \; -\frac{\mu_k m}{r}\sqrt{r^2 g^2 + v^4} = mv\frac{dv}{ds}$$

$$-\frac{\mu_k}{r}\int_0^s ds = \int_{v_0}^{0} \frac{v\,dv}{\sqrt{v^4 + r^2 g^2}} = \int_{v_0}^{0} \frac{\frac{1}{2}dx}{\sqrt{x^2 + r^2 g^2}}$$

$$\Rightarrow\; -\frac{\mu_k}{r}s = \frac{1}{2}\ln\left[x + \sqrt{x^2 + r^2 g^2}\right]_{v_0^2}^{0} \;\Rightarrow\; s = \frac{r}{2\mu_k}\ln\left[\frac{v_0^2 + \sqrt{v_0^4 + r^2 g^2}}{rg}\right]$$

۳-۱۰۳.

(الف) $U_{1\text{-}2} = \frac{1}{2}k(x_1^2 - x_2^2) = \frac{1}{2}(500)[(-0.15)^2 - (0.08)^2] = 4.03\,\text{J}$

(ب) $U_{1\text{-}2} = -mgh = -6(9.81)(0.23\sin 15°) = -3.50\,\text{J}$

۳-۱۰۴.

$$T_A + U_{A-B} = T_B$$

$$\frac{1}{2} m v_A^2 + m g h = \frac{1}{2} m v_B^2 \Rightarrow v_B^2 = v_A^2 + 2gh = 4^2 + 2(9.81)(1.8)$$

$$\Rightarrow v_B = 7.16 \text{ m/s}$$

دانستن شکل هندسی مسیر ضروری نیست. کافیست بدانیم که گاری در بالاترین نقطه قرار دارد.

۳-۱۰۵.

$$T_A + U_{A-B} = T_B$$

$$\frac{1}{2} m v_A^2 + U_f + mgh = \frac{1}{2} m v_B^2 \Rightarrow U_f = m\left(\frac{v_B^2}{2} - \frac{v_A^2}{2} - gh\right)$$

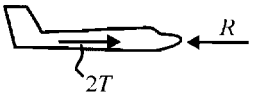
$$= 3\left(\frac{6^2}{2} - \frac{4^2}{2} - 7.81(1.8)\right) = -23.0 \text{ J}$$

۳-۱۰۶.

$$\mathbf{U} = \Delta T$$

$$[(2 \times 44000) - R]\,1450 = \frac{1}{2} 40000 \left[\frac{260}{3.6}\right]^2 - 0$$

$$\Rightarrow R = 16054 \text{ N} \quad \text{یا} \quad R = 16.05 \text{ kN}$$

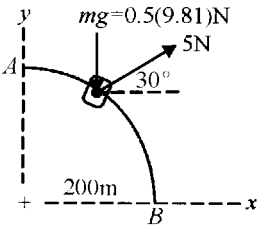


۳-۱۰۷.

$U = \Delta T$:

$$5\cos 30°(0.2) - 5\sin 30°(0.2) + 0.5(9.81)(0.2) = \frac{1}{2} 0.5 (v^2 - 0)$$

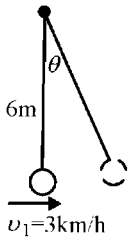
$$\Rightarrow v^2 = 5.39 \Rightarrow v = 2.32 \text{ m/s}$$



۳-۱۰۸.

$$T_1 + U_{1\text{-}2} = T_2 : \frac{1}{2} m v_1^2 - mgh = 0$$

$$\Rightarrow \frac{1}{2}\left(\frac{3}{3.6}\right)^2 - 9.81[6(1 - \cos\theta)] = 0 \Rightarrow \theta = 6.23°$$



۳-۱۰۹. برای طوقه : $U_{1\text{-}2} = \Delta T = 0$

$$U_{1\text{-}2} = 200(1500 - 900) - 15(9.81)(1200 \sin 30°) - \frac{1}{2} k (180)^2 = 0$$

$$\Rightarrow \quad k = 1.957 \text{ N/mm} \quad \text{یا} \quad k = 1957 \text{ N/m}$$

۳-۱۱۰.

$$U_{1\text{-}2} = \Delta T \quad : \quad 2\left(\frac{1}{2} k x^2\right) = \frac{1}{2} m v^2 - 0$$

$$\Rightarrow \quad k = \frac{1}{2} \frac{m v^2}{x^2} \cdot \frac{1500 \left(\frac{8}{3.6}\right)^2}{(0.15)^2} = 164609 \text{ N/m} \quad \text{یا} \quad 164.6 \text{ kN/m}$$

۳-۱۱۱.

$$U_{1\text{-}2} = \Delta T \quad : \quad m g h = \frac{1}{2} m (v^2 - 0) \quad \Rightarrow \quad v = \sqrt{2 g h}$$

۳-۱۱۲.

$$U_{1\text{-}2} = \Delta T \quad : \quad m g h + Q = \frac{1}{2} m (v_B{}^2 - 0^2)$$

$$0.5(9.81)(1.5) + Q = \frac{1}{2}(0.5)(4.7^2 - 0^2) \quad \Rightarrow \quad Q = -1.835 \text{ J}$$

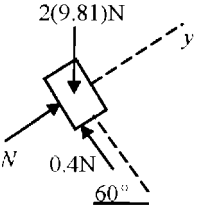
انرژی مکانیکی تلف شده به حرارت تبدیل می‌شود.

۳-۱۱۳.

$$\Sigma F_y = 0 \quad : \quad N - 2(9.81) \cos 60° = 0 \quad \Rightarrow \quad N = 9.81 \text{ N}$$

(الف) $U_{1\text{-}2} = \Delta T$: $2(9.81)(0.5 \sin 60°) - 0.4(9.81)(0.5) = \frac{1}{2} 2 v^2$

$$\Rightarrow \quad v = 2.56 \text{ m/s}$$



(ب) $U_{1\text{-}2} = \Delta T$:

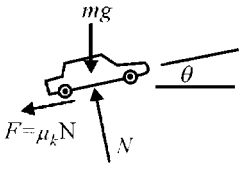
$$2(9.81)(0.5 + x) \sin 60° - 0.4(9.81)(0.5 + x) - \frac{1}{2}(1600) x^2 = 0$$

$$\Rightarrow \quad 800 x^2 - 13.07 x - 6.53 = 0$$

$$\Rightarrow \quad x = 0.0989 \text{ m} \quad \text{یا} \quad x = 98.9 \text{ mm}$$

۳-۱۱۴.

$$\theta = \tan^{-1}\frac{6}{100} = 3.43^\circ$$



$$T_A + U_{A-B} = T_B \Rightarrow \frac{1}{2}m v_0^2 - \mu_k m g\cos\theta\, s - m g s\sin\theta = 0$$

$$\Rightarrow \frac{1}{2}\left(\frac{105}{3.6}\right)^2 - 9.81\,s(0.6\cos 3.43^\circ + \sin 3.43^\circ) = 0$$

$$\Rightarrow s = 65.8\,\text{m}$$

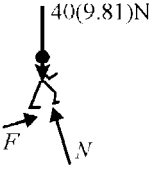
حرکت رو به پائین : $T_B + U_{B-A} = T_A$

$$\frac{1}{2}m v_0^2 - \mu_k m g\cos\theta\, s + m g s\sin\theta = 0 \Rightarrow s = 80.4\,\text{m}$$

۳-۱۱۵.

$$v_B = \frac{8}{3.6} = 2.22\,\text{m/s}$$

$$v_B^2 = 2as \Rightarrow a = \frac{2.22^2}{2(15)} = 0.165\,\text{m/s}^2$$



$$\theta = \tan^{-1}\left(\frac{10}{100}\right) = 5.71^\circ$$

$$\overset{+}{\nearrow}\Sigma F = ma : F - 40(9.81)\sin 5.71^\circ = 40(0.165) \Rightarrow F = 45.6\,\text{N}$$

$$P = Fv = 45.6(2.22) = 101.4\,\text{W}$$

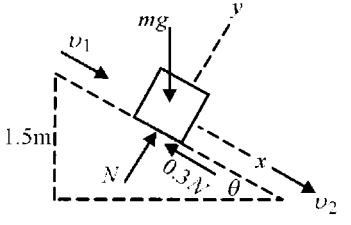
۳-۱۱۶.

$$P = \frac{Wh}{\Delta t} = \frac{54(9.81)(2.75)}{5} = 291.3\,\text{W}$$

۳-۱۱۷.

$$\Sigma F_y = 0 : N - mg\cos\theta = 0$$

$U = \Delta T$:



$$(mg\sin\theta - 0.3\,mg\cos\theta)\frac{1.5}{\sin\theta} = \frac{1}{2}m(0.14^2 - 0.40^2)$$

$$\Rightarrow 15(9.81)\left(1 - \frac{0.3}{\tan\theta}\right) = -0.0702$$

$$\Rightarrow \tan\theta = 0.299 \Rightarrow \theta = 16.62^\circ$$

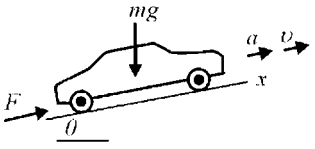
۳-۱۱۸.

$$\text{توان خالص لازم} = 30\,\frac{65(9.81)(7)}{60} = 2232\,\text{W} \quad \text{یا} \quad 2.23\,\text{kW}$$

$$\text{راندمان مکانیکی} = \frac{\text{توان لازم}}{\text{توان خروجی}} = \frac{2.23}{3} = 0.744$$

۳-۱۱۹. $\theta = \tan^{-1}0.1 \Rightarrow \sin\theta = 0.0995$

$$\left.\begin{aligned}\Sigma F_x &= m a_x \\ v^2 &= 2ax\end{aligned}\right\} \Rightarrow F - mg\sin\theta = m\frac{v^2}{2x}$$



$$P = Fv = mgv\sin\theta + \frac{mv^3}{2x}$$

$$= 1500(9.81)\left(\frac{50}{3.6}\right)(0.0995) + \frac{1500\left(\frac{50}{3.6}\right)^3}{2(100)} = 40430\,\text{W} \quad \text{یا} \quad P = 40.4\,\text{kW}$$

۳-۱۲۰. نرخ انجام کار = توان خروجی

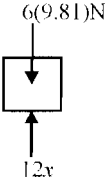
$$= 300(9.81)(2) - 100(9.81)(4) = 1962\,\text{W} \quad \text{یا} \quad 1.962\,\text{kW}$$

راندمان $e = \dfrac{\text{توان خروجی}}{\text{توان ورودی}} = \dfrac{1.962}{2.20} = 0.892$

۳-۱۲۱. $U_{1\text{-}2} = \Delta T$:

$$6(9.81)(0.500 + 0.050) - \frac{1}{2}12000(0.050)^2 = \frac{1}{2}(6)v^2$$

$$\Rightarrow v = 2.41\,\text{m/s}$$



۳-۱۲۲. $x = 75\,\text{mm}$ برای : $U = \Delta T$

$$\frac{1}{2}(0.075)R_{\max} = \frac{1}{2}(0.25)(600)^2 \Rightarrow R_{\max} = 1.2\,\text{MN}$$

$x = 25\,\text{mm}$ برای : $R = \dfrac{25}{75}(1.2) = 0.4\,\text{MN}$ یا $0.4(10^6)\,\text{N}$

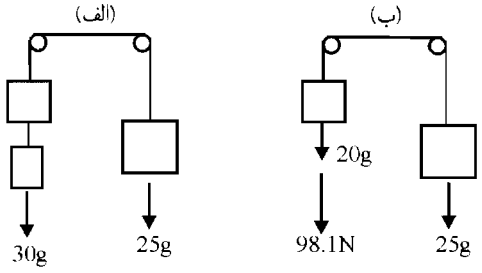
$$U = \Delta T : \frac{1}{2}(0.025)(0.4)10^6 = \frac{1}{2}(0.25)(600^2 - v^2) \Rightarrow v = 566\,\text{m/s}$$

۳-۱۲۳. کار انجام شده توسط وزن $= mg(r_{\pi/2} - 0) = 0.5(9.81)\left(0.3\frac{\pi}{2}\right) = 2.31\,\text{J}$

کار انجام شده توسط $T = T(r_\pi - r_{\pi/2}) = 10\left(0.3\pi - 0.3\frac{\pi}{2}\right) = 4.71\,\text{J}$

$$U = \Delta T : 2.31 + 4.71 = \frac{1}{2}(0.5)(v^2 - 0) \Rightarrow v = 5.30\,\text{m/s}$$

۳-۱۲۴. دیاگرام نیروهای فعال برای دستگاه:



برای دستگاه $U = \Delta T$:

(الف) $(30 - 25)(9.81)2 = \frac{1}{2}(30 + 25)v^2$

$\Rightarrow v = 1.889\,\text{m/s}$

(ب) $[(20 - 25)(9.81) + 98.1]2 =$

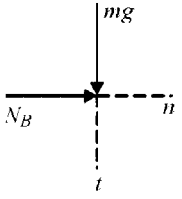
$= \frac{1}{2}(20 + 25)v^2$

$\Rightarrow v = 2.09\,\text{m/s}$

۳-۱۲۵.

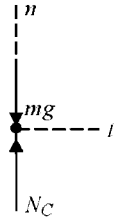
(الف) $T_A + U_{A-B} = T_B : 0 + 2mgR = \frac{1}{2}mv_B^2 \Rightarrow v_B^2 = 4gR$

$\Sigma F_n = ma_n : N_B = m\frac{4gR}{R} = 4mg$



(ب) $T_A + U_{A-C} = T_C : 0 + 3mgR = \frac{1}{2}mv_C^2 \Rightarrow v_C^2 = 6gR$

$\Sigma F_n = ma_n : N_C - mg = m\frac{6gR}{R} \Rightarrow N_C = 7mg$



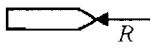
(ج) نقطه توقف لغزنده E : $T_A + U_{A-E} = T_E$

$\Rightarrow 0 + 2mgR - mg\left(\frac{1}{2}s\right) - \mu_k\frac{\sqrt{3}}{2}mgs = 0 \Rightarrow S = \frac{4R}{1 + \mu_k\sqrt{3}}$

(توجه: نیروی عمود بر سطح شیبدار برابر است با $N = mg\cos 30° = \frac{\sqrt{3}}{2}mg$)

۳-۱۲۶.

$U = \Delta T$



$-R \times 0.4 = \frac{1}{2}(0.012)(300^2 - 600^2) \Rightarrow R = 4050\,\text{N}$ یا $4.05\,\text{kN}$

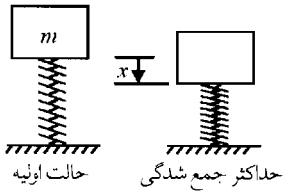
$\Delta Q = |\Delta T| = \frac{1}{2}(0.012)(600^2 - 300^2) = 1620\,\text{J}$ به حرارت تبدیل می‌شود

۳-۱۲۷. حداکثر نیرو $F = kx$ زمانی وارد می‌شود که x با $\dot{x} = 0$ به حداکثر خود برسد.

$$U_{1\text{-}2} = \Delta T$$

$$m g x - \frac{1}{2} k x^2 = 0 \Rightarrow x = \frac{2 m g}{k}$$

$$F = k x = 2 m g$$



۳-۱۲۸. $U_{1\text{-}2} = \Delta T \quad : \quad m g(0.8 - 1.2\cos 60°) = \frac{1}{2} m (v_C^2 - 3^2)$

$$\Rightarrow v_C = 3.59 \text{ m/s}$$

۳-۱۲۹. فرض کنید s مسافت طی شده روی سطح شیبدار به سمت پائین قبل از تغییر جهت باشد.

$$U_{1\text{-}2} = 550(2)(3 + s - s) - 150(9.81)(3 + s - s)\left(\frac{5}{13}\right) = 1602$$

$$\Delta T = \frac{1}{2} 150 [v^2 - (\pm 3)^2] = 75 v^2 - 675$$

$$U_{1\text{-}2} = \Delta T \quad : \quad 1602 = 75 v^2 - 675 \Rightarrow v = 5.51 \text{ m/s}$$

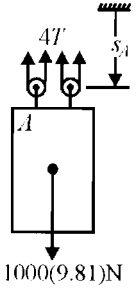
۳-۱۳۰. $+\downarrow \Sigma F = 0 \quad : \quad 9810 - 4T = 0 \Rightarrow T = 2450 \text{ N}$

طول کابل $L = 4 s_A + \text{Const.}$

$$\dot{L} = 4 v_A = 4(-3) = -12 \text{ m/s}$$

$$P_{out} = -T \dot{L} = -2450(-12) = 29400 \text{ W} = 29.4 \text{ kW}$$

$$e = \frac{P_{out}}{P_{in}} \Rightarrow P_{in} = \frac{P_{out}}{e} = \frac{29.4}{0.8} \Rightarrow P_{in} = 36.8 \text{ kW}$$



۳-۱۳۱. توان خروجی سیستم محرک اتومبیل $P_{out} = Fv = 560\left(\frac{90}{3.6}\right) = 14000 \text{ W}$

توان ورودی سیستم محرک اتومبیل $P_{in} = \frac{P_{out}}{e} = \frac{14000}{0.70} = 20000 \text{ W}$

$\Rightarrow$ توان موتور $P = 20 \text{ kW}$

۳-۱۳۲.

$$\theta = \tan^{-1}\frac{6}{100} = 3.43°$$

$$U_{1\text{-}2} = \Delta T : \quad U_f + mgh = \frac{1}{2}m(v_2^2 - v_1^2)$$

$$U_f = -1400(9.81)(200\sin 3.43°) + \frac{1}{2}1400\left[\left(\frac{20}{3.6}\right)^2 - \left(\frac{100}{3.6}\right)^2\right] = -683000\,\text{J} \text{ یا } -683\,\text{kJ}$$

انرژی تلف شده $Q = 683\,\text{kJ}$

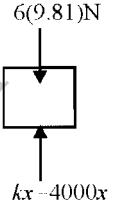
۳-۱۳۳.

$$U_{1\text{-}2} = \Delta T = 0$$

$$6(9.81)(0.1 + \delta) - \int_{0.05}^{0.05+\delta} 4000\,x\,dx = 0$$

$$2000\,\delta^2 + 141.1\,\delta - 5.89 = 0$$

$$\Rightarrow \delta = 0.0294\,\text{m} \quad \text{یا} \quad \delta = 29.4\,\text{mm}$$



۳-۱۳۴.

$$F_1 = 300x \;,\; F_2 = 300x + 150(x - s) = 450x - 150s$$

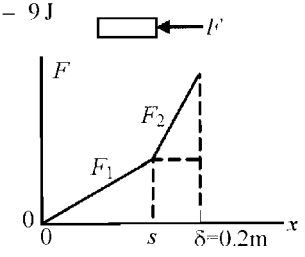
$$U_{1\text{-}2} = -\frac{1}{2}300s^2 - \int_s^{0.2\text{m}}(450x - 150s)\,dx = -75s^2 + 30s - 9\,\text{J}$$

$$U_{1\text{-}2} = \Delta T : \quad -75s^2 + 30s - 9 = \frac{1}{2}(0.5)(0 - 5^2)$$

$$75s^2 - 30s + 2.75 = 0 \quad \Rightarrow s = 0.1423\,\text{m} \text{ یا } 0.257\,\text{m}$$

$0.257\,\text{m} > 200\,\text{mm}$ غیرممکن

$$\Rightarrow s = 142.3\,\text{mm}$$



۳-۱۳۵.

برای سیستم طوقه و کابل $U_{1\text{-}2} = \Delta T$

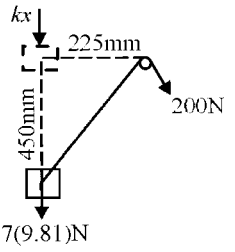
$$U_{1\text{-}2} = 200\left[\sqrt{(0.450)^2 + (0.225)^2} - 0.225\right]$$

$$- 7(9.81)(0.450) - \frac{1}{2}k(0.075)^2$$

$$= 55.6 - 30.9 - 0.00281\,k$$

$$\Delta T = 0$$

$$\Rightarrow 0.00281\,k = 55.6 - 30.9 \Rightarrow k = 8790\,\text{N/m} \quad \text{یا} \quad k = 8.79\,\text{kN/m}$$



۳-۱۳۶.

$$F_1 + F_2 = \mu_k (A) P$$

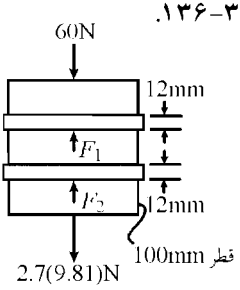
$$= 0.15 \times 2 \times 0.1\pi \times 0.012 P$$

$$= 1.131 (10^{-3}) P$$

$U = \Delta T$:

$$(60 + 2.7(9.81) - 1.131 (10^{-3}) P)\, 0.25 = \frac{1}{2}(2.7)(2.5^2 - 0^2)$$

$$\Rightarrow P = 46629 \text{ Pa} \quad \text{یا} \quad 46.6 \text{ kPa}$$



۳-۱۳۷.

$U = \Delta T$

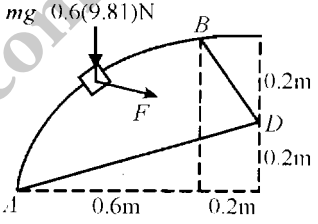
$$U = F(\overline{AD} - \overline{BD}) - 0.6(9.81)\, 0.4$$

$$= F(0.825 - 0.283) - 2.35$$

$$= 0.542 F - 2.35 \text{ J}$$

$$\Delta T = \frac{1}{2} m (v_B^2 - v_A^2) = \frac{1}{2} 0.6 (4^2 - 0) = 4.8 \text{ J}$$

$$0.542 F - 2.35 = 4.8 \quad \Rightarrow \quad F = 13.21 \text{ N}$$



$$\overline{AD} = \sqrt{0.8^2 + 0.2^2} = 0.825 \text{ m}$$

$$\overline{BD} = \sqrt{0.2^2 + 0.2^2} = 0.283 \text{ m}$$

۳-۱۳۸.

$$\theta = 0 \quad \text{یا} \quad \theta = \tan^{-1} \frac{6}{100} = 3.43°$$

$$F_D = k v^2 : \quad 250 = k \left(\frac{100}{3.6}\right)^2 \quad \Rightarrow \quad k = 0.324$$

$$\Rightarrow F_D = 0.324 v^2$$

$$\Sigma F_x = 0 : \quad F_P - F_R - F_D - m g \sin\theta = 0$$

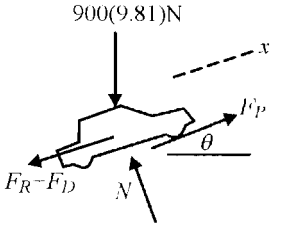
$$\Rightarrow F_P = F_R + F_D + m g \sin\theta$$

(الف) $\theta = 0$

$$v = 50 \text{ km/h} : \quad F_D = 0.324 \left(\frac{50}{3.6}\right)^2 = 62.5 \text{ N}$$

$$F_P = F_R + F_D = 250 + 62.5 = 312.5 \text{ N}$$

$$P_{50} = F v = 312.5 \left(\frac{50}{3.6}\right) = 4340 \text{ W} \quad \text{یا} \quad 4.34 \text{ kW}$$



$v = 100\text{km/h}$: $F_D = 0.324\left(\frac{100}{3.6}\right)^2 = 250\,\text{N}$

$F_P = F_R + F_D = 250 + 250 = 500\,\text{N}$

$P_{100} = Fv = 500\left(\frac{100}{3.6}\right) = 13889\,\text{W}$ یا $13.89\,\text{kW}$

(ب) $\theta = 3.43°$

رو به بالا $F_P = 250 + 250 + 900(9.81)\sin 3.43° = 1028\,\text{N}$

$P_{up} = 1028\left(\frac{100}{3.6}\right) = 28556\,\text{W}$ یا $28.6\,\text{kW}$

رو به پائین $F_P = 250 + 250 - 900(9.81)\sin 3.43° = -28\,\text{N}$

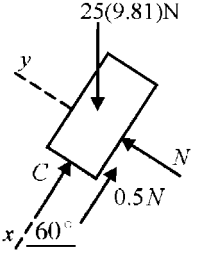
$P_{down} = -28\left(\frac{100}{3.6}\right) = -784\,\text{W}$

(ج) $\Sigma F_x = 0$: $250 + kv^2 - 900(9.81)\sin 3.43° = 0 \Rightarrow v = 29.3\,\text{m/s}$ یا $105.5\,\text{km/h}$

۳-۱۳۹.

$\Sigma F_y = 0$: $N - 25(9.81)\cos 60° = 0 \Rightarrow N = 122.6\,\text{N}$

جابجایی $= 1 + 0.1 = 1.1\,\text{m}$



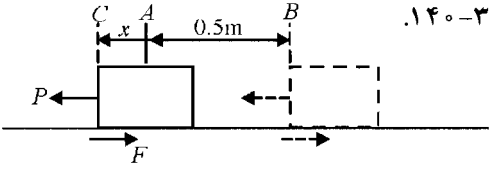
$U_{1-2} = [25(9.81)\sin 60° - 0.5(122.6)]\,1.1 - \int_0^{0.1}(9x + 62x^2)(10^3)\,dx$

$= 55.45$

$U_{1-2} = \Delta T$: $55.45 = \frac{1}{2}25(v^2 - 0.6^2) \Rightarrow v = 2.19\,\text{m/s}$

۳-۱۴۰.

$F = \mu_k N = 0.3(10)(9.81) = 29.43\,\text{N}$



(الف) از A تا B : $U_{1-2} = \Delta T$

$\frac{1}{2}(300)(0.5)^2 - 29.43(0.5) = \frac{1}{2}(10)v^2 \Rightarrow v = 2.13\,\text{m/s}$

(ب) از A تا C : $U_{1-2} = \Delta T$

$-\frac{1}{2}(300)x^2 - 29.43x = 0 - \frac{1}{2}(10)(2.13)^2$

$x^2 + 0.1962x - 0.1519 = 0$

$x = 0.304\,\text{m}$

۳-۱۴۱.

$$(U_{1\text{-}2})_s = -\int_{x_1}^{x_2} (4x - 120x^3)\,dx = (-2x^2 + 30x^4)\Big|_{x_1}^{x_2}$$

$$= -2(x_2^2 - x_1^2) + 30(x_2^4 - x_1^4)$$

$x_1 = 0.1\,\text{m}$, $x_2 = 0$ $\Rightarrow$ $(U_{1\text{-}2})_s = 0.017\,\text{kN.m}$ یا $(U_{1\text{-}2})_s = 17\,\text{N.m} = 17\,\text{J}$

$$(U_{1\text{-}2})_f = -\mu_k m g d = -0.2(10)(9.81)(0.1) = -1.962\,\text{J}$$

$$T_1 + U_{1\text{-}2} = T_2 \;:\; 0 + 17 - 1.962 = \frac{1}{2}(10)v^2 \;\Rightarrow\; v = 1.734\,\text{m/s}$$

برای فنر خطی : $(U_{1\text{-}2})_s = 2(x_1^2 - x_2^2) = 2(0.1)^2 = 0.02\,\text{kN.m} = 20\,\text{J}$ $\Rightarrow$ $v' = 1.899\,\text{m/s}$

۳-۱۴۲.

$$\left.\begin{aligned} P &= Fv \\ F &= ma \end{aligned}\right\} \Rightarrow P = mav \Rightarrow a = \frac{P}{mv}$$

$$v\,dv = a\,ds \Rightarrow mv^2\,dv = P\,ds$$

$$\int_{v_1}^{v_2} mv^2\,dv = \int_0^s P\,ds \Rightarrow \frac{m}{3}(v_2^2 - v_1^2) = Ps \Rightarrow v_2 = \left(\frac{3Ps}{m} + v_1^3\right)^{\frac{1}{3}}$$

۳-۱۴۳.

$$U'_{1\text{-}2} = 0 = \Delta T + \Delta V_g + \Delta V_e$$

$$\Delta T = \frac{1}{2}3(v^2 - 0) = \frac{3}{2}v^2$$

$$\Delta V_g = -3(9.81)(0.8) = -23.5\,\text{J}$$

$$\Delta V_e = \frac{1}{2}200\left[(\sqrt{0.8^2 + 0.6^2} - 0.4)^2 - (0.8 - 0.4)^2\right] = 20\,\text{J}$$

$$0 = \frac{3}{2}v^2 - 23.5 + 20 \Rightarrow v = 1.537\,\text{m/s}$$

۳-۱۴۴.

مبنا نقطهٔ A :

(الف) $T_A + V_A = T_B + V_B$

$$0 + 0 = \frac{1}{2}mv_B^2 - mgh_B \Rightarrow v_B = \sqrt{2gh_B} = \sqrt{2(9.81)(4.5)} = 9.40\,\text{m/s}$$

(ب) حالت فشردگی کامل فنر $= F$

$$T_A + V_A = T_F + V_F$$

$$0 + 0 = 0 - mgh_f + \frac{1}{2}k\delta^2 \Rightarrow \delta = \sqrt{\frac{2mgh_f}{k}} = \sqrt{\frac{2(1.2)(9.81)(3)}{24000}} = 0.0542\,\text{m}$$

یا $\delta = 54.2\,\text{mm}$

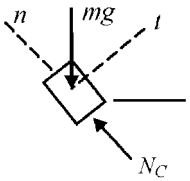
۳-۱۴۵.

مبنا نقطهٔ A : $T_A - V_A = T_C + V_A$

$$0 + 0 = \frac{1}{2} m v_C^2 - m g h_C \quad \Rightarrow \quad v_C = \sqrt{2 g h_C} = \sqrt{2(9.81)(3 + 1.5\cos 30°)} = 9.18\ \text{m/s}$$

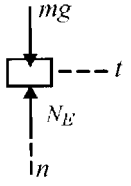
(الف) $\Sigma F_n = m \dfrac{v^2}{\rho}$: $N_C - 1.2(9.81)\cos 30° = 1.2 \dfrac{(9.18)^2}{1.5}$

$$N_C = 77.7\ \text{N}$$



(ب) $\Sigma F_n = 0$: $N_C - 1.2(9.81)\cos 30° = 0$

$$N_C = 10.19\ \text{N}$$



$T_A + V_A = T_E + V_E$: $0 + 0 = \dfrac{1}{2} m v_E^2 - m g h_E$

$$V_E = \sqrt{2 g h_E} = \sqrt{2(9.81)(3)} = 7.67\ \text{m/s}$$

$\Sigma F_n = m \dfrac{v^2}{\rho}$: $-N_E + 1.2(9.81) = 1.2 \dfrac{(7.67)^2}{1.5}$

$N_E = -35.3\ \text{N}$ به طرف پائین

۳-۱۴۶.

برای مجموعه : $U'_{1\text{-}2} = 0 \quad \Rightarrow \Delta V_g + \Delta T = 0$

$$-m g(0.500) + \frac{1}{2}(2m)(v^2 - 0) = 0 \quad \Rightarrow \quad v = 2.21\ \text{m/s}$$

۳-۱۴۷.

$\Delta T + \Delta V_e + \Delta V_g = 0$, $\Delta T = 0$

$$\Delta V_e = \frac{1}{2} k (x_2^2 - x_1^2) = \frac{1}{2} 500 (0.050^2 - 0.100^2) = -1.875\ \text{J}$$

$$\Delta V_g = m g \Delta h = 2(9.81) h = 19.62 h$$

$$\Rightarrow \quad 0 - 1.875 + 19.62 h = 0 \quad \Rightarrow \quad h = 0.0956\ \text{m} \quad \text{یا} \quad h = 95.6\ \text{mm}$$

۳-۱۴۸.

مبنا B : $T_A + V_A = T_B + V_B$

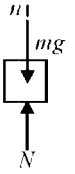
$$0 + m g R + \frac{1}{2} k \left[R\sqrt{2} - R\right]^2 = \frac{1}{2} m v_B^2 + 0 \quad \Rightarrow \quad v_B = \sqrt{2 g R + \frac{k R^2}{m}(3 - 2\sqrt{2})}$$

مبنا C : $T_A + V_A = T_C + V_C$

$$0 + 2 m g R + \frac{1}{2} k \left[R\sqrt{2} - R\right]^2 = \frac{1}{2} m v_C^2 + 0 \quad \Rightarrow \quad v_C = \sqrt{4 g R + \frac{k R^2}{m}(3 - 2\sqrt{2})}$$

سینتیک در C : $\Sigma F_n = m a_n$: $N - mg = m \dfrac{v_C^2}{R}$

$$\Rightarrow N = m\left[5g + \frac{kR}{m}(3 - 2\sqrt{2})\right]$$



۳-۱۴۹. (الف) $\Delta T + \Delta V_g = 0$

$$\frac{1}{2}(2)v^2 + \frac{1}{2}(4)\left(\frac{300}{450}v\right)^2 + 2(9.81)0.45\sin 60° - 4(9.81)0.3\sin 60° = 0$$

$$1.89v^2 = 2.55 \Rightarrow v = 1.162\,\text{m/s}$$

(ب) $\Delta T = 0 \Rightarrow \Delta V_g + \Delta V_e = 0$

$$-2.55 + \frac{1}{2}(35)\,10^3 x^2 = 0 \Rightarrow x = 0.01207\,\text{m} \quad \text{یا} \quad x = 12.07\,\text{mm}$$

۳-۱۵۰. مبنا نقطه رها کردن : $T_A + V_A = T_B + V_B$

$$0 = \frac{1}{2}k_A x_A^2 = 0 + mg(x_A + d + x_B) + \frac{1}{2}k_B x_B^2$$

$$\frac{1}{2}(8400)(0.125)^2 = 6(9.81)(0.125 + 0.35 + x_B) + \frac{1}{2}(1750)x_B^2$$

$$\Rightarrow x_B = 0.1766 \quad \text{یا} \quad x_B = 176.6\,\text{mm}$$

(این نکته که $x_B > x_A$ می‌باشد مربوط به اختلاف در سختی فنرهاست)

۳-۱۵۱. برای دستگاه (مبنا موقعیت اولیه) : $T_1 + V_1 + U'_{1\text{-}2} = T_2 + V_2$

$$\frac{1}{2}m v_1^2 + \frac{1}{2}k x_1^2 + 0 = \frac{1}{2}m v^2 + \frac{1}{2}k x_2^2 - mgh \quad (h = \text{ارتفاع سقوط})$$

توجه کنید که تغییر طول فنر دو برابر وزنه خواهد بود.

$$\frac{1}{2}(1050)(0.075)^2 = \frac{1}{2}(45)v^2 + \frac{1}{2}(1050)(0.075 + 2\times 0.012)^2 - 45(9.81)(0.012)$$

$$\Rightarrow v = 0.371\,\text{m/s}$$

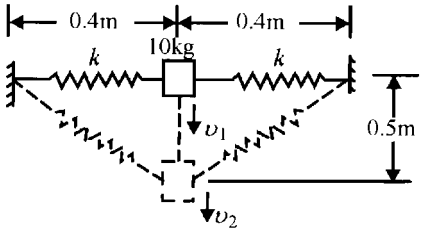
۳-۱۵۲. $k = 800\,\text{N/m}$, $v_1 = 2\,\text{m/s}$, کشیدگی اولیه $\delta_1 = 0.1\,\text{m}$

$\delta_2 = 0.5 - 0.3 = 0.2\,\text{m}$

$\Delta T + \Delta V_g + \Delta V_e = 0$

$\frac{1}{2}(10)(v_2^2 - 2^2) - 10(9.81)(0.3) + 2\frac{1}{2}(800)(0.2^2 - 0.1)^2 = 0$

$\Rightarrow v_2 = 2.26\,\text{m/s}$



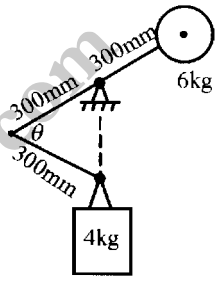
۳-۱۵۳.

$\theta = 180°$ تا $\theta = 60°$ برای حرکت از : $\Delta V_g + \Delta T = 0$

$(\Delta V_g)_{6\text{kg}} = 6(9.81)(0.3)(1 - \sin 30°) = 8.829\,\text{J}$

$(\Delta V_g)_{4\text{kg}} = -4(9.81)(0.3)(1 - \sin 30°) = -11.772\,\text{J}$

$8.829 - 11.772 + \frac{1}{2}6v^2 + 0 = 0 \Rightarrow v = 0.990\,\text{m/s}$



۳-۱۵۴.

(برای دستگاه) $\Delta T + \Delta V_g = 0$ اولین بازه حرکت $(0.4\,\text{m})$

$\frac{1}{2}(4 + 6 + 8)v^2 + 9.81(0.4)(8 - 4 - 6) = 0$

$\Rightarrow v^2 = 0.872 \Rightarrow v = 0.934\,\text{m/s}$

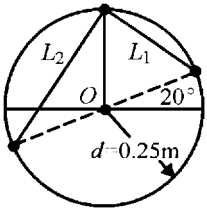
دومین بازه برای وزنه‌های 6 و 8 کیلوگرمی $\Delta T + \Delta V_g = 0$

$0 - \frac{1}{2}(6 - 8)(0.872) + 9.81(h - 0.4)(8 - 6) \Rightarrow h = 0.711\,\text{m}$ یا $h = 711\,\text{mm}$

انرژی جنبشی طوقه حین برخورد با بست به حرارت و صدا تبدیل می‌شود.

۳-۱۵۵.

$$\begin{cases} L_1 = 2d\sin\left(\frac{90° - 20°}{2}\right) = 0.287\,\text{m} \\ \delta_1 = 0.25\sqrt{2} - L_1 = 0.0668\,\text{m} \\ L_2 = 2d\sin\left(\frac{90° + 20°}{2}\right) = 0.410\,\text{m} \\ \delta_2 = L_2 - 0.25\sqrt{2} = 0.0560\,\text{m} \end{cases}$$



می‌توان از تغییر انرژی پتانسیل مربوط به گویها که مساوی و برخلاف هم هستند صرفنظر نمود.

مبنا نقطه O : $T_1 + V_1 = T_2 + V_2$

$0 - mgd\cos 20° + \frac{1}{2}k\delta_1^2 + \frac{1}{2}k\delta_2^2 = 3\left(\frac{1}{2}md^2\dot{\theta}^2\right) - mgd$

$$0 - 3(9.81)(0.25)\cos 20° + \frac{1}{2}1200(0.0668)^2 + \frac{1}{2}1200(0.0560)^2 = \frac{3}{2}3(0.25)^2\dot{\theta}^2 - 3(9.81)(0.25)$$

$$\Rightarrow \dot{\theta} = 4.22 \text{ rad/s}$$

.۱۵۶-۳

$$\overline{AO} = \sqrt{450^2 + 750^2} = 875 \text{ mm}$$

$$U'_{1\text{-}2} = \Delta T + \Delta V_e + \Delta V_g$$

$$\Delta T = \frac{1}{2}(0.9)v^2 - 0 = 0.45v^2$$

$$\Delta V_e = \frac{1}{2}(24)\left[(0.500 - 0.375)^2 - (0.875 - 0.375)^2\right] = -2.81$$

$$\Delta V_g = 0.9(9.81)(0.25) = 2.21$$

$$U'_{1\text{-}2} = 0 \Rightarrow 0.45v^2 - 2.81 + 2.21 = 0 \Rightarrow v = 1.156 \text{ m/s}$$

.۱۵۷-۳ تحلیل نیروها نشان می‌دهد که A به سمت پائین و B به سمت بالا حرکت خواهد کرد.

سینماتیک : $3v_A = 2v_B$

مبنا موقعیت اولیه : $T_1 + V_1 = T_2 + V_2$

$$0 + 0 = \frac{1}{2}m_A v_A{}^2 + \frac{1}{2}m_B\left(\frac{3}{2}v_A\right)^2 + m_B g h_B - m_A g h_A$$

$$0 = \frac{1}{2}(40)v_A{}^2 + \frac{1}{2}(8)\frac{9}{4}v_A{}^2 + 8(9.81)(1) - 40(9.81)\left(\frac{2}{3}(1)\sin 20°\right)$$

$$\Rightarrow v_A = 0.616 \text{ m/s} \quad , \quad v_B = \frac{3}{2}v_A = 0.924 \text{ m/s}$$

.۱۵۸-۳

$$U'_{1\text{-}2} = \Delta T + \Delta V_e + \Delta V_g$$ (برای دستگاه)

$$U'_{1\text{-}2} = 50(1.5)\cos 30° = 64.95 \text{ J}$$

$$\Delta T = \frac{1}{2}(2)v^2 = v^2$$

$$\Delta V_e = \frac{1}{2}30\left[(\sqrt{2^2 + 1.5^2} - 1.5)^2 - (2 - 1.5)^2\right] = 11.25 \text{ J}$$

$$\Delta V_g = 2(9.81)1.5 = 29.43 \text{ J}$$

$$\Rightarrow 64.95 = v^2 + 11.25 + 29.43 \Rightarrow v = 4.93 \text{ m/s}$$

۳-۱۵۹. مبنا را در $\theta = 0$ (حالت A) در نظر می‌گیریم و $\theta = 30°$ را در حالت B

$$T_A + V_A = T_B + V_B$$

$$0 + 0 = 2\left(\frac{1}{2} m v_B^2\right) - 2 m g L (1 - \cos\theta) + \frac{1}{2} k \left[2 \frac{L}{2} \sin\theta\right]^2$$

عددگذاری : $0 = 1.5 v^2 - 2(1.5)(9.81)(0.48)(1 - \cos 30°) + \frac{1}{2} 60 [0.48 \sin 30°]^2$

$$\Rightarrow \quad v = 0.331 \text{ m/s}$$

۳-۱۶۰.

$$\Delta T + \Delta V_g + \Delta V_e - U_f = 0$$

تغییر طول فنر در A $\quad \delta_A = \sqrt{1.2^2 + 1.6^2} - 0.5 = 1.5 \text{ m}$

تغییر طول فنر در B $\quad \delta_B = \sqrt{1.2^2 + 1.2^2} - 0.5 = 1.197 \text{ m}$

$$\Delta V_e = \frac{1}{2} k (\delta_B^2 - \delta_A^2) = \frac{1}{2} 30 (1.197^2 - 1.5^2) = -12.26 \text{ J}$$

$$\Delta T = \frac{1}{2} m (v_B^2 - v_A^2) = \frac{1}{2} (2)(5^2 - 3^2) = 16 \text{ J}$$

$$\Delta V_g = W \Delta z = 2(9.81)(0 - 1.6) = -31.39 \text{ J}$$

$$\Rightarrow \quad 16 - 31.39 - 12.29 + U_f = 0 \quad \Rightarrow \quad U_f = 27.68 \text{ J}$$

$$U_f = F_{av} s \quad \Rightarrow \quad F = \frac{27.68}{2} = 13.82 \text{ N}$$

۳-۱۶۱.

$$U'_{1\text{-}2} = \Delta T + \Delta V_g = 0$$

$$\Delta T = \frac{1}{2} m \left[v^2 - \left(\frac{3000}{3.6}\right)^2\right]$$

$$\Delta V_g = -m g R^2 \left(\frac{1}{R} - \frac{1}{2R}\right) = -\frac{m g R}{2} = -\frac{1}{2} m (1.62)(1738) 10^3$$

$$\Rightarrow \quad v^2 - \left(\frac{3000}{3.6}\right)^2 = (1.62)(1738) 10^3 \quad \Rightarrow \quad v = 1873.5 \text{ m/h} \quad \text{یا} \quad v = 6744 \text{ km/h}$$

۳-۱۶۲.

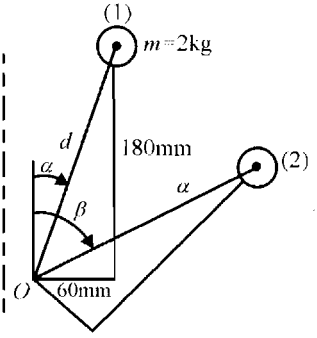
$$\alpha = \tan^{-1} \frac{60}{180} = 18.43°$$

$$\beta = \alpha + 45° = 63.4°$$

کشیدگی فنر $\delta = 2d(\sin\beta - \sin\alpha)$

$= 2\sqrt{0.060^2 + 0.180^2}(\sin 63.4° - \sin 18.43°)$

$= 0.219\,\text{m}$



مبنا نقطهٔ O : $T_1 + V_1 = T_2 + V_2$

$0 + 2mgd\cos\alpha = 0 + 2mgd\cos\beta + \frac{1}{2}k\delta^2$

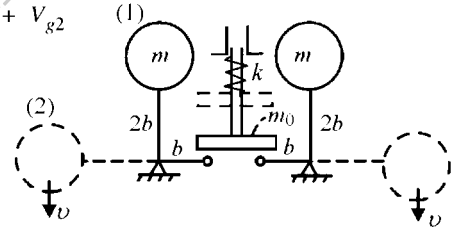
$2(2)(9.81)(0.1897)\cos 18.43° = 2(2)(9.81)(0.1897)\cos 63.4° + \frac{1}{2}k(0.219)^2$

$\Rightarrow k = 155.1\,\text{N/m}$

۳-۱۶۳. $m = 10\,\text{kg}$, $m_0 = 8\,\text{kg}$, $b = 0.080\,\text{m}$, $k = 2000\,\text{N/m}$

سیستم پایستار است. : $E_1 = E_2$

$T_1 + V_{e1} + V_{g1} = T_2 + V_{e2} + V_{g2}$



$T_1 = 0$

$T_2 = 2\left(\frac{1}{2}mv^2\right) = 10v^2$

$V_{e1} = 0$

$V_{e2} = \frac{1}{2}kb^2 = \frac{1}{2}(2000)(0.080)^2 = 6.40\,\text{J}$

$V_{g1} = 2(mg(2b)) = 4(10)(9.81)(0.080) = 31.39\,\text{J}$

$V_{g2} = m_0 gb = 8(9.81)(0.080) = 6.28$

$\Rightarrow 0 + 0 + 31.39 = 10v^2 + 6.40 + 6.28 \Rightarrow v = 1.368\,\text{m/s}$

۳-۱۶۴. $U'_{1\text{-}2} = \Delta T + \Delta V_g + \Delta V_e$ (برای مجموعه)

$U'_{1\text{-}2} = 0$

$\Delta T = \frac{1}{2}mv^2 = \frac{1}{2}(5)v^2 = 2.5v^2\,\text{J}$

$\Delta V_g = mgh = -5 \times 9.81(0.100 + x)\,\text{J}$ (x برحسب متر)

$$\Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}(1.8)(10^3)x^2 = 900x^2\,\text{J}$$

برای $x_{\max}$: $\Delta T = 0$

$$0 = 0 - 5\times 9.81(0.100 + x_{\max}) + 900x_{\max}^2$$

$$x_{\max}^2 - 0.0545x_{\max} - 0.00545 = 0$$

$$x_{\max} = 0.1059\,\text{m} \quad \text{یا} \quad x_{\max} = 105.9\,\text{mm}$$

برای $v_{\max}$: $0 = 2.5v^2 - 5\times 9.81(0.100 + x) + 900x^2$

$$v^2 = 1.962 + 19.62x - 360x^2$$

$$\frac{d(v^2)}{dx} = 19.62 - 720x = 0 \Rightarrow x = 0.0272\,\text{m} \quad \text{یا} \quad x = 27.2\,\text{mm}$$

$$v_{\max} = \sqrt{1.962 + 19.62(0.0272) - 360(0.0272)^2} = 1.493\,\text{m/s}$$

۳-۱۶۵. برای سیستم : $\Delta V_g + \Delta V_e + \Delta T = 0$

$$\Delta V_g = -mgx$$

$$\Delta V_e = \frac{1}{2}kx^2 - 0$$

$$\Delta T = \frac{1}{2}m(v^2 - 0)$$

$$\Rightarrow -mgx + \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = 0 \Rightarrow v^2 = 2gx - \frac{k}{m}x^2$$

$$\frac{d(v^2)}{dx} = 2g - 2\frac{k}{m}x = 0 \quad (\text{برای } v_{\max}^2) \Rightarrow x = \frac{mg}{k}$$

$$v^2 = \frac{2mg^2}{k} - \frac{k}{m}\left(\frac{mg}{k}\right)^2 = \frac{mg^2}{k} \Rightarrow v_{\max} = g\sqrt{\frac{m}{k}}$$

۳-۱۶۶. $U'_{1\text{-}2} = 0 \Rightarrow T_1 + V_{g1} = T_2 + V_{g2}$ ($V_g = 0$ مبنا را در سطح زمین در نظر بگیرید)

$$T_1 = \frac{1}{2}(80 + 4.5)v^2 = 42.25v^2$$

$$T_2 = 0$$

$$V_{g1} = (80 + 4.5)(9.81)(1.1) = 911.84\,\text{J}$$

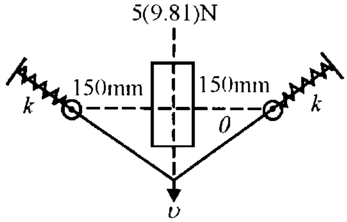
$$V_{g2} = 80(9.81)(5.5) + 4.5(9.81)\left(\frac{4.9}{2}\right) = 4424.56\,\text{J}$$

$$\Rightarrow 42.25v^2 + 911.84 = 0 + 4424.56$$

$$\Rightarrow v = 9.12\,\text{m/s} \quad \text{یا} \quad v = 32.8\,\text{km/h}$$

۳-۱۶۷. برای بازهٔ $\theta = 0$ تا $\theta = 30°$: $\Delta V_g = -5(9.81)(0.15 \tan 30°) = -4.25\,\text{J}$

فشردگی فنر $x = \dfrac{0.150}{\cos 30°} - 0.150 = 0.023\,\text{m}$



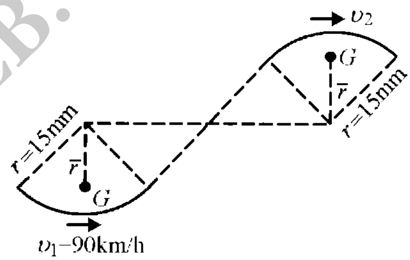
$$\Delta V_e = 2\left\{\frac{1}{2}(3.5)10^3(0.023)^2\right\} = 1.88\,\text{J}$$

$$\Delta T = \frac{1}{2}(5)\,v^2 = 2.5v^2$$

$$\Delta T + \Delta V_g + \Delta V_e = 0$$

$$2.5v^2 - 4.25 + 1.88 = 0 \quad \Rightarrow \quad v = 0.972\,\text{m/s}$$

۳-۱۶۸. فرض کنید m جرم کل قطار باشد.



$$\bar{r} = \frac{2\sqrt{2}}{\pi}\,r = \frac{2\sqrt{2}\,(15)}{\pi} = 13.50\,\text{m}$$

برای کل مجموعه واگنها : $\Delta T + \Delta V_g = 0$

$$\frac{1}{2}m\,(v_2^2 - v_1^2) + mg\,(2\bar{r}) = 0$$

$$v_2^2 = v_1^2 - 4g\bar{r}$$

$$v_2^2 = \left(\frac{90}{3.6}\right)^2 - 4(9.81)(13.5) \Rightarrow$$

$v_2 = 9.76\,\text{m/s}$ یا $v_2 = 35.1\,\text{km/h}$

۳-۱۶۹. $U' = \Delta T + \Delta V_g = 0$

$$\Delta V_g = \frac{-mgR^2}{r} = -9.825\,m\,[6371(10^3)]^2\left(\frac{1}{(2500 + 6371)10^3} - \frac{1}{(2200 + 6371)10^3}\right)$$

$$= 1.573(10^6)\,\text{m}$$

$$\Delta T = \frac{1}{2}m\left(v_B^2 - \left[\frac{25000}{3.6}\right]^2\right)$$

$\Rightarrow \dfrac{1}{2}v_B^2 - \dfrac{1}{2}\left(\dfrac{25000}{3.6}\right)^2 + 1.573(10^6) = 0 \Rightarrow v_B = 6714\,\text{m/s}$ یا $v_B = 24170\,\text{km/h}$

۳-۱۷۰. کل انرژی که ثابت می‌باشد $E = T_A + V_{gA} = T_P + V_{gP}$

$$\frac{1}{2}m v_A^2 - \frac{m g R^2}{r_A} = \frac{1}{2}m v_P^2 - \frac{m g R^2}{r_P}$$

$$v_A^2 = v_P^2 - 2gR^2\left(\frac{1}{r_P} - \frac{1}{r_A}\right) \Rightarrow v_A = \sqrt{v_P^2 - 2gR^2\left(\frac{1}{r_P} - \frac{1}{r_A}\right)}$$

۳-۱۷۱.

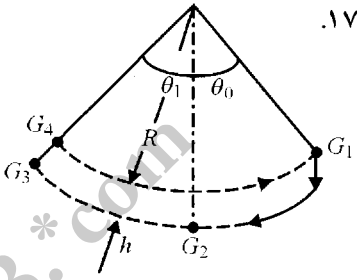
$$U'_{1\text{-}3} = \Delta T + \Delta V_g$$

$$U'_{1\text{-}2} = U'_{2\text{-}3} = 0$$

$$\Delta T = 0 \Rightarrow V_{g1} = V_{g3}$$

$$\Rightarrow R\cos\theta_0 = (R + h)\cos\theta_1$$

$$\Rightarrow \theta_1 = \cos^{-1}\left(\frac{R}{R + h}\cos\theta_0\right)$$



۳-۱۷۲.

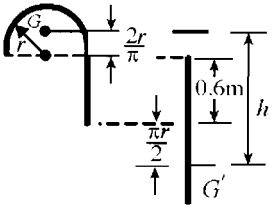
$$r = 0.5\,\text{m}$$

$$\frac{2r}{\pi} = \frac{2(0.5)}{\pi} = 0.3183\,\text{m}$$

$$\pi r = 1.571\,\text{m}$$

$$h = 0.318 + 0.6 + 0.785 = 1.704\,\text{m}$$

$$\Delta V_g + \Delta T = 0$$



فرض کنید ρ جرم واحد طول زنجیر باشد

$$\Delta V_g = -\rho g\pi rh = -\rho(9.81\pi \times 0.5 \times 1.704) = -26.25\rho$$

$$\Delta T = \frac{1}{2}\rho(1.571 + 0.6)v^2 = 1.0855\rho v^2$$

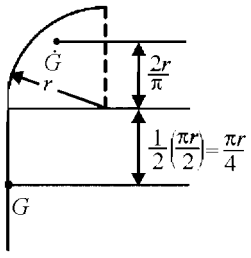
$$\Rightarrow 1.0855v^2 - 26.25 = 0 \Rightarrow v = 4.92\,\text{m/s}$$

۳-۱۷۳. برای کل زنجیر : $U'_{1\text{-}2} = \Delta(T + V_g) = 0$

$$\Delta T = \frac{1}{2}m v^2 = \frac{1}{2}\rho\left(\frac{\pi r}{2}\right)v^2 = \frac{1}{4}\rho\pi r v^2$$

$$\Delta V_g = -\rho\left(\frac{\pi r}{2}\right)g\left[\frac{\pi r}{4} + \frac{2r}{\pi}\right] = -\frac{1}{2}\rho g r^2\pi\left(\frac{\pi}{4} + \frac{2}{\pi}\right)$$

$$\Rightarrow \frac{1}{4}\rho\pi r v^2 - \frac{1}{2}\rho g r^2\pi\left(\frac{\pi}{4} + \frac{2}{\pi}\right) = 0 \Rightarrow v = \sqrt{gr\left(\frac{\pi}{2} + \frac{4}{\pi}\right)}$$



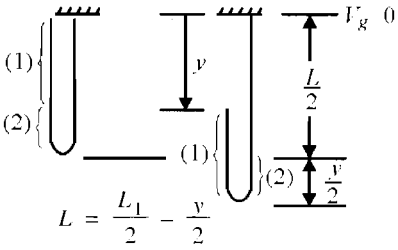
۳-۱۷۴ ◀

$$(\Delta V_g)_{L1} = -\rho g L_1 y = -\rho g \left(\frac{L}{2} - \frac{y}{2}\right) y$$

$$(\Delta V_g)_{L2} = -\rho g \frac{y}{2}\frac{y}{2} = -\rho g \frac{y^2}{4}$$

$$\Delta V_g = -\rho g \left(\frac{Ly}{2} - \frac{y^2}{4}\right)$$

$$\Delta T = \frac{1}{2}\rho L_1 v^2 = \frac{1}{2}\rho \left(\frac{L}{2} - \frac{y}{2}\right) v^2$$



$$L = \frac{L_1}{2} - \frac{y}{2}$$

$$U'_{1\text{-}2} = \Delta V_g = \Delta T = 0 \;:\; -\rho g \left(\frac{Ly}{2} - \frac{y^2}{4}\right) - \frac{1}{2}\rho \left(\frac{L}{2} - \frac{y}{2}\right) v^2 = 0$$

$$\Rightarrow v = \sqrt{2 g y \frac{L - \frac{y}{2}}{L - y}}$$

وقتی $y = L$ باشد، $v \to \infty$.

اتلاف انرژی پتانسیل در آخرین المان متحرک طناب متمرکز می‌شود.

کل اتلاف انرژی برابر $\frac{\rho g L^2}{4}$ می‌باشد (که به حرارت و صدا تبدیل می‌شود)

۳-۱۷۵ ◀

$$U'_{1\text{-}2} = \Delta T + \Delta V$$

$$U'_{1\text{-}2} = M\frac{x}{r}$$

$$\Delta V_e = 0$$

$$V_{g2} = -g\left[m(L - x) + mx + \rho(L - x)\frac{L - x}{2} + \rho x \frac{x}{2}\right]\sin\theta$$

$$= -g\sin\theta\left\{mL + \frac{\rho}{2}[(L - x)^2 + x^2]\right\}$$

$$V_{g1} = -g\sin\theta\left\{mL + \rho L\frac{L}{2}\right\}$$

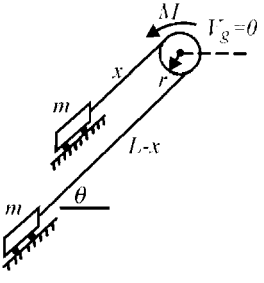
$$\Delta V_g = -g\sin\theta\left\{mL + \frac{\rho}{2}[(L - x)^2 + x^2] - mL - \frac{\rho L^2}{2}\right\}$$

$$= -g\sin\theta\left\{\frac{\rho}{2}[2x^2 - 2Lx]\right\}$$

$$\Delta T = \frac{1}{2}(2m + \rho L)v^2$$

$$\Rightarrow M\frac{x}{r} = \frac{1}{2}(2m + \rho L)v^2 - g\sin\theta\left\{\frac{\rho}{2}[2x^2 - 2Lx]\right\}$$

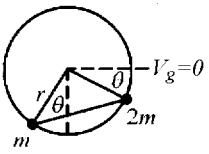
$$\Rightarrow v = \sqrt{\frac{2}{2m + \rho L}}\sqrt{\frac{Mx}{r} - \rho g x(L - x)\sin\theta}$$



۳-۱۷۶. ◀ برای کل واحد $U' = \Delta T + \Delta V_g = 0$

$$\Delta V_g = (-2mgr\sin\theta - mgr\cos\theta) - (-mgr + 0)$$

$$= mgr(-2\sin\theta - \cos\theta + 1)$$



$$\Rightarrow \frac{1}{2}3mv^2 = 0 + mgr(-2\sin\theta - \cos\theta + 1) = 0$$

$$\Rightarrow \frac{v^2}{gr} = \frac{2}{3}(2\sin\theta + \cos\theta - 1)$$

(الف) وقتی میله افقی است که $\theta = 45°$

$$\frac{v^2}{gr} = \frac{2}{3}(2\sin 45° + \cos 45° - 1) = 0.748 \Rightarrow v_{45°} = 0.865\sqrt{gr}$$

(ب) $\frac{d}{d\theta}\left(\frac{v^2}{gr}\right) = \frac{2}{3}(2\cos\theta - \sin\theta) = 0$ (برای v_{max})

$$\Rightarrow \tan\theta = 2 \Rightarrow \theta = \tan^{-1}2 = 63.4°$$

$$\Rightarrow \frac{v_{max}^2}{gr} = \frac{2}{3}(2\sin 63.4° + \cos 63.4° - 1) = 0.824 \Rightarrow v_{max} = 0.908\sqrt{gr}$$

(ج) $T = \Delta T = 0$ وقتی $\theta = \theta_{max}$

$$\Rightarrow 2\sin\theta + \cos\theta - 1 = 0$$

$$2\sqrt{1 - \cos^2\theta} = 1 - \cos\theta \Rightarrow 5\cos^2\theta - 2\cos\theta - 3 = 0$$

$$\Rightarrow \cos\theta = 0.2 \pm 0.8 = 1 \text{ یا } 0.6 \Rightarrow \theta = 0 \text{ یا } \theta_{max} = 126.9°$$

۳-۱۷۷.

$$\int \Sigma F\,dt = \Delta G$$

$$(20000)(3\times 60) = 30000(v - 24000)\frac{1000}{3600}$$

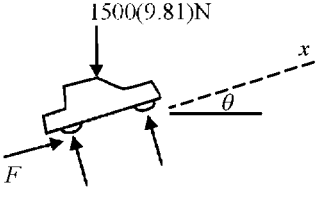
$$\Rightarrow v = 24432\ \text{km/h}$$

۳-۱۷۸.

$$\theta = \tan^{-1}\frac{1}{10} = 5.71°$$

$$\int \Sigma F_x\,dt = \Delta G_x:$$

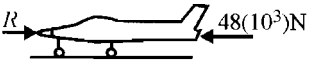
$$\left[F - 1500(9.81)\sin 5.71°\right]8 = 1500(60 - 30)\frac{1000}{3600}$$



$$\Rightarrow F = 3030\ \text{N} \quad \text{یا} \quad F = 3.03\ \text{kN}$$

۳-۱۷۹. $\int \Sigma F\,dt = m\,\Delta v$

$[48(10^3) - R]\,10 = 6450\left(\frac{250}{3.6} - 0\right)$



$\Rightarrow R = 3208\,\text{N}$ یا $R = 3.21\,\text{kN}$

۳-۱۸۰. بقای مومنتوم خطی :

$\xrightarrow{+} 0.075(600) = 50.075\,v_f \Rightarrow v_f = 0.899\,\text{m/s}$

انرژی اولیه $T_1 = \frac{1}{2}(0.075)(600)^2 = 13500\,\text{J}$

انرژی نهایی $T_2 = \frac{1}{2}(50.075)(0.899)^2 = 20.2\,\text{J}$

اتلاف انرژی مطلق $|\Delta E| = T_1 - T_2 = 13480\,\text{J}$

درصد اتلاف $n = \frac{|\Delta E|}{T_1}(100\%) = 99.9\%$

۳-۱۸۱. $\Delta G = 0 : 80000(3) + 60000(5) = (80000 + 60000)\,v$

$\Rightarrow v = 3.857\,\text{km/h}$

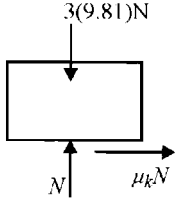
$|\Delta E| = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 - \frac{1}{2}(m_A + m_B)v^2$

$= \frac{1}{2}(80000)\left(\frac{3}{3.6}\right)^2 + \frac{1}{2}(60000)\left(\frac{5}{3.6}\right)^2 - \frac{1}{2}(140000)\left(\frac{3.857}{3.6}\right)^2 = 5297\,\text{J}$

۳-۱۸۲. برای مجموعه گلوله و قطعه چوبی $\Delta G = 0 \Rightarrow G_1 = G_2$

$(0.060)(600) = (0.060)(400) + 3\,v$

$\Rightarrow v = 4\,\text{m/s}$ سرعت اولیه قطعه چوبی



برای قطعه چوبی $U = \Delta T$:

$-\mu_k(3\times 9.81)(2.70) = \frac{1}{2}3(0 - 4^2) \Rightarrow \mu_k = 0.302$

۳-۱۸۳.

$$\int F\,dt = m\,\Delta v$$

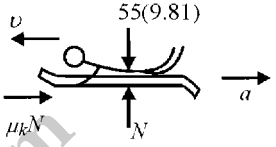
$$2(26)10^3 t = 90(10^3)\,\frac{[28100 - 28000]}{3.6} \quad \Rightarrow \quad t = 48.1\,\text{s}$$

۳-۱۸۴.

$$\Delta G = 0 \;:\; 45(4.6) = (45 + 10)\,v \quad \Rightarrow \quad v = 3.76\,\text{m/s}$$

$$v^2 = 2as \;:\; a = \frac{v^2}{2s} = \frac{3.76^2}{2(25)} = 0.283\,\text{m/s}^2$$

$$\Sigma F = m\,a \;:\; \mu_k(55 \times 9.81) = 55(0.283) \quad \Rightarrow \quad \mu_k = 0.0288$$

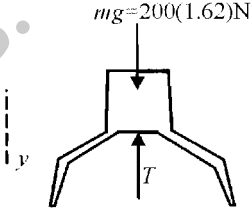


۳-۱۸۵.

$$\int \Sigma F\,dt = m\,\Delta v_y$$

$$200(1.62)(5) - \left[\frac{1}{2}2(800) + 2(800)\right] = 200\,(v - 6)$$

$$\Rightarrow \quad v = 2.10\,\text{m/s}$$

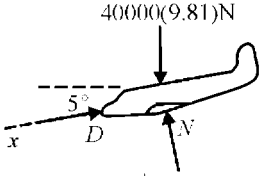


۳-۱۸۶.

$$\int \Sigma F_x\,dt = \Delta G_x$$

$$[(40000)(9.81)\sin 5° - D]120 = 40000\left(\frac{600 - 650}{3.6}\right)$$

$$\Rightarrow \quad D = 38829\,\text{N} \quad \text{یا} \quad D = 38.8\,\text{kN}$$

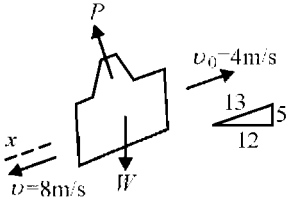


۳-۱۸۷.

$$\int \Sigma F_x\,dt = m\,\Delta v_x$$

$$W\left(\frac{5}{13}\right)t = \frac{W}{9.81}(8 - [-4])$$

$$\Rightarrow \quad t = 3.18\,\text{s}$$



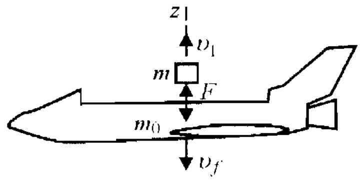
۳-۱۸۸.

$$\Delta G = 0$$

$$600(18000) - \left\{400\,v_3 + 200(18060)\right\} = 0 \quad \Rightarrow \quad v_3 = 17970\,\text{km/h}$$

.۱۸۹-۳

سرعت نسبی : $v_1 + v_f = 0.3 \text{ m/s}$ (1)



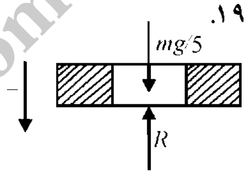
$$\left.\begin{array}{l}\int F\,dt = m v_1 \\ \int -F\,dt = m_f(-v_f)\end{array}\right\} \Rightarrow m v_1 = m_0 v_f$$

$$\Rightarrow 800 v_1 = 90000 v_f \qquad (2)$$

حل (1) و (2) : $v_f = 0.3 - \dfrac{90000}{800} v_f \Rightarrow v_f = 0.00264 \text{ m/s}$

$$F_{av} \int_0^4 dt = 90000(0.00264) \Rightarrow F_{av} = \frac{90(2.64)}{4} = 59.5 \text{ N}$$

.۱۹۰-۳



واشر : $\dfrac{m}{5} v + \left[\dfrac{mg}{5} - R\right] \Delta t = 0$

$$\Rightarrow R = \frac{m}{5}\left(\frac{v}{\Delta t} + g\right)$$

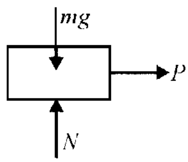
انرژی اولیه : $\dfrac{1}{2} \cdot \dfrac{6m}{5} v^2 = \dfrac{3}{5} m v^2$

انرژی نهایی : $\dfrac{1}{2} m v^2$

$$\Rightarrow n = \frac{\frac{3}{5} - \frac{1}{2}}{\frac{3}{5}} (100\%) = 16.67\%$$

.۱۹۱-۳

$$\xrightarrow{+} \quad m v_1 + \int \Sigma F\,dt = m v_2$$



$$0 + \int_0^t F_0 e^{-bt}\,dt = m v$$

$v = \dfrac{F_0}{mb}(1 - e^{-bt}) \Rightarrow v \to \dfrac{F_0}{mb}$ وقتی $t \to \infty$

$$\frac{ds}{dt} = \frac{F_0}{mb}(1 - e^{-bt}) \Rightarrow \int_{s_0=0}^{s} ds = \int \frac{F_0}{mb}(1 - e^{-bt})\,dt$$

$$\Rightarrow s = \frac{F_0}{mb}\left[t + \frac{1}{b}(1 - e^{-bt})\right]$$

.۱۹۲-۳

$$\Delta G = 0$$

$$(0.140)(600) - [0.140 + 3 \times 0.100] v = 0 \Rightarrow v = 190.9 \text{ m/s}$$

$$|\Delta E| = \frac{1}{2}(0.140)(600)^2 - \frac{1}{2}(0.140 + 0.300)(190.9)^2 = 17.18(10^3) \text{ J}$$

۳-۱۹۳.

$\Delta G = 0$

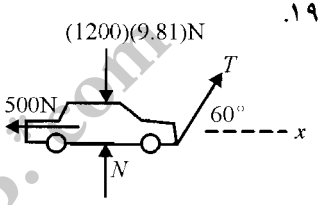
$320(28) - (320 + 20\times 18)\,v = 0 \Rightarrow v = 13.18\,\text{m/s}$ سرعت اولیه زنجیر

$\int \Sigma F\,dt = m\,\Delta v$

$(20\times 18)\,9.81\,(0.7)\,t = (320 + 20\times 18)(13.18) \Rightarrow t = 3.62\,\text{s}$

۳-۱۹۴.

$m\,v_{x1} + \int_{t_1}^{t_2} \Sigma F_x\,dt = m\,v_{x2}$

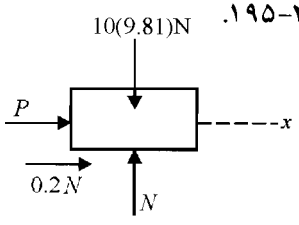


$1200\left(\frac{30}{3.6}\right) + [-500 + T\cos 60°]15 = 1200\left(\frac{70}{3.6}\right)$

$\Rightarrow T = 2780\,\text{N}$

۳-۱۹۵.

$\xrightarrow{+} m\,v_1 + \int_0^t \Sigma F\,dt = 0$

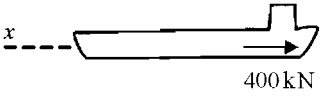


$-10(1.2) + 20(0.2) + 10(t - 0.2) + [0.2(10)(9.81)]\,t = 0$

$\Rightarrow t = 0.338\,\text{s}$

۳-۱۹۶.

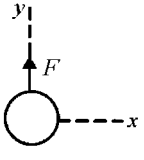
$\int \Sigma F_x\,dt = \Delta G_x$



$-400(10^3)\,t = [150(10^6)][-2 - 2]\frac{1.852}{3.6} \Rightarrow t = 771.7\,\text{s}$ یا $t = 12.86\,\text{min}$

۳-۱۹۷.

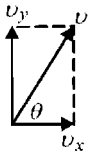
$\int \Sigma F_y\,dt = \Delta G_y$



$\int_0^4 \left(2 + \frac{3t^2}{4}\right) dt = 2.4\left(v_y - \left[-\frac{3}{5}5\right]\right)$

$2t + \frac{t^3}{4}\Big|_0^4 = 2.4(v_y + 3) \Rightarrow v_y = 7\,\text{m/s}$

$$\int \Sigma F_x\,dt = \Delta G_x$$



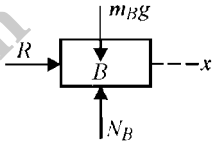
$$0 = 2.4\left(v_x - \frac{4}{5}5\right) \Rightarrow v_x = 4\ \text{m/s} \quad \text{ثابت}$$

$$v = \sqrt{4^2 + 7^2} = 8.06\ \text{m/s} \quad , \quad \theta = \tan^{-1}\frac{7}{4} = 60.3^\circ$$

۳-۱۹۸.

(الف) $m_A v_A = (m_A + m_B)v' \Rightarrow v' = \dfrac{m_A}{m_A + m_B}v_A = \dfrac{1800}{1800 + 900}30 = 20\ \text{km/h}$

(ب) اتومبیل B



$$m_B v_B + R\,\Delta t = m_B v'$$

$$0 + R(0.1) = 900\left(\frac{20}{3.6}\right) \Rightarrow R = 50000\ \text{N} \quad \text{یا} \quad R = 50\ \text{kN}$$

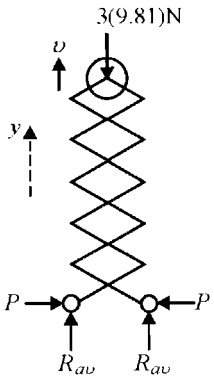
(طبق قانون سوم نیوتن، نیرویی که اتومبیل B روی A وارد می‌کند 50 kN به سمت چپ است)

(ج) $$a_A = \frac{\Delta v}{\Delta t} = \frac{\frac{(20 - 30)}{3.6}}{0.1} = -27.8\ \text{m/s}^2$$

$$a_B = \frac{\Delta v}{\Delta t} = \frac{\frac{(20 - 0)}{3.6}}{0.1} = 55.6\ \text{m/s}$$

۳-۱۹۹.

$v_1 = 2\ \text{m/s} \quad , \quad v_2 = 4\ \text{m/s} \quad , \quad \Delta t = 2\ \text{s}$



$$\int_0^2 \Sigma F_y\,dt = m\,\Delta v_y$$

$$[2R_{av} - 3(9.81)]2 = 3(4 - 2) \Rightarrow R_{av} = 16.22\ \text{N}$$

۳-۲۰۰.

$$\Delta G_x = 0 \quad : \quad 1500(48) = (1500 + 1600)v_x$$

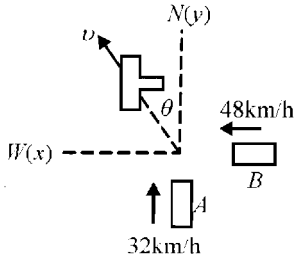
$$\Rightarrow v_x = 23.2\ \text{km/h}$$

$$\Delta G_y = 0 \quad : \quad 1600(32) = (1500 + 1600)\, v_y$$

$$\Rightarrow \quad v_y = 16.5\,\text{km/h}$$

$$v = \sqrt{(23.2)^2 + (16.5)^2} = 28.5\,\text{km/h}$$

$$\theta = \tan^{-1}\frac{v_x}{v_y} = \tan^{-1}\frac{23.2}{16.5} = 54.6°$$



۳-۲۰۱.

$$\mathbf{G}_1 = \mathbf{G}_2 \quad : \quad m_s \mathbf{v}_s + m_m \mathbf{v}_m = (m_s + m_m)\,\mathbf{v}$$

$$1000(2000)\mathbf{j} + 10(5000)\left[\frac{+5\,\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}}{\sqrt{5^2 + 4^2 + 2^2}}\right] = (1000 + 10)\mathbf{v}$$

$$\Rightarrow \quad \mathbf{v} = 36.9\mathbf{i} + 1951\mathbf{j} - 14.76\mathbf{k} \ \text{m/s}$$

زاویه بین $\mathbf{v}$ و $\mathbf{v}_s$ $\beta = \cos^{-1}\dfrac{\mathbf{v}\cdot\mathbf{v}_s}{v\,v_s}$

$$= \cos^{-1}\left[\frac{(36.9\mathbf{i} + 1951\mathbf{j} - 14.76\mathbf{k})\cdot 2000\,\mathbf{j}}{\sqrt{36.9^2 + 1951^2 + 14.76^2}\cdot 2000}\right] = 1.167°$$

۳-۲۰۲.

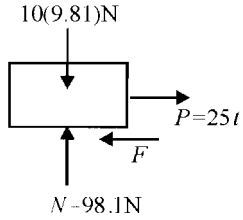
$$F_s = \mu_s N = 0.6\,(98.1) = 58.9\ \text{N}$$

$$F_k = \mu_k N = 0.4\,(98.1) = 39.2\ \text{N}$$

بلوک حرکت نمی‌کند تا اینکه $P = F_s$ یا $25t = 58.9$ یعنی $t = 2.35$ باشد

$$\int \Sigma F\, dt = m\Delta v \quad : \quad \int_{2.35}^{4} (25t - 39.2)\, dt = 10(v - 0)$$

$$\frac{25t^2}{2} - 39.2\,t\,\Big|_{2.35}^{4} = 10\,v \quad \Rightarrow \quad v = 6.61\ \text{m/s}$$



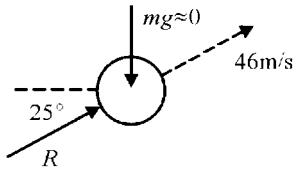
۳-۲۰۳.

$$\overset{+}{\nearrow} \quad R\Delta t = mv$$

$$R(0.001) = 0.0459(46) \Rightarrow R = 2111.4\ \text{N}$$

$$\overset{+}{\nearrow} \quad R = ma \quad : \quad 2111.4 = 0.0459\,a \Rightarrow a = 46000\ \text{m/s}^2$$

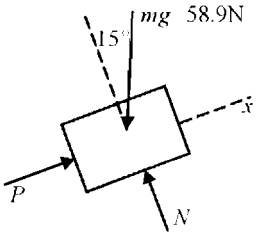
$$v^2 - v_0^2 = 2ad : 46^2 - 0^2 = 2(46000)\,d \Rightarrow d = 0.023\ \text{m} \ \text{یا} \ d = 23\ \text{mm}$$



۳-۲۰۴.

$0 \le t \le 5s \;:\; P = 2t^2$ N

$t \ge 5s \;:\; P = 50$ N



$$\int_0^{5s} P dt = \int_0^{5s} 2t^2 dt = \frac{2t^3}{3}\Big|_0^{5s} = 83.3 \text{ N.s}$$

$$mv_{x1} + \int_{t_1=0}^{t_2} \Sigma F_x \, dt = mv_{x2}$$

$$-6(20) + 83.3 + 50(8 - 5) - 58.9 \sin 15°(8) = 6v_2$$

$$\Rightarrow v_2 = -1.423 \text{ m/s}$$

برای رسیدن گاری به سرعت صفر :

$$-6(20) + 83.3 + 50(t - 5) - 58.9 \sin 15° (t) = 0$$

$$\Rightarrow t = 8.25 \text{ s}$$

۳-۲۰۵.

$$\int \Sigma F_x \, dt = m\Delta v_x$$

$$\int_0^1 -40 \sin \pi t \, dt = 30 (v_1 - 1.5) \Rightarrow \frac{+40}{\pi} \cos \pi t \Big|_0^1 = 30 (v_1 - 1.5)$$

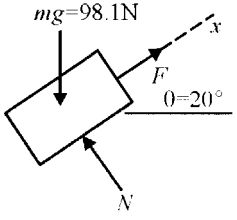
$$\Rightarrow \frac{40}{\pi} (-1 - 1) = 30 (v_1 - 1.5) \Rightarrow v_1 = 0.651 \text{ m/s}$$

برای $t = 2s$: ضربه خالص $= 0 \Rightarrow v_2 = 1.5$ m/s

۳-۲۰۶.

$F = b + 45 \sin 6t$

(الف) $mv_{x1} + \int_{t_1}^{t_2} \Sigma F_x \, dt = mv_{x2}$:



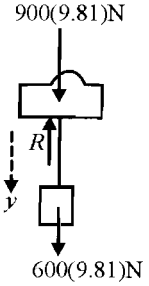
$$0 - (mg \sin \theta) \Delta t + \int_0^{\Delta t} F \, dt = mv$$

$$-98.1 \sin 20° (1.5) + \left[22t - \frac{45}{6} \cos 6t\right]_0^{1.5} = 10v$$

$$\Rightarrow v = -0.299 \text{ m/s}$$

(ب) b باید برابر $mg \sin \theta$ باشد یعنی $b = 98.1 \sin 20° = 33.55$ N

۳-۲۰۷.



کل سیستم : $\int \Sigma F_y \, dt = \Delta G_y$

$$[(600 + 900)9.81 - R]\,6 = (0.5 - 3)$$

$$\Rightarrow R = 14.96 \text{ kN}$$

۳-۲۰۸.

$$\Delta G = 0 \Rightarrow G_1 = G_2$$

$$0.060(600) = (0.060 + 20)\, v_2 \Rightarrow v_2 = 1.795 \text{ m/s}$$

$$U = \Delta T \; : \; v_2 = \sqrt{2gh} \Rightarrow (1.795)^2 = 2(9.81)(2)(1 - \cos\theta)$$

$$\Rightarrow \cos\theta = 0.918 \Rightarrow \theta = 23.4°$$

$$\text{درصد اتلاف انرژی} = \frac{\frac{1}{2} m_1 v_1^2 - (m_1 + m_2)\, gh}{\frac{1}{2} m_1 v_1^2} \times 100\% = \left(1 - \frac{m_1 + m_2}{m_1} \frac{2\, gh}{v_1^2}\right) 100\%$$

$$= \left[1 - \frac{0.060 + 20}{0.060} \frac{2(9.81)(2)(1 - 0.918)}{600^2}\right] 100\% = 99.7\ \%$$

۳-۲۰۹.

$$\int \mathbf{F}\, dt = \mathbf{F}t = m\Delta\mathbf{v}$$

$$\mathbf{F} = \frac{0.20}{0.04}\left([18 \cos 20°]\mathbf{i} + [18 \sin 20°]\mathbf{j} - [-12\ \mathbf{i}]\right)$$

$$= 5(18 \times 0.9397\mathbf{i} + 18 \times 0.3420\mathbf{j} + 12\mathbf{i})$$

$$= 30(4.819\mathbf{i} + 1.026\mathbf{j}) \text{ N}$$

$$F = 30\sqrt{4.819^2 + 1.026^2} = 147.8 \text{ N}$$

$$\beta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{1.026}{4.819} = 12.02°$$

۳-۲۱۰.

$$m v_{x1} + \int_{t_1}^{t_2} \Sigma F_x \, dt = m v_{x2}$$

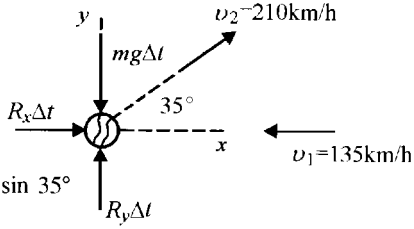
$$-0.146\ \frac{135}{3.6} + R_x\,(0.005) = 0.146\ \frac{210}{3.6} \cos 35°$$

$\Rightarrow R_x = 2490$ N

$mv_{y1} + \int_{t_1}^{t_2} \Sigma F_y\, dt = mv_{y2}$

$0 + R_y(0.005) - 0.146(9.81)(0.005) = 0.146\,\frac{210}{3.6}\sin 35°$

$\Rightarrow R_y = 978$ N



اگر در طی برخورد از وزن صرف‌نظر شود، $R_y = 978$ N فرض خوبی است!
مقدار mg را نمی‌توان بعد از ضربه در نظر نگرفت.

۳-۲۱۱.

$mv_{x1} + \int_{t_1}^{t_2} \Sigma F_x\, dt = mv_{x2}$:

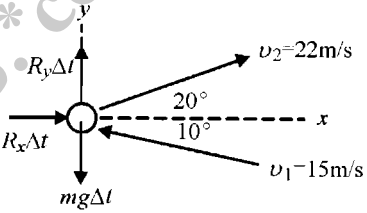
$0.060(-15\cos 10°) + R_x(0.05)$

$= 0.060(22\cos 20°) \Rightarrow R_x = 42.5$ N

$mv_{y1} + \int_{t_1}^{t_2} \Sigma F_y\, dt = mv_{y2}$:

$0.060(15\sin 10°) + R_y(0.05) - 0.060(9.81)(0.05) = 0.060(22\sin 20°) \Rightarrow R_y = 6.49$ N

$mg = 0.060(9.81) = 0.589$ N



نیروی وزن حدود ۹ درصد R_y می‌باشد و نیازی به حذف آن از تحلیل مسأله نمی‌باشد.

$R = \sqrt{R_x^2 + R_y^2} = 43.0$ N

$\beta = \tan^{-1}\frac{R_y}{R_x} = 8.68°$

۳-۲۱۲.

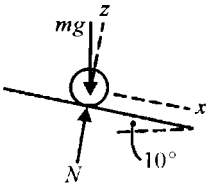
$\int F_x\, dt = m\Delta v_x$

$(mg\sin 10°)\,2 = m\,[v_x - (-3\sin 15°)] \Rightarrow v_x = 2.63$ m/s

$\int F_y\, dt = m\Delta v_y$

$0 = m\,[v_y - 3\cos 15°] \Rightarrow v_y = 2.90$ m/s

$v = \sqrt{v_x^2 + v_y^2} = 3.91$ m/s



۳-۲۱۳.

$$\int F_x\,dt = m\Delta v_x$$

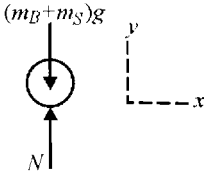
$$0.9\,t = 0.500\,[v_x - (-3\ \sin 30°)] \Rightarrow v_x = \frac{dx}{dt} = 1.8t - 1.5$$

$$\int_0^0 dx = \int_0^t (1.8\,t - 1.5)\,dt \Rightarrow t = 1.667\ \text{s}$$

۳-۲۱۴.

$$m_B v_{Bx} + \cancel{m_S v_{Sx}}^{0} = (m_B + m_S)\,v$$

$$v = \frac{m_B v_{Bx}}{m_B + m_S} = \frac{40(5)\cos 30°}{40 + 5} = 3.85\ \text{m/s}$$



$$m_B v_{By} + \cancel{m_S v_{Sy}}^{0} + \int_0^{\Delta t} [N - (m_B + m_S)\,g]\,dt = 0$$

$$-40\ (5)\ \sin 30° + N(0.05) - (40 + 5)\ 9.81\ (0.05) = 0$$

$$\Rightarrow N = 2441\ \text{N}\ \text{یا}\ N = 2.44\ \text{kN}$$

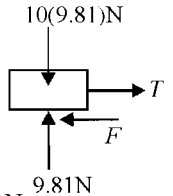
۳-۲۱۵.

$T = F = \mu W = 0.5\ (98.1) = 49.05$ N : بلوک وقتی شروع به حرکت می‌کند که:

$t_1 = \frac{49.05}{100}\,4 = 1.96$ s شروع حرکت زمانی رخ می‌دهد که:

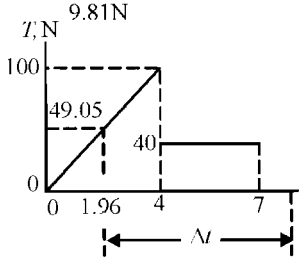
$$\int \Sigma F dt = m\Delta v$$

حداکثر سرعتی که بلوک کسب می‌کند در $t = 4$s رخ می دهد.



$$\frac{100 - 49.05}{2}\,(4 - 1.96) = 10(v - 0) \Rightarrow v_{\max} = 5.19\ \text{m/s}$$

$\Delta v = 0$ برای کل حرکت



$$\Rightarrow \frac{100 + 49.05}{2}\,(4 - 1.96) + 40(7 - 4) - 49.05\,\Delta t = 0$$

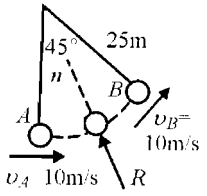
$$\Rightarrow \Delta t = 5.54\ \text{s}$$

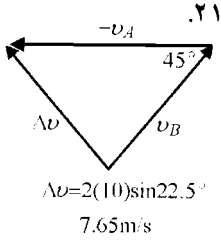
۳-۲۱۶.

$$t_{A+B} = \frac{s}{v} = \frac{r\theta}{v} = \frac{25(\pi/4)}{10} = 1.963 \text{ s}$$

$$\int_0^{1.963s} \mathbf{R}_{n_{av}}\, dt = \mathbf{G}_B - \mathbf{G}_A$$

$$R_{av}(1.963) = 3(7.65) \Rightarrow R_{av} = 11.69 \text{ N}$$





مقدار دقیق لحظه‌ای نیرو $\Sigma F_n = ma_n$: $R = m\frac{v^2}{r} = 3\frac{10^2}{25} = 12$ N

۳-۲۱۷.

$$T_y = 600\cos\theta$$

$$\dot{\theta} = \frac{\pi}{10} \text{ rad/s} \Rightarrow dt = \frac{10}{\pi}\, d\theta$$

$$\int \Sigma F_y\, dt = m\Delta v_y$$

$$\int_0^{\pi/2} 600\cos\left(\frac{10}{\pi}\, d\theta\right) = 260(v_y - 0) \Rightarrow \frac{6000}{\pi}\sin\theta\Big|_0^{\pi/2} = 260\, v_y \Rightarrow v_y = \frac{600}{260\pi} = 7.35 \text{ m/s}$$

۳-۲۱۸.

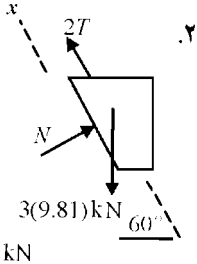
شروع به حرکت : $2T = 3(9.81)\frac{\sqrt{3}}{2} \Rightarrow T = 12.74$ kN

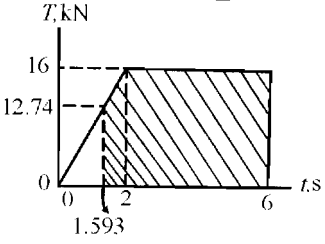
$$t = \frac{12.74}{16} = 1.593 \text{ s}$$

$$\int \Sigma F_x\, dt = m\Delta v_x$$

$$2\left[\frac{16 + 12.74}{2}(2 - 1.593) + 16(6 - 2)\right] - 3(9.81)\frac{\sqrt{3}}{2}(6 - 1.593) = 3(v - 0)$$

$$\Rightarrow v = 9.13 \text{ m/s}$$



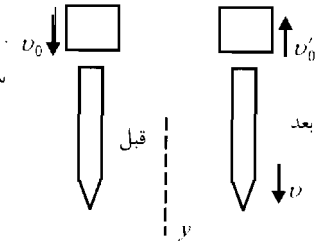


۳-۲۱۹. ◀

سرعت چکش درست قبل از برخورد:

$$v_0 = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1.5} = 5.42 \text{ m/s}$$

$$\int \Sigma F_y\, dt = \Delta G_y$$



برای چکش ضربهٔ وزن ($W_r . \Delta t$) در مقایسه با ضربهٔ نیروی بسیار زیاد برخورد قابل صرف‌نظر است. (بطور مشابه برای شمع نیز جمع برداری وزن و مقاومت نفوذ قابل صرفنظر می‌باشد).

(الف) $\Delta G_y = 0 : (400 + 300)v - 400(5.42) = 0 \Rightarrow v = 3.10$ m/s

(ب) $\Delta G_y = 0 : (400 \times 0) + 300v - 400(5.42) = 0 \Rightarrow v = 7.23$ m/s

(ج) $\Delta G_y = 0$:

در حالت (ج) سرعت روبه‌بالای چکش درست بعد از برخورد برابر است با:

$v_0' = \sqrt{2gh} = \sqrt{2g\,(0.100)} = 1.401$ m/s

$\Rightarrow (300v - 400 \times 1.401) - 400(5.42) = 0 \Rightarrow v = 9.10$ m/s

۳-۲۲۰. ◀

$\mathbf{v}_C = \mathbf{v}_1 + \mathbf{v}_{C/1}$

$v_{Cx} = 50 \cos 15° - v_1 = 48.296 - v_1$ km/h

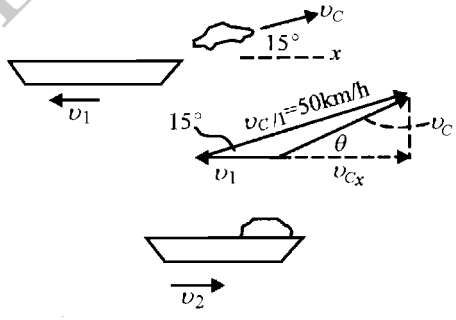
$\Delta G_x = 0 : 500 \times 10^3\, v_1 = 1500\,(48.296 - v_1)$

$\Rightarrow v_1 = 0.144$ km/h یا $v_1 = 40.1$ mm/s

$\Rightarrow v_{Cx} = 48.152$ km/h

$\Delta G_x = 0 : 1500\,(48.152) = 501500\, v_2$

$\Rightarrow v_2 = 0.144$ km/h یا $v_2 = 40.1$ mm/s



۳-۲۲۱.

(الف) $\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$

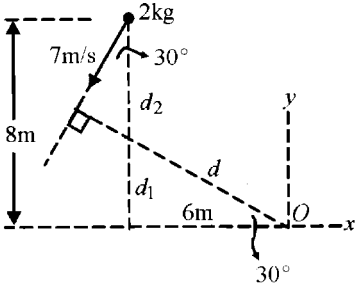
$\mathbf{H}_O = (-6\,\mathbf{i} + 8\mathbf{j}) \times 2\,(7)(-\sin 30°\mathbf{i} - \cos 30°\mathbf{j}) = 128.7\,\mathbf{k}$ kg.m^2/s

(ب) $d_1 = 6 \tan 30° = 3.46$ m

$d_2 = 8 - d_1 = 4.54$ m

$d = \dfrac{6}{\cos 30°} + 4.54 \sin 30° = 9.20$ m

$\curvearrowleft_+ H_O = mvd = 2(7)(9.20) = 128.7$ kgm^2/s



۳-۲۲۲. (الف) $\mathbf{G} = m\mathbf{v} = 3.4\,[\cos 45°\mathbf{i} - \sin 45°\mathbf{j}] = 8.49\mathbf{i} - 8.49\mathbf{j}$ kg.m/s

(ب) $\mathbf{H}_O = \mathbf{r} \times m\mathbf{v} = \mathbf{r} \times \mathbf{G} = 2(\cos 60°\mathbf{i} + \sin 60°\mathbf{j}) \times (8.49\mathbf{i} - 8.49\mathbf{j}) = -23.2\,\mathbf{k}$ kg.m^2/s

(ج) $T = \frac{1}{2} mv^2 = \frac{1}{2}(3)(4)^2 = 24$ J

۳-۲۲۳. $\mathbf{H}_O = \mathbf{r} \times m\mathbf{v} = (a\mathbf{i} + b\mathbf{j} + c\mathbf{k}) \times mv\mathbf{j} = mv(-c\,\mathbf{i} + a\mathbf{k})$

$\dot{\mathbf{H}}_O = \mathbf{M}_O = (a\mathbf{i} + b\mathbf{j} + c\mathbf{k}) \times F\mathbf{k} = F(b\mathbf{i} - a\mathbf{k})$

۳-۲۲۴. مومنتوم زاویه‌ای حول O پایستار است : $H_{O_1} = H_{O_2}$

$$3mv(L) + 2mv(L) = 3mL^2\omega \Rightarrow \omega = \frac{5}{3}\frac{v}{L}$$

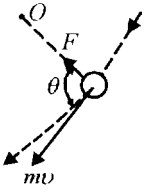
۳-۲۲۵. $H_1 + \int_{t_1}^{t_2} M dt = H_2$

$$0 + 20(0.1)\,t = 4(3)(0.4)^2\left[150\left(\frac{1}{60}\right)(2\pi)\right] \Rightarrow t = 15.08 \text{ s}$$

۳-۲۲۶. $\Sigma M_O = \dot{H}_O = 0 \Rightarrow H_O =$ ثابت

$H_{OA} = H_{OB}$

$m(4)(0.35 \sin 54°) = mv_B(0.230 \sin 65°) \Rightarrow v_B = 5.43$ m/s



۳-۲۲۷. مومنتوم زاویه‌ای ثابت می ماند $\Rightarrow \Sigma M_O = \dot{H}_O = 0$: برای کل سیستم

$H_{O1} = H_{O2} : 2mr^2\omega_0 + 0 = 2mr^2\omega + 2m\,(2r)^2\omega \Rightarrow \omega = \frac{\omega_0}{5}$

اتلاف انرژی جنبشی : $\Delta Q = T_1 - T_2$

$$\Delta Q = 2\left(\frac{1}{2} mr^2\omega_0^2\right) - \left\{2\left(\frac{1}{2} mr^2\omega^2\right) + 2\left(\frac{1}{2} m\,(2r)^2\omega^2\right)\right\}$$

$$= mr^2\omega_0^2 - mr^2\left(5\left(\frac{\omega_0}{5}\right)^2\right) = \frac{4}{5} mr^2\omega_0^2$$

$$n = \frac{\Delta Q}{T_1}(100\%) = \frac{\frac{4}{5} mr^2\omega_0^2}{2\left(\frac{1}{2} mr^2\omega_0^2\right)}(100\%) = 80\%$$

۳-۲۲۸.

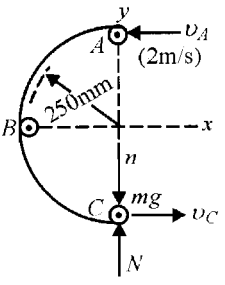
$$T_A + U_{A-C} = T_C$$

$$\frac{1}{2} m v_A^2 + mgh_{A-C} = \frac{1}{2} m v_C^2$$

$$v_C^2 = v_A^2 + 2gh_{A-C} = 2^2 + 2(9.81)(0.500) = 13.81 \text{ m}^2/\text{s}^2$$

$$\Sigma F_y = ma_y : N - 0.110(9.81) = 0.110 \frac{13.81}{0.250} \Rightarrow N = 7.16 \text{ N}$$

$$\dot{\mathbf{H}}_B = \mathbf{M}_B = \left(7.16 - 0.110(9.81)\right)0.25\mathbf{k} = 1.519\mathbf{k} \text{ N.m}$$



۳-۲۲۹.

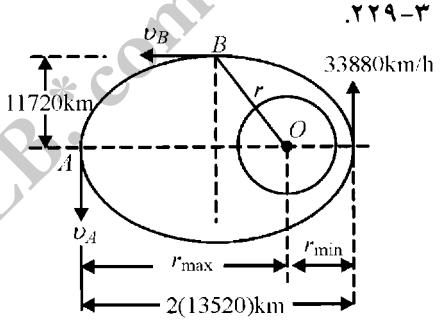
$\Sigma M_O = \dot{H}_O = 0 \Rightarrow H_O =$ ثابت

$r_{min} = 6371 + 390 = 6761$ km

$r_{max} = 2(13520) - 6761 = 20279$ km

ثابت $H_O = 6761(33880) = 11720\, v_B = 20279\, v_A$

$$\Rightarrow \quad v_A = 11300 \text{ km/h}, \quad v_B = 19540 \text{ km/h}$$



۳-۲۳۰.

$\Sigma M_{Oz} = \dot{H}_{Oz}$:

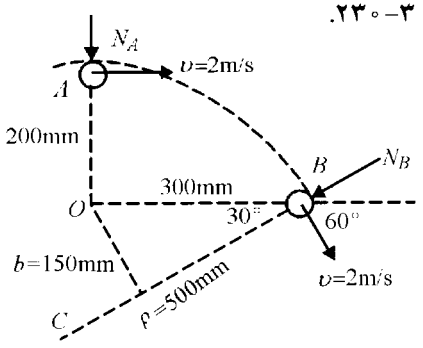
A در نقطه : $\Sigma M_{Oz} = 0 \Rightarrow \dot{H}_{Oz} = 0$

B در نقطه : $\Sigma M_{Oz} = -N_B b$

$$N_B = m\frac{v^2}{\rho} = 0.1\frac{2^2}{0.5} = 0.8 \text{ N} \Rightarrow$$

$$\dot{H}_{Oz} = -N_B b = -0.8(0.150)$$

$$= -0.120 \text{ N.m} \text{ یا } -0.120 \text{ kg.m}^2/\text{s}^2$$



۳-۲۳۱.

$$\Delta H = 0 \Rightarrow 2mr\omega_0(r) - 2m(2r)\omega(2r) = 0 \Rightarrow \omega = \frac{\omega_0}{4}$$

$$\Delta T = 2\left(\frac{1}{2} m[r\omega_0]^2\right) - 2\left(\frac{1}{2} m\left[2r\frac{\omega_0}{4}\right]^2\right) = mr^2\omega_0^2\left(\frac{3}{4}\right)$$

$$n = \frac{\Delta T}{T} = \frac{\frac{3}{4} mr^2\omega_0^2}{mr^2\omega_0^2} = \frac{3}{4}$$

۳-۲۳۲. $\Sigma M_O = \dot{H}_O = 0 \Rightarrow$ بقای مومنتوم زاویه‌ای $\Rightarrow H_{O_1} = H_{O_2}$ (O : هر نقطه‌ای روی مدار)

$\Rightarrow 0.2\,(0.3\cos 30°)^2 4 = 0.2(0.2\cos 30°)\,\omega \Rightarrow \omega = 9$ rad/s

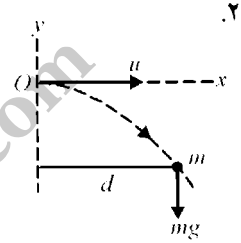
$U'_{1\text{-}2} = \Delta T + \Delta V_g$

$\Delta T = \frac{1}{2}(0.2)\,[(0.2\cos 30° \times 9)^2 - (0.3\cos 30° \times 4)^2] = 0.1350$ J

$\Delta V_g = 0.2(9.81)(0.1\sin 30°) = 0.0981$ J

$U'_{1\text{-}2} = 0.1350 + 0.0981 = 0.233$ J

۳-۲۳۳.



$d = ut$

$\dot{\mathbf{H}}_O = \frac{d\mathbf{H}_O}{dt} = \mathbf{M}_O = -mgd\,\mathbf{k}$

$\int_0^{\mathbf{H}_O} d\mathbf{H}_O = -\int_0^t mgd\mathbf{k}\,dt = -\int_0^t mgut\mathbf{k}\,dt$

$\Rightarrow \mathbf{H}_O = -\frac{1}{2}mgut^2\,\mathbf{k}$

۳-۲۳۴. $\int \Sigma M_O\,dt = \Delta H_O = H_{OB} - H_{OA}$

$H_{OA} = 0.02(4)(0.090)\sin 30° = 0.0036$ kg.m^2/s

$H_{OB} = 0.02(6)(0.180)\sin 60° = 0.01871$ kg.m^2/s

$\Delta H_O = 0.01871 - 0.0036 = 0.01511$ kg.m^2/s

$M_{O_{av}} \times 0.5 = 0.01511 \Rightarrow M_{O_{av}} = 0.0302$ N.m

۳-۲۳۵. بقای مومنتوم زاویه‌ای $= (mv_\theta)_A = (mv_\theta)_B$

$80.47(10^6)(57.45)(10^3) = 120.7(10^6)v_\theta \Rightarrow v_\theta = 38.3(10^3)$ m/s

بقای انرژی : $T_A + V_A = T_B + V_B$

$\frac{1}{2}mv_A^2 - \frac{Gm_s m}{r_A} = \frac{1}{2}mv_B^2 - \frac{Gm_s m}{r_B}$

$\frac{1}{2}(57450)^2 - \frac{1}{2}v_B^2 = 6.673\,(10^{-11})\,333000(5.976)(10^{24})\left|\frac{1}{80.47(10^9)} - \frac{1}{120.7(10^9)}\right|$

$\Rightarrow v_B = 46909$ m/s

$v_r = \sqrt{v_B^2 - v_\theta^2} = \sqrt{46909^2 - 38300^2} = 27084$ یا $27.1\,(10^3)$ m/s

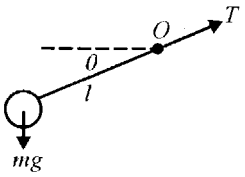
۳-۲۳۶.

$$\Sigma M_O = \dot{H}_O = mg\,l\cos\theta = \frac{d}{dt}(ml^2\dot{\theta}) = ml^2\ddot{\theta}$$

$$\Rightarrow \ddot{\theta} = \frac{g}{l}\cos\theta$$

$$\int \dot{\theta}\,d\dot{\theta} = \int \ddot{\theta}d\theta \Rightarrow \frac{\dot{\theta}^2}{2}\Big|_0^{\dot{\theta}} = \int_0^{\theta}\frac{g}{l}\cos\theta\,d\theta$$

$$\Rightarrow \dot{\theta}^2 = \frac{2g}{l}\sin\theta \Rightarrow \dot{\theta}_{\theta=90} = \sqrt{\frac{2g}{l}}$$



$$\theta = 90^\circ \Rightarrow v = l\dot{\theta} = \sqrt{2gl}$$

اصل کار و انرژی $U = \Delta T$: $mgl = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gl}$

۳-۲۳۷. نیروهای روی ذره هیچ گشتاوری حول محور مرکزی اعمال نمی‌کنند، بنابراین مومنتوم زاویه‌ای حول این محور پایستار است. لذا $\Delta H_z = 0$ بنابراین:

$$mv_0\cos\beta\,(r) = mv\cos\theta\,(r) \Rightarrow v_0\cos\beta = v\cos\theta$$

همچنین انرژی پایستار است. بنابراین:

$$\Delta T + \Delta V_g = 0 \Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 - mgh = 0$$

$$v \text{ حذف} \Rightarrow \cos\theta = \frac{v_0\cos\beta}{\sqrt{v_0^2 + 2gh}} \Rightarrow \theta = \cos^{-1}\frac{\cos\beta}{\sqrt{1 + \frac{2gh}{v_0^2}}}$$

۳-۲۳۸. مومنتوم زاویه‌ای سیستم طی برخورد پایستار است. لذا $(+\circlearrowleft) H_{O_1} = H_{O_2}$

$$0.050(300)(0.4\cos 20^\circ) - 3.2\,(0.2)^2\,6 - 3.2\,(0.4)^2 6 = (0.05 + 3.2)(0.4)^2\omega' + 3.2(0.2)^2\omega'$$

$$\Rightarrow \omega' = 2.77 \text{ rad/s}$$

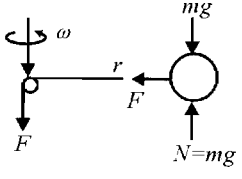
تغییرات انرژی پس از برخورد : $T' + V' = \cancel{T}^{0} + V$ (مبنا نقطه O)

$$\frac{1}{2}(0.05 + 3.2)[0.4(2.77)]^2 + \frac{1}{2}(3.2)\,[0.2(2.77)]^2 + [3.2(0.2) - (3.2 + 0.05)(0.4)]\,9.81$$

$$= 0 + [3.2(0.2) - (3.2 + 0.05)(0.4)]\,9.81\cos\theta$$

$$\Rightarrow \theta = 52.1^\circ$$

۳-۲۳۹. مومنتوم زاویه‌ای حول محور مرکزی پایستار است. لذا:



$$\dot{H} = \frac{d}{dt}(mr\omega^2) = 0$$

$$m(2r\omega dr + r^2 d\omega) = 0 \Rightarrow \frac{d\omega}{dr} = -\frac{2\omega}{r}$$

$\Sigma F_n = ma_n : F = mr\omega^2$

$dU = dT \;: -Fdr = d\left(\frac{1}{2}mr^2\omega^2\right)$

$\Rightarrow -mr\,\omega^2 dr = m(r\omega^2 dr + r^2\omega d\omega) = m\left(r\omega^2 dr + r^2\omega\left[-\frac{2\omega}{r}dr\right]\right) = -mr^2\omega^2 dr$

۳-۲۴۰. معادله مسیر: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a = 1.5 \;,\; b = 1.2)$

بقای مومنتوم زاویه‌ای حول O: $mr_A v_A = mr_B v_B \Rightarrow v_B = \frac{r_A}{r_B}v_A = \frac{a}{b}v_A = \frac{1.5}{1.2}(2) = 2.5$ m/s

$y = b\left[1 - \left(\frac{x}{a}\right)^2\right]^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}b\left[1 - \left(\frac{x}{a}\right)^2\right]^{-1/2}\cdot\left(\frac{2x}{a^2}\right) = -\frac{bx}{a^2}\left[1 - \left(\frac{x}{a}\right)^2\right]^{-1/2}$

$\frac{d^2y}{dx^2} = -\frac{b}{a^2}\left[1 - \left(\frac{x}{a}\right)^2\right]^{-1/2} - \frac{bx}{a^2}\left(-\frac{1}{2}\right)\left[1 - \left(\frac{x}{a}\right)^2\right]^{-3/2}\left(\frac{-2x}{a^2}\right)$

$= -\frac{b}{a^2}\left[1 - \left(\frac{x}{a}\right)^2\right]^{-1/2} - \frac{bx^2}{a^4}\left[1 - \left(\frac{x}{a}\right)^2\right]^{-3/2}$

$\left.\frac{dy}{dx}\right|_{x=0} = 0 \quad , \quad \left.\frac{d^2y}{dx^2}\right|_{x=0} = -\frac{b}{a^2} = \frac{-1.2}{(1.5)^2} = -0.533$

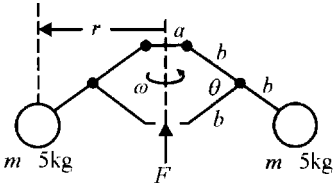
$\rho_{xy} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{[1 + 0]^{3/2}}{-0.533} = -1.875$

$\Sigma F_n = m\frac{v^2}{\rho} \;:\; T_B = 0.7\frac{(2.5)^2}{1.875} = 2.33$ N

۳-۲۴۱. ◀

$\omega_0 = \frac{40(2\pi)}{60} = 4.19$ rad/s

$a = 0.1$ m , $b = 0.3$ m



$\theta = 90° \;,\; r_0 = 0.1 + 2(0.3)\cos 45° = 0.524$ m

$\theta = 60° \;,\; r = 0.1 + 2(0.3)\cos 30° = 0.620$ m

$\Delta H = 0 \;:\; 2mr_0^2\omega_0 - 2mr^2\omega = 0$

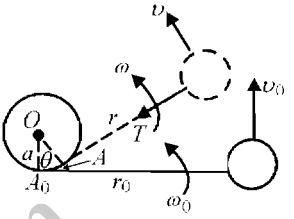
$\Rightarrow \omega = \frac{r_0^2}{r^2}\omega_0 = \left(\frac{0.524}{0.620}\right)^2(4.19) = 3.00$ rad/s یا $\frac{3.00}{2\pi}60 = 28.6$ rev/min

$U = \Delta T + \Delta V_g = 2\left(\frac{1}{2}\text{m}\right)(r^2\omega^2 - r_0^2\omega_0^2) + 2\,mg\,\Delta h$

$\Delta h = 2b(\sin 45° - \sin 30°) = 2(0.3)(0.7071 - 0.5) = 0.1243$ m

$U = 5\left([0.620 \times 3.00]^2 - [0.524 \times 4.19]^2\right) + 2(5)(9.81)(0.1243) = 5.34$ J

◀ **۳-۲۴۲.** گشتاور کشش T حول نقطه مماس A صفر است اما A یک نقطه ثابت نیست. گشتاور حول نقطهٔ ثابت O صفر است. بنابراین مومنتوم زاویه‌ای پایستار نیست. همچنین $\Sigma F \neq 0$ بنابراین مومنتوم خطی نیز پایستار نیست.



بقای انرژی : $\frac{1}{2} m(r_0\omega_0)^2 = \frac{1}{2} m(r\omega)^2$

$$\omega = \frac{r_0}{r}\omega_0 = \frac{r_0}{r_0 - a\theta} \text{ یا } \omega = \frac{\omega_0}{1 - a\theta/r_0}$$

$$\curvearrowleft + \Sigma M_O = \dot{H}_O \; : \; -Ta = \frac{d}{dt}(mvr) = mv\dot{r} \quad (\text{چون } v \text{ ثابت است})$$

$$\dot{r} = \frac{d}{dt}(r_0 - a\theta) = -a\dot{\theta} = -a\omega$$

$$\left.\begin{array}{l} \Rightarrow T = -\frac{m}{a} v(-a\omega) = mv\omega \\ v = v_0 = r_0\omega_0 \end{array}\right\} \Rightarrow T = mr_0\omega_0\omega$$

۳-۲۴۳.

$$\left.\begin{array}{l} v = \sqrt{2gh} \\ v' = \sqrt{2gh'} \end{array}\right\} \Rightarrow e = \frac{v'}{v} = \sqrt{\frac{h'}{h}} = \sqrt{\frac{1100}{2100}} = 0.724$$

$$n = \frac{mgh - mgh'}{mgh}(100\%) = \frac{2100 - 1100}{2100}(100\%) = 47.6\%$$

۳-۲۴۴.

مومنتوم خطی سیستم : $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$

$$2(7) + 3(-5) = 2v_1' + 3v_2'$$

$$e = \frac{v_2' - v_1'}{v_1 - v_2} : 0.6 = \frac{v_2' - v_1'}{7 - (-5)}$$

حل دو معادله : $v_1' = -4.52 \text{ m/s} \;,\; v_2' = 2.68 \text{ m/s}$

انرژی اولیه : $T_1 = \frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2v_2^2 = \frac{1}{2}(2)(7)^2 + \frac{1}{2}(3)(5)^2 = 86.5 \text{ J}$

$$T_2 = \frac{1}{2} m_1v_1'^2 + \frac{1}{2} m_2v_2'^2 = \frac{1}{2}(2)(4.52)^2 + \frac{1}{2}(3)(2.68)^2 = 31.2 \text{ J}$$

$$n = \frac{T_1 - T_2}{T_1}(100\%) = \frac{86.5 - 31.2}{86.5}(100\%) = 63.9\%$$

۳-۲۴۵.

$$\left.\begin{array}{l} \text{مومنتوم سیستم} : 2(7) + 3v_2 = 2v_1' + 0 \\ \text{ضریب بازگشت} : \dfrac{-v_1'}{7 - v_2} = 0.6 \end{array}\right\} \Rightarrow \begin{array}{l} v'_1 = -11.66 \ \text{m/s} \\ v_2 = -12.44 \ \text{m/s} \end{array}$$

۳-۲۴۶.

$$\text{مومنتوم خطی سیستم} : m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \xrightarrow{+}$$

$$mv + 0 = mv_A' + pmv_B' \qquad (1)$$

$$\text{ضریب بازگشت} \quad e = \frac{v_B' - v_A'}{v_A - v_B} : 0.1 = \frac{v_B' - v_A'}{v - 0} \qquad (2)$$

$$\text{حل (1) و (2)} : v_A' = \left(\frac{1 - 0.1p}{1 + p}\right)v \quad , \quad v_B' = \frac{1.1}{1 + p}\,v$$

$$p = \frac{1}{2} : v_A' = 0.633\,v \ , \ v_B' = 0.733\,v$$

۳-۲۴۷. حالتی را در نظر بگیرید که $v'_2 = v_1$. با بقای مومنتوم خطی سیستم خواهیم داشت:

$$m_1 v_1 + m_2 \overbrace{v_2}^{0} = m_1 v_1' + m_2 v_2' = m_1 v_1' + m_2 v_1 = \Rightarrow v_1' = \frac{(m_1 - m_2)}{m_1}\,v_1$$

$$\text{ضریب بازگشت} \ e = \frac{v_2' - v_1'}{v_1 - v_2} = \frac{v_1 - \left(\dfrac{m_1 - m_2}{m_1}\right)v_1}{v_1} \Rightarrow \frac{m_1}{m_2} = \frac{1}{e}$$

بنابراین برای آنکه $v'_2 > v_1$ شود باید $\dfrac{m_1}{m_2} > \dfrac{1}{e}$ باشد.

۳-۲۴۸.

$$\Delta G = 0 : m_A v_A + 0 = m_A v_A' + m_B v_B'$$

$$e = 0 \ : \ v_A' = v_B' \ \Rightarrow \ m_A v_A = (m_A + m_B)v_A'$$

$$|\Delta T| = -\frac{1}{2}m_A v_A'^2 - \frac{1}{2}m_B v_B'^2 + \frac{1}{2}m_A v_A^2$$

$$= -\frac{1}{2}m_A\left(\frac{m_A}{m_A + m_B}v_A\right)^2 - \frac{1}{2}m_B\left(\frac{m_A}{m_A + m_B}v_A\right)^2 + \frac{1}{2}m_A v_A^2$$

$$= -\frac{1}{2}\left(\frac{m_A}{m_A + m_B}v_A\right)^2(m_A + m_B) + \frac{1}{2}m_A v_A^2 = \frac{1}{2}\frac{m_A m_B}{m_A + m_B}v_A^2 \qquad \text{(اتلاف)}$$

$$\frac{|\Delta T|}{T} = \frac{1}{2}\frac{m_A m_B}{m_A + m_B}v_A^2 \cdot \frac{1}{\frac{1}{2}m_A v_A^2} = \frac{m_B}{m_A + m_B}$$

۳-۲۴۹.

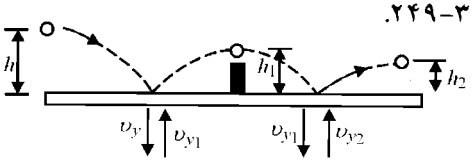
$v_y = \sqrt{2g(h - r)}$

$v_{y1} = ev_y = e\sqrt{2g(h - r)} = \sqrt{2g(h_1 - r)}$

$\Rightarrow 0.9^2(2g)(h - 18.75) = (2g)(225 - 18.75)$

$\Rightarrow h = 273.4$ mm

$v_{y2} = e\,v_{y1} : \sqrt{2g(h_2 - r)} = 0.9\sqrt{2g(h_1 - r)} \Rightarrow h_2 = 185.8$ mm



(توجه: فاصله‌های عمودی به اندازه‌ی $r = 18.75$ mm کم می‌شوند)

۳-۲۵۰.

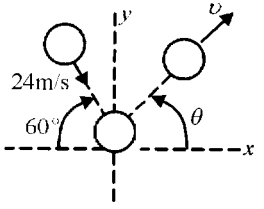
هیچ تغییری در مؤلفه x سرعت رخ نمی‌دهد $\Rightarrow \Sigma F_x = 0$ طی برخورد

$v\cos\theta = 24(0.5) = 12$ m/s

در جهت y : $e = \dfrac{v\sin\theta}{24\cos 30°} = 0.8 \Rightarrow \tan\theta = \dfrac{16.63}{12} = 1.386$

$\Rightarrow \theta = 54.2°$

$v = \dfrac{12}{\cos 54.2°} = 20.5$ m/s



۳-۲۵۱.

$\tan\alpha = \dfrac{d/2}{x}$, $\tan\beta = \dfrac{d}{d - x}$

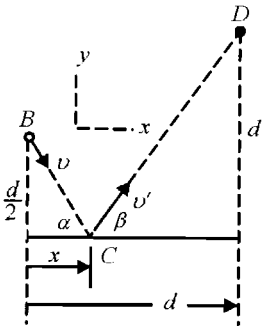
(الف) $e = 1$: $\alpha = \beta$

$\dfrac{d/2}{x} = \dfrac{d}{d - x} \Rightarrow x = \dfrac{d}{3}$

(ب) $e = 0.8$: $\alpha \neq \beta$

$\tan\alpha = \dfrac{|v_y|}{v_x}$, $\tan\beta = \dfrac{v_y'}{v_x'} = \dfrac{0.8\,|v_y|}{v_x} \Rightarrow \tan\alpha = \dfrac{1}{0.8}\tan\beta$

$\Rightarrow \dfrac{d/2}{x} = \dfrac{1}{0.8}\dfrac{d}{d - x} \Rightarrow x = 0.286\,d$



۳-۲۵۲.

سرعت برخورد $v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1.2} = 4.85$ m/s

$\Delta G = 0 : 200(4.85) + 0 = 0 + 320\,v' \Rightarrow v' = 3.03$ m/s

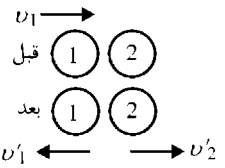
$e = \dfrac{v'}{v} = \dfrac{3.03}{4.85} = 0.625$

۳-۲۵۳.

$\Delta G = 0 : mv_1 = -m\,v_1' + mv_2' \Rightarrow v_2' = v_1 + v_1'$

$e = \dfrac{v_2' + v_1'}{v_1} \Rightarrow v_1' = ev_1 - v_2'$

ترکیب روابط فوق : $v_2' = v_1 + ev_1 - v_2' \Rightarrow v_2' = \dfrac{1+e}{2} v_1$



به همین ترتیب : $v_3' = \dfrac{1+e}{2} v_2' = \left(\dfrac{1+e}{2}\right)^2 v_1$

$$v_4' = \frac{1+e}{2} v_3' = \left(\frac{1+e}{2}\right)^3 v_1$$
$$\vdots$$
$$v_n = \left(\frac{1+e}{2}\right)^{n-1} v_1$$

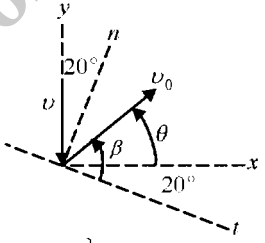
۳-۲۵۴.

$v = \sqrt{2gh} = \sqrt{2(9.81)(0.75)} = 3.84$ m/s

$v_{0t} = 3.84 \sin 20° = 1.312$ m/s

$v_{0n} = (3.84 \cos 20°)(0.85) = 3.06$ m/s

$v_0 = \sqrt{{v_{0t}}^2 + {v_{0n}}^2} = 3.33$ m/s , $\beta = \tan^{-1}\dfrac{v_{0n}}{v_{0t}} = 66.8°$



$\theta = \beta - 20° = 46.8°$

$$\left.\begin{aligned} x &= x_0 + v_{x_0}t : R\cos 20° = 0 + 3.33\cos 46.8°\, t \\ y &= y_0 + v_{y_0}t - \frac{1}{2}gt^2 : -R\sin 20° = 0 + 3.33\sin 46.8° t - \frac{9.81}{2}t^2 \end{aligned}\right\} \Rightarrow R = 1.613 \text{ m}$$

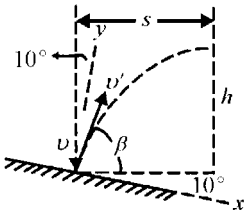
۳-۲۵۵.

$v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.9} = 4.2$ m/s

در برخورد $\Sigma F_x = 0 \Rightarrow \Delta G_x = 0$

$\Rightarrow v' \cos(\beta + 10°) - 4.2 \sin 10° = 0 \quad (1)$

$e = 0.7 = \dfrac{v' \sin(\beta + 10°)}{4.2 \cos 10°} \quad (2)$



ترکیب (1) و (2) : $\tan(\beta + 10°) = 3.97 \Rightarrow \beta + 10° = 75.9° \Rightarrow \beta = 65.9°$

رابطه (1) : $v' = \dfrac{4.2 \sin 10°}{\cos 75.9°} = 2.99$ m/s

طبق مثال ۲-۶ : $h = \dfrac{v'^2 \sin^2\beta}{2g} = \dfrac{(2.99)^2 \sin^2 65.9°}{2 \times 9.81} = 0.379$ m یا 379 mm

$$s = \frac{v'^2 \sin\beta}{2g} = \frac{(2.99)^2 \sin 131.7°}{2 \times 9.81} = 0.34 \text{ m یا } 340 \text{ mm}$$

۳-۲۵۶.

$$G_{1x} = G_{2x} : m_B v_B + 0 = (m_A + m_B)\, v' \sin 30°$$

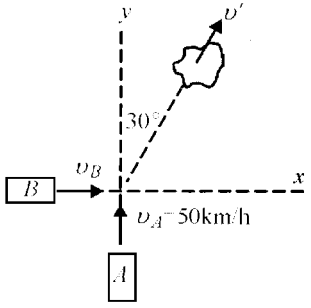
$$1600\, v_B = 2800\, v' \left(\frac{1}{2}\right) \quad (1)$$

$$G_{1y} = G_{2y} : m_A v_A + 0 = (m_A + m_B)\, v' \cos 30°$$

$$1200(50) = 2800\, v' (0.866) \quad (2)$$

(2) رابطه : $v' = 24.7$ km/h

(1) رابطه : $v_B = 21.7$ km/h



۳-۲۵۷.

برای ① و ② با هم : $mv_1 \cos 45° = mv_1' \sin\theta + mv_2'$

$$v_1' \sin\theta + v_2' = \frac{v_1}{\sqrt{2}} \quad (1)$$

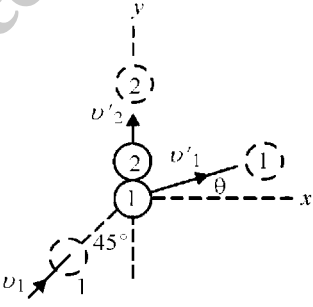
برای ① به تنهایی : $mv_1 \sin 45° = mv_1' \cos\theta$

$$v_1' \cos\theta = \frac{v_1}{\sqrt{2}} \quad (2)$$

ضریب بازگشت : $v_2' - v_1' \sin\theta = e\, v_1 \cos 45°$

$$v_2' - v_1' \sin\theta = \frac{0.9\, v_1}{\sqrt{2}} \quad (3)$$

(3) و (1) : $v_1' \sin\theta = 0.0354\, v_1$ ⇒ (2) تقسیم بر : $\theta = 2.86°$



$$n = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} = 1 - \frac{\frac{1}{2} mv_2'^2 + \frac{1}{2} mv_1'^2}{\frac{1}{2} mv_1^2} = 1 - \frac{v_2'^2 + v_1'^2}{v_1^2}$$

$$v_1' = 0.708\, v_1 \;,\; v_2' = 0.672\, v_1 \Rightarrow n = 1 - \frac{0.672^2 + 0.708^2}{1} = 0.0475$$

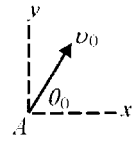
۳-۲۵۸. فرض کنید v_s و v_b معرف سرعت بازگشت از روی صفحات فولادی و برنجی باشند.

سرعت برخورد $= \sqrt{2gh} = \sqrt{2(9.81)(0.15)} = 1.716$ m/s

$$\left.\begin{aligned} 0.6 &= \frac{v_s}{1.716} \Rightarrow v_s = 1.029 \text{ m/s} \\ 0.4 &= \frac{v_b}{1.716} \Rightarrow v_b = 0.686 \text{ m/s} \end{aligned}\right\} \Rightarrow \omega = \frac{1.029 - 0.686}{0.60} = 0.572 \text{ rad/s CCW}$$

۳-۲۵۹. فرض کنید سرعت پرتاب در نقطه A برابر v_0 و زاویه پرتاب θ_0 باشد.

$$\text{برد } L_1 = \frac{2\,v_0^2 \sin\theta_0 \cos\theta_0}{g}$$



$$\text{مؤلفه‌های سرعت در نقطه } B \text{ قبل از برخورد} \begin{cases} v_x = v_0 \cos\theta_0 \\ v_y = -v_0 \sin\theta_0 \end{cases}$$

$$\text{مؤلفه‌های سرعت در نقطه } B \text{ پس از برخورد} \begin{cases} v_x = v_0 \cos\theta_0 \\ v_y = e\,v_0 \sin\theta_0 \end{cases} \Rightarrow L_2 = \frac{2e\,v_0^2 \sin\theta_0 \cos\theta_0}{g} \Rightarrow L_2 = eL_1$$

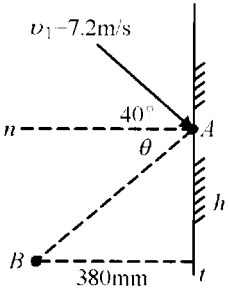
۳-۲۶۰.

$$(v_1)_t = 7.2 \sin 40° = 4.63 \text{ m/s}$$

$$(v_1)_n = -7.2 \cos 40° = -5.51 \text{ m/s}$$

$$(v_1')_t = (v_1)_t = 4.63 \text{ m/s}$$

$$(v_1')_n = -e\,(v_1)_n = -0.84\,(-5.51) = 4.63 \text{ m/s}$$

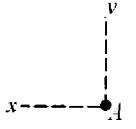


(الف) $\tan\theta = \dfrac{(v_1')_t}{(v_1')_n} = \dfrac{h}{380} \Rightarrow h = 380 \text{ mm}$

(ب) حرکت پرتابه از A تا B

$$x_B = x_A + (v_1')_n\,t : 0.380 = 0 + 4.63\,t \Rightarrow t = 0.082 \text{ s}$$

$$y_B = y_A + (-v_1')_t\,t - \frac{1}{2}\,gt^2$$



$$-h = 0 + (-4.63)(0.082) - \frac{1}{2}\,(9.81)(0.082)^2 \Rightarrow h = 0.412 \text{ m یا } 412 \text{ mm}$$

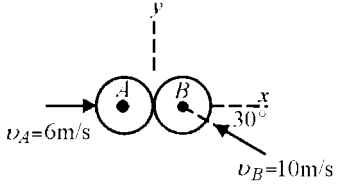
۳-۲۶۱.

$$v'_{Ay} = v_{Ay} = 0$$

$$v'_{By} = v_{By} = 10 \sin 30° = 5 \text{ m/s}$$

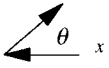
$$m_A v_{Ax} + m_B v_{Bx} = m_A v'_{Ax} + m_B v'_{Bx}$$

$$\Rightarrow 6 - 10 \cos 30° = v'_{Ax} + v'_{Bx} \qquad (1)$$



$$e = \frac{v'_{Bx} - v'_{Ax}}{v_{Ax} - v_{Bx}} : 0.75 = \frac{v'_{Bx} - v'_{Ax}}{6 - (-10 \cos 30°)} \qquad (2)$$

(1) و (2) :
$$\begin{cases} v'_{Ax} = -6.83 \text{ m/s} \Rightarrow v'_A = 6.83 \text{ m/s} \ @ \ \theta_A = 180° \\ v'_{Bx} = 4.17 \text{ m/s} \Rightarrow v'_B = 6.51 \text{ m/s} \ @ \ \theta_B = 50.2° \end{cases}$$



حالت اولیه : $T_1 = m\,(6^2 + 10^2) = 68 \text{ m}$

حالت نهایی : $T_2 = \frac{1}{2} m\,(6.83^2 + 6.51^2) = 44.5 \text{ m}$

$$\Rightarrow n = \frac{68 - 44.5}{68}(100\%) = 34.6\%$$

۳-۲۶۲.

برای سیستم : $\Delta G_n = 0$

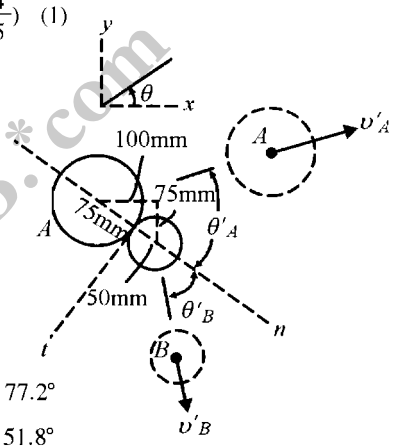
$$23\, v_A' \cos\theta'_A + 4\, v_B' \cos\theta'_B = 23(4)(\tfrac{4}{5}) - 4(12)(\tfrac{4}{5}) \quad (1)$$

برای هر گوی : $\Delta G_t = 0$

گوی A : $4\left(\frac{3}{5}\right) = v_A' \sin\theta'_A \quad (2)$

گوی B : $12\left(\frac{3}{5}\right) = v_B' \sin\theta'_B \quad (3)$

$$0.4 = \frac{v_B' \cos\theta'_B - v_A' \cos\theta'_A}{12\left(\frac{4}{5}\right) + 4\left(\frac{4}{5}\right)} \quad (4)$$



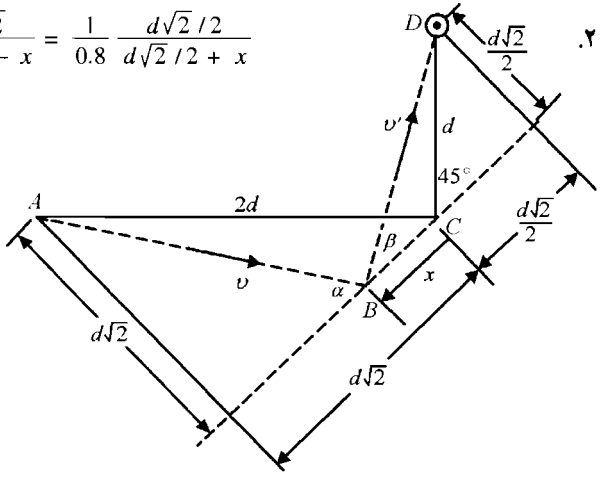
حل معادلات (1) تا (4) :
$$\begin{cases} v_A' = 2.46 \text{ m/s} \ , \ \theta'_A = 77.2° \\ v_B' = 9.16 \text{ m/s} \ , \ \theta'_B = 51.8° \end{cases}$$

با توجه به جهت محور x، جهت سرعت‌هایی نهایی عبارتست از:
$$\begin{cases} \theta_A = 77.2° - 36.9° = 40.3° \\ \theta_B = -51.8° - 36.9° = -88.7° \end{cases}$$

۳-۲۶۳.

$$\tan\alpha = \frac{1}{e}\tan\beta = \frac{d\sqrt{2}}{d\sqrt{2} - x} = \frac{1}{0.8}\,\frac{d\sqrt{2}/2}{d\sqrt{2}/2 + x}$$

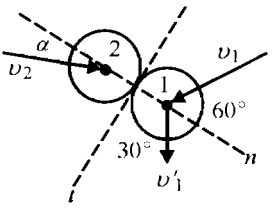
$$\Rightarrow x = 0.1088\, d$$



۳-۲۶۴.

پایستاری مومنتوم n – : $m(-v_1 \cos 60°) + m(v_2 \cos \alpha) = mv'_{1n} + mv'_{2n}$ (1)

ضریب بازگشت $e = 0.8 = \dfrac{v'_{2n} - v'_{1n}}{-v \cos 60° - v_2 \cos \alpha}$ (2)

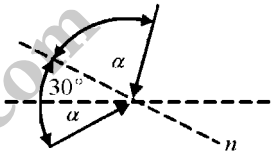


حل همزمان (1) و (2): $v'_{1n} = v_1 [0.9 \cos \alpha - 0.05]$

$$v'_{1t} = v_{1t} = v_1 \sin 60° = \frac{\sqrt{3}}{2} v_1$$

$$\tan 30° = \frac{v'_{1n}}{v'_{1t}} = \frac{v_1 [0.9 \cos \alpha - 0.05]}{\sqrt{3}/2\, v_1} \Rightarrow \cos \alpha = 0.611 \Rightarrow \alpha = \pm 52.3°$$

$$\Rightarrow \begin{cases} \theta = 30° + 52.3° = 82.3° \\ \text{یا} \\ \theta = 30° - 52.3° = -22.3° \end{cases}$$



۳-۲۶۵.

$$\alpha = \tan^{-1} \frac{256.7}{328.6} = 38.0°$$

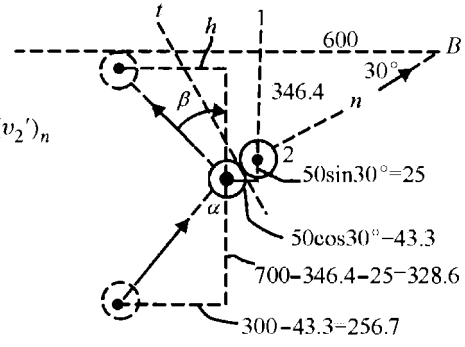
$$\theta_1 = \alpha + 30° = 68.0°$$

مومنتوم: $m_1(v_1)_n + m_2(v_2)_n = m_1(v_1')_n + m_2(v_2')_n$

$$v_1 \sin 68° = (v'_1)_n + (v'_2)_n$$

$$e = \frac{(v_2')_n - (v_1')_n}{(v_1)_n - (v_2)_n} : 0.9 = \frac{(v_2')_n - (v_1')_n}{v_1 \sin 68° - 0}$$

$$\Rightarrow (v_1')_n = 0.0464\, v_1$$



$$(v_1')_t = (v_1)_t = v_1 \cos 68.0° = 0.375\, v_1$$

$$\tan \theta_1' = \frac{(v_1')_t}{(v_1')_n} = \frac{0.0464\, v_1}{0.375\, v_1} \Rightarrow \theta'_1 = 7.05°$$

$$\beta = 30 - \theta' = 22.95° \quad , \quad \tan \beta = \frac{h}{346.4 - 25 + 25} = 0.423 \Rightarrow h = 146.7 \text{ mm}$$

$$\Rightarrow x = 600 - 43.3 - 146.7 = 410 \text{ mm}$$

۳-۲۶۶.

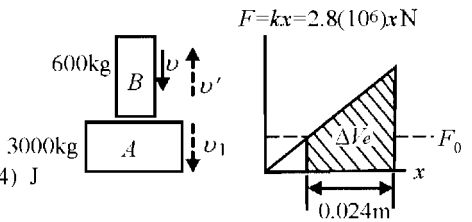
چکش : $v = \sqrt{2gh} = \sqrt{2(9.81)(0.5)} = 3.13$ m/s

سندان : $\Delta T + \Delta V_e + \Delta V_g = 0$

$\Delta T = 0 - \frac{1}{2}\,3000\,v_1^2$ J

$\Delta V_e = \frac{1}{2}(2.8\times10^6)(0.024)^2 + 29.4\,(10^3)(0.024)$ J

$\Delta V_g = -29.4(10^3)(0.024)$ J



$F_0 = mg = 3000\,(9.81) = 29.4\,(10^3)$ N

جای‌گذاری $\Rightarrow -\frac{1}{2}(3000)\,v_1^2 + \frac{1}{2}(2.8\times10^6)(0.024)^2 = 0 \Rightarrow v_1 = 0.733$ m/s

برخورد چکش باسندان : $\Delta G_x = 0 : 3000\,(0.733) - 600\,v' - 600\,(3.13) = 0$

$\Rightarrow v' = 0.534$ m/s

چکش بعد از برخورد : $v' = \sqrt{2gh} \Rightarrow h = \frac{0.534^2}{2\times9.81} = 0.01453$ m یا $h = 14.53$ mm

$e = \frac{0.534 - (-0.733)}{3.13} = 0.405$

۳-۲۶۷. ◀

$v_{xA} = 15\cos\alpha$, $v_{yA} = 15\sin\alpha$

$t_{AB} = \frac{3}{v_{xA}} = \frac{3}{15\cos\alpha} = \frac{1}{5\cos\alpha}$

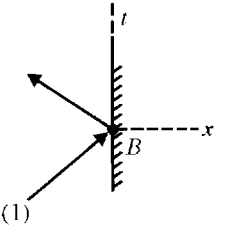
$v_{xB} = v_{xA} = 15\cos\alpha$

$v_{yB} = v_{yA} - gt = 15\sin\alpha - \frac{g}{5\cos\alpha}$

$y_B = y_A + v_{yA}t - \frac{1}{2}gt^2 = 0 + (15\sin\alpha)\left(\frac{1}{5\cos\alpha}\right) - \frac{g}{2}\left(\frac{1}{25\cos^2\alpha}\right) = 3\tan\alpha - \frac{3}{50\cos^2\alpha}$

برخورد در B : $v'_{yB} = v_{yB} = 15\sin\alpha - \frac{g}{5\cos\alpha}$

$e = \frac{v'_{2x} - v'_{1x}}{v_{1x} - v_{2x}} : \frac{0 - v'_{1x}}{15\cos\alpha - 0} = 0.5 \Rightarrow v'_{1x} = -7.5\cos\alpha$



$t_{BA} = \frac{3}{7.5\cos\alpha} = \frac{2}{5\cos\alpha}$

$y_A = y_B + v'_{yB}\,t - \frac{1}{2}gt^2$

$0 = \left(3\tan\alpha - \frac{g}{50\cos^2\alpha}\right) + \left(15\sin\alpha - \frac{g}{5\cos\alpha}\right)\left(\frac{2}{5\cos\alpha}\right) - \frac{g}{2}\left(\frac{2}{5\cos\alpha}\right)^2$

$\Rightarrow 9\tan\alpha - \frac{9g}{50}\frac{1}{\cos^2\alpha} = 0$

$$\left(\frac{1}{\cos^2\alpha} = \tan^2\alpha + 1\right) \Rightarrow \tan^2\alpha - 5.097\tan\alpha + 1 = 0$$

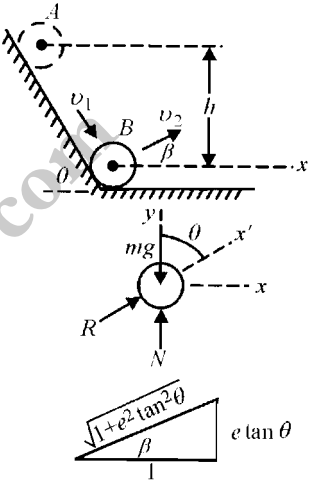
$$\Rightarrow \begin{cases} \tan\alpha = 0.2044 \Rightarrow \alpha = 11.55^\circ \\ \text{یا} \\ \tan\alpha = 4.89 \Rightarrow \alpha = 78.4^\circ \end{cases}$$

◀ ۳-۲۶۸.

از A تا B :

$$\Delta T + \Delta V_g = 0 : \frac{1}{2}mv_1^2 - mgh = 0$$

$$\Rightarrow v_1 = \sqrt{2gh}$$



طی برخورد :

$$\Sigma F_{x'} \geq 0 : R - mg\cos\theta + N\cos\theta \geq 0$$

$$\Rightarrow R < mg\cos\theta$$

طی زمان کوتاه برخورد، ضربه‌های R و mg قابل اغماض هستند بنابراین :

$$\int \Sigma F_x\, dt \approx 0 \quad , \quad \Delta G_x \approx 0$$

$$mv_1\cos\theta = mv_2\cos\beta \qquad (1)$$

$$\int \Sigma F_y\, dt \neq 0 \quad , \quad v_2\sin\beta = ev_1\sin\theta \qquad (2)$$

تقسیم (2) بر (1) : $e\tan\theta = \tan\beta$

$$v_x = v_2\cos\beta = v_1\cos\theta \Rightarrow v_x = \sqrt{2gh}\cos\theta$$

$$n = \frac{|\Delta T|}{T} = \frac{\frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2}{\frac{1}{2}mv_1^2} = 1 - \frac{v_2^2}{v_1^2} = 1 - \frac{(ev_1\sin\theta/\sin\beta)^2}{v_1^2}$$

$$\Rightarrow n = 1 - \frac{e^2\sin^2\theta}{\sin^2\beta} = 1 - (\cos^2\theta + e^2\sin^2\theta) \qquad \left(\text{که در آن } \sin^2\beta = \frac{e^2\tan^2\theta}{1 + e^2\tan^2\theta}\right)$$

برای گوشه‌های گرد با شعاع بزرگتر از شعاع کره، هیچ ناپیوستگی در مقدار سرعت وجود نخواهد داشت لذا ضربه‌ای رخ نمی‌دهد.

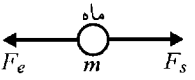
۲۶۹-۳. برای یک مدار دایروی $r_{max} = r_{min}$ و $a = R + h$ بنابراین معادله ۳-۴۴ چنین می‌شود:

$$U = R\sqrt{\frac{g}{g + h}} = 6371(10^3)\sqrt{\frac{9.825}{(6371 + 590)10^3}} = 7569 \text{ m/s} \quad \text{یا} \quad 27250 \text{ km/h}$$

۲۷۰-۳.

$$\frac{F_s}{F_e} = \frac{G\, m_s m / d^2_{m\text{-}s}}{G\, m_e m / d^2_{m\text{-}e}} = \left(\frac{d_{m\text{-}e}}{d_{m\text{-}s}}\right)^2 \frac{m_s}{m_e}$$

$$= \left(\frac{384398}{149.6(10^6) - 384398}\right)^2 333000 = 2.21$$

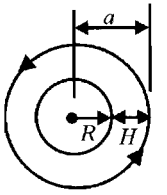


بنابراین شتاب ماه به سمت خورشید است و از اینرو تقعر مسیر حرکت ماه به سمت خورشید است.

۲۷۱-۳. $r = a = R + H$ با ۳-۴۳ معادله: $v^2 = 2gR^2\left(\frac{1}{a} - \frac{1}{2a}\right) = \frac{gR^2}{R + H}$

$$= \frac{1.62\left(\frac{3476}{2}\right)^2}{\frac{3476}{2} + 80}(1000)$$

$$\Rightarrow v = 1641 \text{ m/s} \quad \text{یا} \quad 5910 \text{ km/h}$$



۲۷۲-۳. طبق معادله ۳-۴۳ برای یک مدار دایره‌ای با ارتفاع H و $a = r = R + H$:

$$v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2r}\right) = gR^2/(R + H) \Rightarrow v = R\sqrt{g/(R + H)}$$

$$v^2_{escape} = 2gR^2\left(\frac{1}{r} - \frac{1}{\infty}\right) = 2gR^2/(R + H) \Rightarrow v = R\sqrt{2g/(R + H)}$$

$$\Delta v = v_{escape} - v = R\sqrt{\frac{g}{R + H}}(\sqrt{2} - 1) = (\sqrt{2} - 1)\, v = 0.414\, v$$

برای $R = 6371$ km ، $g = 9.825$ m/s^2 ، $H = 320$ km:

$$v = 6371\ (10^3)\sqrt{\frac{9.825}{(6371 + 320)10^3}} = 7720 \text{ m/s}$$

$$\Delta v = 0.414\ (7714) = 3196 \text{ m/s} \quad \text{یا} \quad 3.2 \text{ km/s}$$

۲۷۳-۳. $r_{min} = 2R \quad , \quad r_{max} = 3R$

$$a = \frac{r_{min} + r_{max}}{2} = 2.5\,R$$

$$v_P = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{max}}{r_{min}}} = R\sqrt{\frac{g}{2.5\,R}}\sqrt{\frac{3R}{2R}}\sqrt{\frac{3gR}{5}}$$

سرعت در مدار دایره‌ای اولیه $v_C = R\sqrt{\frac{g}{a}} = R\sqrt{\frac{g}{2R}} = \sqrt{\frac{1}{2}gR}$

$$\Delta v = v_P - v_C = \sqrt{gR}\left(\sqrt{\frac{3}{5}} - \sqrt{\frac{1}{2}}\right) = 0.0675\sqrt{gR}$$

عدد گذاری : $\Delta v = 0.0675\sqrt{9.825(6371)(1000)} = 534$ m/s

۲۷۴-۳. سرعت ناظر : $v_O = R\omega = 6371(10^3) \times 0.7292(10^{-4}) = 464.6$ m/s

سرعت ماهواره $v_s = R\sqrt{\frac{g}{R+H}}$ (طبق معادله ۳-۴۴) , $a = R + H$, $r_{min} = r_{max}$

$$= 6371(10^3)\sqrt{\frac{9.825}{(6371 + 300)10^3}} = 7732 \text{ m/s}$$

(الف) غرب به شرق $v_{rel} = 7732 - 464.6 = 7267.4$ m/s یا 26163 km/h

(ب) شرق به غرب $v_{rel} = 7732 + 464.6 = 8196.6$ m/s یا 29508 km/h

مدار غرب به شرق از مزیت سرعت اولیهٔ غرب به شرق برخوردار است لذا انرژی کمتری برای رسیدن به سرعت نسبی کمتر لازم است.

۲۷۵-۳. $r = a = 6371 + 300 = 6671$ km $= 6.671\,(10^6)$ m

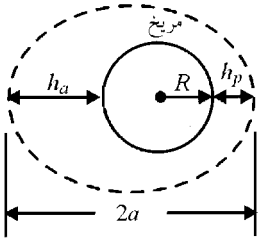
$$\tau = \frac{2\pi a^{3/2}}{R\sqrt{g}} = \frac{2\pi(6.671 \times 10^6)^{3/2}}{6.371 \times 10^6\sqrt{9.825}} = 5421 \text{ s}$$

سرعت نقطه زمین روی استوا : $v_e = R_e\omega_e = (6378)(7.292 \times 10^{-5}) = 0.4651$ km/s

فاصله لازم $d = v_e\tau = (0.4651)(5421) = 2520$ km

۲۷۶-۳. $2R = 6788$ km , $g = 3.73$ m/s^2

$h_p = 1508$ km , $\tau = 24$ h 37 min 23 s $= 88643$ s



۴۰-۳ معادله : $\tau = 2\pi \dfrac{a^{3/2}}{R\sqrt{g}} \Rightarrow a = \left(\dfrac{\tau R\sqrt{g}}{2\pi}\right)^{2/3}$

$$a = \left[\frac{(88643)\left(\frac{6788}{2} \times 10^3\right)\sqrt{3.73}}{2\pi}\right]^{2/3} = (924.8 \times 10^8)^{2/3}$$

$$\Rightarrow a^3 = 8552 \times 10^{18}\ \text{m}^3 \Rightarrow a = 20.45 \times 10^6\ \text{m} \text{ یا } 20.45 \times 10^3\ \text{km}$$

$$h_a = 2a - 2R - h_p = 2(20.45 \times 10^3) - 6788 - 1508 = 32600\ \text{km}$$

۲۷۷-۳.

(الف) $v = R\sqrt{\dfrac{g}{r}} = 6371(10^3)\sqrt{\dfrac{9.825}{(6371 + 637)10^3}} = 7544\ \text{m/s}$

(ب) $r_{\min} = a(1 - e) \Rightarrow a = \dfrac{r_{\min}}{1 - e} = \dfrac{1.1(6371)}{1 - 0.1} = 7787\ \text{km}$

$$v_P = R\sqrt{\frac{g}{a}}\sqrt{\frac{1 + e}{1 - e}} = 6371(10^3)\sqrt{\frac{9.825}{7787(10^3)}}\sqrt{\frac{1 + 0.1}{1 - 0.1}} = 7912\ \text{m/s} = v$$

(ج) $a = \dfrac{r_{\min}}{1 - e} = \dfrac{1.1(6371)}{1 - 0.9} = 70081\ \text{km}$

$$v_P = 6371(10^3)\sqrt{\frac{9.825}{70081(10^3)}}\sqrt{\frac{1 + 0.9}{1 - 0.9}} = 10398\ \text{m/s} = v$$

(د) $a \to \infty$ معادله : ۴۳-۳ $\Rightarrow v = R\sqrt{\dfrac{2g}{r}}$

این مقدار $\sqrt{2}$ برابر جواب قسمت الف می‌باشد بنابراین:

$v = \sqrt{2}\,7544 = 10668\ \text{m/s}$

۲۷۸-۳.

$$v_A = R\sqrt{\frac{g}{r}} = 6371(10^3)\sqrt{\frac{9.825}{(6371 + 1200)(10^3)}} = 7258\ \text{m/s}$$

$$v_B = R\sqrt{\frac{g}{a}} = \sqrt{\frac{r_{\max}}{r_{\min}}} = 6371(10^3)\sqrt{\frac{9.825}{\frac{2(6371) + 2700}{2}(10^3)}} = \sqrt{\frac{6371 + 1500}{6371 + 1200}} = 7328\ \text{m/s}$$

بقای مومنتوم طی برخورد : $m_A v_A + m_B v_B = (m_A + m_B)v_C$

$$m_A = m_B \Rightarrow v_C = \frac{1}{2}(v_A + v_B) = 7293\ \text{m/s}$$

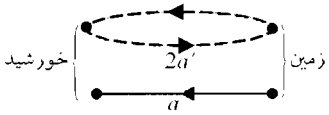
$$v_P = R\sqrt{\frac{g}{a}} = \sqrt{\frac{r_{max}}{r_{min}}} \Rightarrow r_{max} = \frac{r_{min}}{\left(\frac{2gR^2}{v_P^2 r_{min}} - 1\right)} = 7720 \text{ km}$$

$$\Rightarrow h_{max} = r_{max} - R = 7720 - 6371 = 1349 \text{ km}$$

۳-۲۷۹. شعاع مدار واقعی پیرامون خورشید a است که محور $2a'$ از بیضی سقوط است.

R = شعاع خورشید

g = شتاب جاذبه روی سطح خورشید



تناوب مدار طبق معادله ۳-۴۰ برای مدار واقعی $\tau = 2\pi \frac{a^{3/2}}{R\sqrt{g}}$

برای بیضی سقوط $\tau' = 2\pi \frac{(a/2)^{3/2}}{R\sqrt{g}} \Rightarrow \frac{\tau'}{\tau} = \frac{\left(\frac{1}{2}\right)^{3/2}}{1}$

زمان سقوط: $t = \frac{1}{2}\tau' = \frac{1}{2}\left(\frac{1}{2}\right)^{3/2}\tau = \frac{1}{4\sqrt{2}}\,365.26 = 64.6$ روز

۳-۲۸۰.

سرعت اوج در نقطه C $v_a = R\sqrt{\frac{g}{a}} = \sqrt{\frac{r_{min}}{r_{max}}} = 6371(10^3)\sqrt{\frac{9.825}{(2\times6371+240+320)/2}} = \sqrt{\frac{6371+240}{6371+320}}$

$= 7697$ m/s

سرعت مدار دایره‌ای در $h = 320$ km $v_{circ} = R\sqrt{\frac{g}{r_{max}}} = 7720$ m/s

$\Delta v = v_{circ} - v_a = 7720 - 7697 = 23.25$ m/s

$F\Delta t = m\Delta v : 2(30000)\ \Delta t = 85{,}000(23.25) \Rightarrow \Delta t = 32.9$ s

۳-۲۸۱. ضربه‌های خطی ناشی از دراگ و پیشرانه باید از نظر مقدار برابر باشند. یعنی

$\Delta t = \Sigma T t_{burn}$

$t = 10\tau$

$\tau = 2\pi \frac{a^{3/2}}{R\sqrt{g}} = 2\pi \frac{(6.571\times10^6)^{3/2}}{6.371\times10^6\sqrt{9.825}} = 5300$ s

$t = 10\,\tau = 53000$ s

$\Delta = \frac{\Sigma T t_{burn}}{t} = \frac{2(300)}{53{,}000} = 0.01132$ N

۲۸۲-۳. $r_{min} = a(1 - e)$

$(6371 + 6400) = (6400 + 6371 + 25600)(1 - e) \Rightarrow e = 0.667$

$b = a\sqrt{1 - e^2} = 38371\sqrt{1 - (0.667)^2} = 28583$ km

در B : $r = \sqrt{25600^2 + 28583^2} = 38371$ km

$$v_B^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right) = 2(9.81)(6371 \times 10^3)^2\left(\frac{1}{38371(10^3)} - \frac{1}{2(38371)10^3}\right)$$

$\Rightarrow v_B = 3224$ m/s یا 3.224 km/s یا 11606 km/h

۲۸۳-۳. سرعت در مدار دایره‌ای $v = R\sqrt{\frac{g}{a}} = 6371(10^3)\sqrt{\frac{9.825}{(6371+800)(10^3)}} = 7457.5$ m/s

زمان لازم برای B تا به C برسد : $t = \frac{2\pi r - 2000\ (10^3)}{v} = 5774$ s

$$\tau = \frac{2\pi a^{3/2}}{R\sqrt{g}} \Rightarrow a = \left(\frac{\tau R\sqrt{g}}{2\pi}\right)^{2/3}$$

$$a = \left(\frac{5774(6371 \times 10^3)\sqrt{9.825}}{2\pi}\right)^{2/3} = 6.957(10^6)\ \text{m}$$

در اوج : $v_{C'} = \sqrt{2gR^2\left[\frac{1}{r} - \frac{1}{2a}\right]} = 7342$ m/s

$\Delta v = v - v_{C'} = 7457.5 - 7342 = 115.5$ m/s

(با پیدا کردن $r_{min} = 6742$ km $> R = 6371$ km می‌توان چک کرد و اطمینان حاصل نمود که C به زمین برخورد نمی‌کند)

۲۸۴-۳. طبق جواب مسئله قبل، سرعت مدار دایره‌ای برابر است با 7457.5 m/s

زمان لازم برای B تا به C برسد $t = \frac{4\pi r - (2000)(10^3)}{v} = 11815$ s

(پس از دو دور گردش در مدار دایره‌ای)

$$a = \left(\frac{\tau R\sqrt{g}}{2\pi}\right)^{2/3} = \left[\frac{\left(\frac{11815}{2}\right)(6371 \times 10^3)\sqrt{9.825}}{2\pi}\right]^{2/3} = 7.0644(10^6)\ \text{m}$$

در اوج : $v_{C'} = \sqrt{2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)} = 7401$ m/s

$\Delta v = v - v_{C'} = 7457.5 - 7401 = 56.5$ m/s یا 0.0565 km/s یا 203.4 km/h

۳-۲۸۵. سرعت مدار دایره‌ای $v_O = R\sqrt{\frac{g}{a}} = R\sqrt{\frac{g}{3R}} = \sqrt{\frac{1}{3}gR}$

سرعت در A(اوج) در مدار بیضوی $v_A = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{\min}}{r_{\max}}} = R\sqrt{\frac{g}{2R}}\sqrt{\frac{R}{3R}} = \sqrt{\frac{1}{6}gR}$

$$v_r = v_O - v_A = \sqrt{gR}\left[\sqrt{\frac{1}{3}} - \sqrt{\frac{1}{6}}\right] = 0.1691\sqrt{gR}$$

عدد گذاری: $v_r = 0.1691\sqrt{1.62\,\frac{3476}{2}(1000)} = 284\ \text{m/s}$ (به سمت عقب)

دوره تناوب مدار دایره‌ای را τ_O و دوره تناوب مدار بیضوی را τ_{AB} فرض کنید.

$$\tau_O = \frac{2\pi a^{3/2}}{R\sqrt{g}} = \frac{2\pi(3R)^{3/2}}{R\sqrt{g}}$$

$$\tau_{AB} = \frac{2\pi(2R)^{3/2}}{R\sqrt{g}}$$

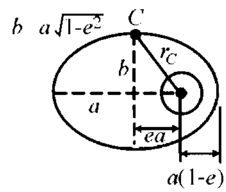
$$\theta = \left(\frac{\tau_{AB}/2}{\tau_0/2}\right)\pi = \left(\frac{2}{3}\right)^{2/3}\pi = 1.710\ \text{rad}$$ یا $98°$

۳-۲۸۶. طبق معادله ۳-۴۳: $v_C^2 = 2gR^2\left(\frac{1}{r_C} - \frac{1}{2a}\right)$

$$r_C^2 = b^2 + (ea)^2 = a^2(1-e^2) + a^2e^2 = a^2 \Rightarrow r_C = a$$

$$\Rightarrow v_C^2 = 2gR^2\left(\frac{1}{a} - \frac{1}{2a}\right) = g\frac{R^2}{a} \Rightarrow v_C = R\sqrt{\frac{g}{a}}$$

مشابه مدار دایره‌ای با شعاع $r = a$



۳-۲۸۷. $\tau = 3600 + 39(60) + 22 = 5962\ \text{s}$

از معادله ۳-۴۰: $\tau = 2\pi\frac{a^{3/2}}{R\sqrt{g}} \Rightarrow a^{3/2} = 5962(6371)(10^3)\sqrt{9.825}/2\pi \Rightarrow a = 7.1076(10^6)\ \text{m}$

معادله ۳-۳۹: $r_{\min} = R + H_P = a(1-e) \Rightarrow (6371 + 500)10^3 = 7.1076(10^6)(1-e)$

$$\Rightarrow e = 0.0333$$

$$H_A + H_P + 2R = 2a \Rightarrow H_A + 500(10^3) + 2(6371)(10^3) = 2(7.1076)(10^6)$$

$$\Rightarrow H_A = 973200\ \text{m}$$

معادله ۳-۴۴: $v_A = R\sqrt{\frac{g}{a}}\sqrt{\frac{1-e}{1+e}} = 6371(10^3)\sqrt{\frac{9.825}{7.1076(10^6)}}\sqrt{\frac{1-0.0333}{1+0.0333}} = 7245\ \text{m/s}$

یا $26082\ \text{km/h}$

۲۸۸-۳.

مدار دایره‌ای : $v = R\sqrt{\dfrac{g}{r}} = 6371(10^3)\sqrt{\dfrac{9.825}{(6371+320)10^3}} = 7720 \text{ m/s}$

$$a_t = \frac{F}{m} = \frac{2(27{,}000)}{80{,}000} = 0.675 \text{ m/s}^2$$

$$v_a = v - a_t t = 7720 - 0.675(150) = 7619 \text{ m/s}$$

$$v^2 = 2gR^2\left[\frac{1}{r} - \frac{1}{2a}\right]$$

جای‌گذاری شرایط در B : $a = 6.52(10^6)\text{m}$

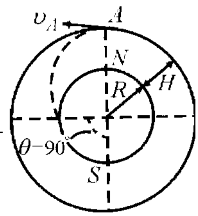
$$v_A = R\sqrt{\frac{g}{a}}\sqrt{\frac{1-e}{1+e}} \Rightarrow e = 0.026$$

$r = \dfrac{a(1-e^2)}{1+e\cos\theta}$ $\overset{\text{در نقطه } C}{\Rightarrow}$ $6371(10^3) = \dfrac{6.52(10^6)(1-0.026^2)}{1+0.026\cos\theta} \Rightarrow \theta = 28.7°$

$$\beta = 180 - \theta = 151.3°$$

۲۸۹-۳.

طبق معادله ۳-۳۹ : $\dfrac{1}{r} = \dfrac{1+e\cos\theta}{a(1-e^2)}$



وقتی $\theta = 90° \Rightarrow r = R$

وقتی $\theta = 180° \Rightarrow r = R + H$

$$\Rightarrow \begin{cases} \dfrac{1}{R} = \dfrac{1}{a(1-e^2)} \\[2mm] \dfrac{1}{R+H} = \dfrac{1-e}{a(1-e^2)} = \dfrac{1}{a(1+e)} \end{cases}$$

$$\Rightarrow 1 - e = \frac{R}{R+H}, \quad 1 + e = R + H$$

طبق معادله ۳-۴۴ : $v_A = R\sqrt{\dfrac{g}{a}\dfrac{(1-e)}{(1+e)}} = R\sqrt{g}\sqrt{\dfrac{R}{R+H}\dfrac{1}{R+H}} = \dfrac{R\sqrt{gR}}{R+H}$

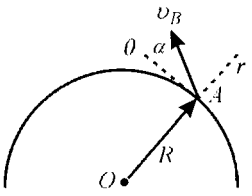
برای مدار دایره‌ای : $v = R\sqrt{\dfrac{g}{R+H}}$

$$\Delta v_A = R\sqrt{\frac{g}{R+H}} - \frac{R\sqrt{gR}}{R+H} = R\sqrt{\frac{g}{R+H}}\left(1 - \sqrt{\frac{R}{R+H}}\right)$$

۲۹۰-۳.

$$v_\theta = v\cos\alpha = 2000\cos 30^\circ = 1732 \text{ m/s}$$

$$v_r = v\sin\alpha = 2000\sin 30^\circ = 1000 \text{ m/s}$$



$$v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$$

با استفاده از شرایط در B: $a = 3.2906\times 10^6$ m

$$T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}m(2000)^2 = 2\times 10^6\, m$$

$$V_B = \frac{-mgR^2}{r} = \frac{-m(9.825)(6.371\times 10^6)^2}{6.371\times 10^6} = -6.2595\times 10^7\, m$$

$$E = T_B + V_B = -6.0595\times 10^7\, m$$

$$h = rv_B = 6.371(10^6)(1732) = 1.1035\times 10^{10}$$

$$e = \sqrt{1 + \frac{2Eh^2}{mg^2R^4}} \Rightarrow e = 0.9525$$

$$r_{max} = a(1+e) = 3.2906(10^6)(1+0.9525) = 6.4249\times 10^6 \text{ m}$$

$$h_{\max} = r_{\max} - R = 53900 \text{ m} \quad \text{یا} \quad 53.9 \text{ km}$$

۲۹۱-۳.

نقطه A نقطهٔ اوج است: $r_{\max} = \frac{3R}{2} = a(1+e)$

در نقطه B: $r = \frac{a(1-e^2)}{1+e\cos\theta} \Rightarrow R = \frac{a(1-e^2)}{1+e\cos 135^\circ}$

$$\Rightarrow \quad e = 0.6306 \ , \ a = 0.9199\, R$$

$$v_B^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$$

در نقطه A: $v_B^2 = 2(9.825)(6.371\times 10^6)^2\left(\frac{1}{6.371\times 10^6} - \frac{1}{2(0.9199)(6.371\times 10^6)}\right)$

$$\Rightarrow v_B = 7560 \text{ m/s}$$

۲۹۲-۳.

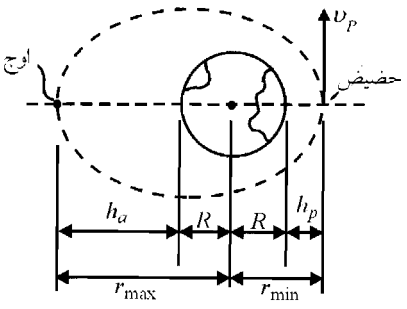
در اوج: $a = a_n = \frac{v_P^2}{\rho_P} \Rightarrow \rho_P = \frac{v_P^2}{a_n}$

طبق معادله ۴۴-۳: $v_P^2 = R^2\frac{g}{a}\frac{r_{\max}}{r_{\min}}$

طبق معادله ۳۹-۳: $r_{\min} + r_{\max} = 2a$

$$\Rightarrow \quad v_P^2 = gR^2\frac{r_{\max}}{r_{\min}}\frac{2}{r_{\min}+r_{\max}}$$

$$a_n = g_{\text{حضیض}} = g\left(\frac{R}{r_{\min}}\right)^2 \quad \text{(طبق فصل ۱)}$$



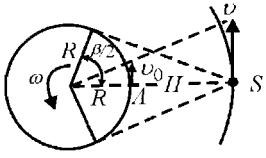
$$\Rightarrow \rho_p = 2gR^2 \frac{r_{\max}}{r_{\min}} \frac{1}{r_{\min} + r_{\max}} \Big/ g\frac{R^2}{r^2_{\min}}$$

$$= 2\frac{r_{\max} r_{\min}}{r_{\min} + r_{\max}}$$

$$\text{یا} \quad \rho_p = 2\frac{(R + h_a)(R + h_p)}{2R + h_a + h_p}$$

۳-۲۹۳. مسیر محدود به یک مدار استوایی است تا بالای یک نقطه A روی استوا بماند.

$$\frac{v}{R + H} = \omega$$



$(a = r = R + H)$: برای مدار دایره‌ای از معادله ۳-۴۳

$$v = R\sqrt{\frac{g}{R + H}}$$

$R + H = \sqrt[3]{\frac{gR^2}{\omega^2}}$: ترکیب دو رابطه $\Rightarrow H = \sqrt[3]{\frac{9.825(6371 \times 10^3)^2}{(0.7292 \times 10^{-4})^2}} - 6371 \times 10^3$

$$\Rightarrow H = 35800 \text{ km}$$

$$\frac{\beta}{2} = \cos^{-1}\frac{R}{R + H} = \cos^{-1}\frac{6371}{42170} = 81.3° \Rightarrow \beta = 162.6°$$

۳-۲۹۴.

$$\tan\beta = \frac{\dot{r}}{r\dot{\theta}}$$

معادله ۳-۳۹ : $r = \frac{a(1 - e^2)}{1 + e\cos\theta} \Rightarrow \dot{r} = \frac{a(1 - e^2)\, e\dot{\theta}\sin\theta}{(1 + e\cos\theta)^2}$

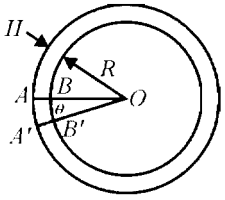
$$\Rightarrow \tan\beta = \frac{a(1 - e^2)\sin\theta}{(1 + e\cos\theta)^2} \Big/ \frac{a(1 - e^2)\dot{\theta}}{1 + e\cos\theta} \Rightarrow \tan\beta = \frac{e\sin\theta}{1 + e\cos\theta}$$

۳-۲۹۵.

τ = دوره تناوب ماهواره = زمان برای دوران OA به مقدار 2π

τ' = زمان دوران خط شعاعی ناظر به مقدار θ

ω_e = سرعت زاویه زمین و خط OB



OA سرعت زاویه‌ای $\omega_s = \dfrac{v_A}{R + H}$

$$\left.\begin{aligned}\theta &= \omega_e \tau' \\ \theta + 2\pi &= \omega_s \tau'\end{aligned}\right\} \Rightarrow \tau' = \frac{2\pi}{\omega_s - \omega_e}$$

معادله ۳-۴۳ : $v_A = R\sqrt{\dfrac{g}{R+H}} = 6371\sqrt{\dfrac{9.825\times10^{-3}}{6371+300}} = 7.73$ km/s $\quad (a = r = R + H)$

$$\omega_s = \frac{7.73}{6371 + 300} = 11.59 \times 10^{-4} \text{ rad/s}$$

از جدول د-۲ : $\omega_e = 0.7292 \times 10^{-4}$ rad/s

$$\Rightarrow \tau' = \frac{2\pi \times 10^4}{11.59 - 0.7292} = 5790 \text{ s} \quad \text{یا} \quad \tau' = 1\text{h}\ 36\text{min}\ 25\text{s}$$

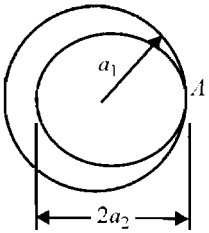
$$\tau = \frac{2\pi}{\omega_s} = \frac{2\pi \times 10^4}{11.59} = 5420\text{s} \quad \text{یا} \quad \tau = 1\text{h}\ 30\text{min}\ 21\text{s}$$

$$\tau' - \tau = 6\text{min}\ 4\text{s}$$

۳-۲۹۶.

$Ft = m\Delta v$

برای مدار دایره‌ای $v_1 = R\sqrt{\dfrac{g}{a}} = 6371(10^3)\sqrt{\dfrac{9.825}{12371(10^3)}} = 5768$ m/s



برای مدار بیضوی در اوج A : $v_A = R\sqrt{\dfrac{g}{a_2}}\sqrt{\dfrac{r_{\min}}{r_{\max}}}$

$r_{\min} = 6371 + 3000 = 9371$ km , $a_1 = R + 6000 = 12371$ km

$r_{\max} = 6371 + 6000 = 12371$ km , $2a_2 = 2a_1 - 3000 = 21742 \Rightarrow a_2 = 10871$ km

$$v_A = 6371(10^3)\sqrt{\frac{9.825}{10871(10^3)}}\sqrt{\frac{9371}{12371}} = 5271 \text{ m/s}$$

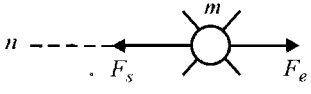
$\Delta v = 5678 - 5271 = 406$ m/s

$2000\ t = 800(406) \Rightarrow t = 162$ s

۳-۲۹۷. *

$F_s =$ نیروی وارد بر فضاپیما از طرف خورشید

$F_e =$ نیروی وارد بر فضاپیما از طرف زمین



$$\Sigma F_n = ma_n : F_s - F_e = m\frac{v^2}{\rho} = m\rho\omega^2 = m(D-h)\left(\frac{2\pi}{T}\right)^2$$

که D فاصلهٔ زمین تا خورشید و T دوره تناوب مدار زمین می‌باشد

$$\frac{G\,m_s\,m}{(D-h)^2} - \frac{G\,m_e\,m}{h^2} = m(D-h)\left(\frac{2\pi}{T}\right)^2$$

$G = 6.673(10^{-11})\ \text{m}^3/\text{kg.s}^2$

$m_s = 333{,}000\ m_e$

$m_e = 5.976(10^{24})\ \text{kg}$

$D = 149.6 \times 10^6\ \text{km}$

$T = 365.26(24)(3600)\ \text{s}$

حل عددی $\Rightarrow$ $h = 1.482(10^6)\ \text{km}$

◀ ۳-۲۹۸.

برای 1 : $v_1 = R\sqrt{\frac{g}{r_1}}$ ، برای 2 : $v_2 = R\sqrt{\frac{g}{r_2}}$

برای بیضی انتقال در A : $v_1' = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_2}{r_1}}$

برای بیضی انتقال در B : $v_2' = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_1}{r_2}}$

$\Rightarrow a = \frac{r_1 + r_2}{2}$ (معادله ۳-۴۴)

در A : $\Delta v_A = v_1' - v_1 = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_2}{r_1}} - R\sqrt{\frac{g}{r_1}} = R\sqrt{\frac{g}{r_1}}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right)$

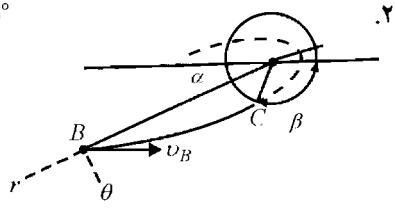
در B : $\Delta v_B = v_2 - v_2' = R\sqrt{\frac{g}{r_2}} - R\sqrt{\frac{g}{a}}\sqrt{\frac{r_1}{r_2}} = R\sqrt{\frac{g}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right)$

$\Delta v_A = 6371(10^3)\sqrt{\frac{9.825}{6871(10^3)}}\left(\sqrt{\frac{2(42171)}{6871+42171}} - 1\right) = 2370\ \text{m/s}$

$\Delta v_B = 6371(10^3)\sqrt{\frac{9.825}{42171(10^3)}}\left(1 - \sqrt{\frac{2(6871)}{6871+42171}}\right) = 1447\ \text{m/s}$

◀ ۳-۲۹۹.

در B : $r = \sqrt{29}\,R$ ، $\alpha = \tan^{-1}\left(\frac{2R}{5R}\right) = 21.8°$



$v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$

در B : $3200^2 = 2(9.825)(6.371 \times 10^6)^2 \left[\frac{1}{\sqrt{29}\ 6.371(10^6)} - \frac{1}{2a}\right] \Rightarrow a = 3.066 \times 10^7$ m

$$T_B = \frac{1}{2} m v_B^2 = \frac{1}{2} m (3200)^2 = 5.120 \times 10^6\ m$$

$$V_B = \frac{-mgR^2}{r_B} = -m \frac{(9.825)(6.371 \times 10^6)^2}{\sqrt{29}\ (6.371 \times 10^6)} = -1.162 \times 10^7\ m$$

$$E = T_B + V_B = -6.504 \times 10^6\ m$$

$$v_B = 3200 \sin \alpha = 1188.5 \text{ m/s}$$

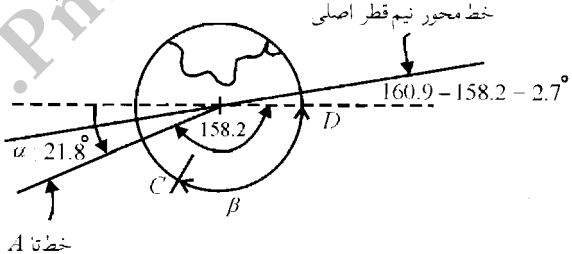
$$h = r v_\theta = \sqrt{29}\ (6.371 \times 10^6)(1188.5) = 4.077 \times 10^{10} \text{ kg-m}^2\text{/s}$$

$$e = \sqrt{1 + \frac{2Eh^2}{mg^2R^4}} = \sqrt{1 + \frac{2(-6.504 \times 10^6\ m)(4.077 \times 10^{10})^2}{m(9.825)^2(6.371 \times 10^6)^4}} = 0.9295$$

$$r = \frac{a(1 - e^2)}{1 + e \cos \theta}$$

در B : $\sqrt{29}\ (6.371 \times 10^6) = \frac{(3.066 \times 10^7)(1 - 0.9295^2)}{1 + 0.9295 \cos \theta} \Rightarrow \theta = 160.9°$

در C : $6.371(10^6) = \frac{(3.066 \times 10^7)(1 - 0.9295^2)}{1 + 0.9295 \cos \theta} \Rightarrow \theta = 111.8°$



$\beta = 111.8° - 2.7° = 109.1°$

◀ ۳-۳۰۰ سرعت شاتل $v_0 = \sqrt{\frac{Gm_e}{r}} = \sqrt{\frac{6.673(10^{-11})(5.976)(10^{24})}{(6371 + 200)(10^3)}} = 7790$ m/s

سرعت ماهواره $v = \sqrt{v_0^2 + v_{S/0}^2} = 7791$ m/s

۴۳-۳ معادله : $v^2 = 2gR^2 \left(\frac{1}{r} - \frac{1}{2a}\right)$

$$(7791)^2 = 2(9.825)(6371 \times 10^3)^2 \left(\frac{1}{6571 \times 10^3} - \frac{1}{2a}\right) \Rightarrow a = 6572 \text{ km}$$

$$\tau = \frac{2\pi a^{3/2}}{R\sqrt{g}} = 5301 \text{ s}$$

$$E = \frac{1}{2} m v^2 - \frac{Gm_e m}{r} = -30.3\ m(10^6)\ \text{J}$$

$$h = rv_\theta = 6571(10^3)7790 = 5.12(10^{10})\ \text{m}^2/\text{s}$$

۴۱-۳ معادله: $e = \sqrt{1 + \dfrac{2Eh^2}{mg^2R^4}} = 0.01284$

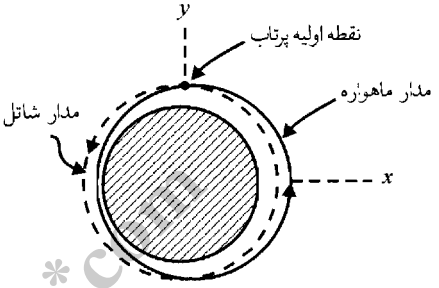
$$\frac{1}{r} = \frac{1 + e\cos\theta}{a(1 - e^2)} \quad , \quad \theta = 90° \quad \text{(نصف قطر اصلی موازی محور } x \text{ است)}$$

$$r_{\min} = a(1 - e) = 6.49(10^6)\ \text{m}$$

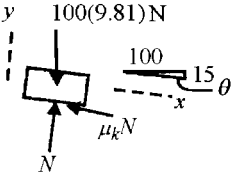
$$r_{\max} = a(1 + e) = 6.66(10^6)\ \text{m}$$

$$v_p = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{\max}}{r_{\min}}} = 7890\ \text{m/s}$$

$$v_a = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{\min}}{r_{\max}}} = 7690\ \text{m/s}$$



۳۰۱-۳. کف کامیون یک چارچوب مرجع سرعت ثابت است بطوریکه $U_{\text{rel}} = \Delta T_{\text{rel}}$



$$\Sigma F_y = 0 : N - 981\cos 8.53° = 0 \Rightarrow N = 970\ \text{N}$$

$$U_{\text{rel}} = \Delta T_{\text{rel}} : (981\sin 8.53° - 970\,\mu_k)2 = \frac{1}{2}(100)(0 - 3^2)$$

$$\Rightarrow \mu_k = 0.382$$

$\theta = \tan^{-1} 0.15 = 8.53°$

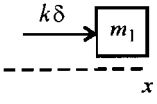
۳۰۲-۳.

$$\Sigma F_x = ma_x : k\delta = m_1 a_{x_1}$$

$$-k\,\delta = m_2 a_{x_2}$$

$$a_{1/2} = a_{x_1} - a_{x_2} = \frac{k\delta}{m_1} - \left(-\frac{k\delta}{m_2}\right)$$

$$a_{1/2} = a_{\text{rel}} = k\delta\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$$



۳۰۳-۳.

$$v_{\text{rel}} = l\dot{\theta} = 0.5(2) = 1\ \text{m/s} \rightarrow$$

$$v = v + v_{\text{rel}} = 2 + 1 = 3\ \text{m/s} \rightarrow$$

$$\mathbf{G} = m\mathbf{v} = 3(3\mathbf{i}) = 9\mathbf{i}\ \text{kg.m/s}$$

$$\mathbf{G}_{\text{rel}} = m\mathbf{v}_{\text{rel}} = 3(1\mathbf{i}) = 3\mathbf{i}\ \text{kg.m/s}$$

$$T = \frac{1}{2}mv^2 = \frac{1}{2}(3)(3)^2 = 13.5\ \text{J}$$

$$T_{rel} = \frac{1}{2}mv_{rel}^2 = \frac{1}{2}(3)(1)^2 = 1.5\ \text{J}$$

$$\mathbf{H}_O = -lm\ v\mathbf{k} = -(0.5)(3)(3)\ \mathbf{k} = -4.5\ \mathbf{k}\ \text{kg.m}^2/\text{s}$$

$$\mathbf{H}_{B_{rel}} = -lm\ v_{rel}\mathbf{k} = -(0.5)(3)(1)\ \mathbf{k} = -1.5\ \mathbf{k}\ \text{kg.m}^2/\text{s}$$

۳۰۴-۳. $U_{rel} = \Delta T_{rel}$ نسبت به ناو

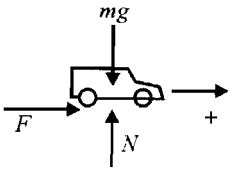
$$(22 + P)(10^3)75 = \frac{1}{2}(3)(10^3)\left[\left(\frac{240}{3.6}\right)^2 - 0\right] \Rightarrow 22 + P = 88.9 \Rightarrow P = 66.9\ \text{kN}$$

۳۰۵-۳. چارچوب ثابت یدک‌کش نیوتنی است.

$$v^2 = v_0^2 + 2a(s - s_0)$$

$$\left(\frac{24}{3.6}\right)^2 = 0 + 2a(25) \Rightarrow a = 0.889\ \text{m/s}^2$$

$$\Sigma F = ma \ : \ F = 2000(0.889) = 1778\ \text{N}$$

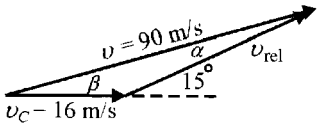


۳۰۶-۳.

$$\frac{\sin 165°}{90} = \frac{\sin\alpha}{16} \Rightarrow \alpha = 2.64°$$

$$\beta = 180 - 165 - \alpha = 12.36°$$

$$\frac{\sin\beta}{v_{rel}} = \frac{\sin 165°}{90} \Rightarrow v_{rel} = 74.4\ \text{m/s}$$



$$U_{rel} = \Delta T_{rel} : Fd = \frac{1}{2}m\left(v_{rel}^2 - 0\right) \Rightarrow F(100) = \frac{1}{2}(7000)(74.4)^2 \Rightarrow F = 194000\ \text{N}$$ یا 194kN

۳۰۷-۳. برای کامیون: $a_T = -0.9\ g$, $t_{stop} = \dfrac{15\ \text{m/s}}{0.9(9.81\ \text{m/s}^2)} = 1.699\ \text{s}$

برای صندوق: $a_C = -0.7\ g$

$a_{C/T} = a_C - a_T = -0.7\ g - (-0.9\ g) = 0.2\ g$ (مادامیکه کامیون حرکت می‌کند)

$t = t_C$ در : $x_{C/T} = (x_{C/T})_0 + (v_{C/T})_0\ t_{stop} + \frac{1}{2}a_{C/T}\ t_{stop}^2$

$$= 0 + 0 + \frac{1}{2}(0.2\ g)(1.699)^2 = 2.83\ \text{m}$$

$$v_{C/T} = (v_{C/T})_0 + a_{C/T}\ t_{stop} = 0 + (0.2\ g)(1.699) = 3.33\ \text{m/s}$$

$$v_C^2 = v_{C0}^2 + 2a_C(x - x_0) = (3.33)^2 + 2(-0.7\ g)(3.2 - 2.83) \Rightarrow v_C = 2.46\ \text{m/s}$$

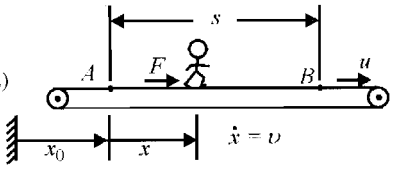
۳-۳۰۸.

مطلق: $U = \Delta T \;:\; F(s + \Delta x_0) = \frac{1}{2} m (u + v)^2 - \frac{1}{2} mu^2$

$$Fs + F\Delta x = \frac{1}{2} mv^2 + muv \quad (1)$$

نسبت به راهرو: $U_{rel} = \Delta T_{rel} \;:\; Fs = \frac{1}{2} mv^2 - 0 \quad (2)$

از (1) تفاضل (2): $F\Delta x_0 = muv$



جملهٔ muv معرف کاری است که توسط نیروی F فقط بخاطر حرکت راهرو انجام می‌شود

۳-۳۰۹.

$$\Sigma F_x = ma_x : -k(x - x_0) = m(\ddot{x} - a_0)$$

$$\dot{x}d\dot{x} = \ddot{x}dx \Rightarrow \int_0^{\dot{x}} \dot{x}d\dot{x} = \int_{x_0}^{x} \left[a_0 - \frac{k}{m}(x - x_0)\, dx\right]$$

$$\frac{1}{2}\dot{x}^2 = \left(a_0 - \frac{kx_0}{m}\right)(x - x_0) - \frac{k}{2m}(x^2 - x_0^2)$$

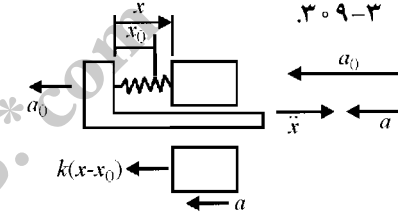
$$\frac{d}{dx}\left(\frac{\dot{x}^2}{2}\right) = a_0 + \frac{kx_0}{m} - \frac{kx}{m} = 0 \qquad \text{(برای ماکزیمم یا ماکزیمم } \frac{\dot{x}}{2}\text{)}$$

$$\Rightarrow \frac{kx}{m} = a_0 + \frac{kx_0}{m} \Rightarrow x = x_0 + \frac{ma_0}{k}$$

$$(v_{rel})^2_{max} = \dot{x}^2_{max} = 2\left(a_0 + \frac{kx_0}{m}\right)\left(x_0 + \frac{ma_0}{k} - x_0\right) - \frac{k}{m}\left(x_0^2 + \frac{2ma_0}{k}x_0 + \frac{m^2a_0^2}{k^2} - x_0^2\right)$$

$$= \frac{ma_0^2}{k}$$

$$\Rightarrow (v_{rel})_{max} = a_0\sqrt{\frac{m}{k}}$$



۳-۳۱۰.

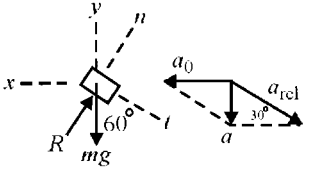
برای داشتن شتاب قائم رو به پایین $a_0 = a_{rel}\cos 30°$

$$a_x = 0 \Rightarrow \Sigma F_x = 0$$

$$-R \sin 30° = 0 \Rightarrow R = 0$$

$$\Sigma F_y = ma_y : mg = ma \Rightarrow a = g$$

$$a_0 = a\sqrt{3} = g\sqrt{3} = 9.81\sqrt{3} = 16.99 \text{ m/s}^2$$

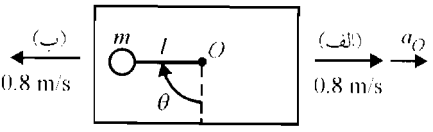


۳-۳۱۱.

$m_b = 10$ kg $\quad l = 0.8$ m $\quad \theta = 90°$ $\quad \dot{\theta} = 3$ rad/s $\quad v_{b/s} = l\dot{\theta} = 0.8(3) = 2.4$ m/s

$m_c = 250$ kg

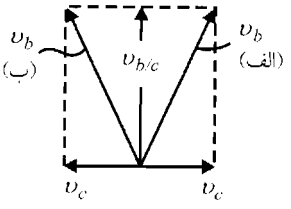
$$T_b = \frac{1}{2} m_b v_b^2 = \frac{1}{2}(10)\,[0.8^2 + 2.4^2] = 32 \text{ J}$$



یکسان برای حالت الف و ب

$$T_c = \frac{1}{2} m_c v_c^2 = \frac{1}{2}(250)(0.8)^2 = 80 \text{ J}$$

$$T = T_b + T_c = 32 + 80 = 112 \text{ J}$$



برای حالت الف و ب

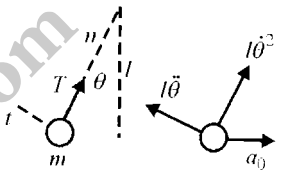
۳-۳۱۲.

$$\Sigma \mathbf{F} = m(\mathbf{a}_0 + \mathbf{a}_{\text{rel}})$$

در جهت t $\quad \Sigma F_t = 0 \Rightarrow a_t = l\ddot{\theta} - a_0 \cos\theta = 0$

$$\ddot{\theta} = \frac{a_0}{l}\cos\theta \qquad (1)$$

در جهت n $\quad \Sigma F_n = ma_n \Rightarrow T = m(l\dot{\theta}^2 + a_0 \sin\theta) \qquad (2)$



(1) انتگرال گیری : $\ddot{\theta} = \dot{\theta}\frac{d\dot{\theta}}{d\theta} = \frac{a_0}{l}\cos\theta \Rightarrow \int_0^{\dot{\theta}} \dot{\theta}\, d\dot{\theta} = \int_0^{\theta} \frac{a_0}{l}\cos\theta\, d\theta \Rightarrow \frac{1}{2}\dot{\theta}^2 = \frac{a_0}{l}\sin\theta$

(2) رابطه : $T = m\left[2a_0 \sin\theta + a_0 \sin\theta\right] = 3ma_0 \sin\theta$

$\theta = \frac{\pi}{2}$: $T_{\pi/2} = 3ma_0 = 3(10)(3) = 90$ N

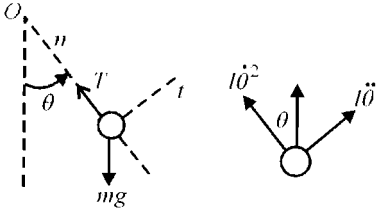
۳-۳۱۳.

$$\Sigma F_t = ma_t$$

$$-mg \sin\theta = m(l\ddot{\theta} + a_0 \sin\theta) \Rightarrow \ddot{\theta} = -\left(\frac{a + g}{l}\right)\sin\theta$$

$$\int_0^{\dot{\theta}} \dot{\theta}\, d\dot{\theta} = -\int_{\theta_0}^{\theta} \left(\frac{a_0 + g}{l}\right)\sin\theta\, d\theta$$

$$\dot{\theta}^2 = 2\left(\frac{a_0 + g}{l}\right)(\cos\theta - \cos\theta_0)$$



$$\Sigma F_n = ma_n$$

$$T - mg\cos\theta = m(l\dot{\theta}^2 + a_0 \cos\theta)$$

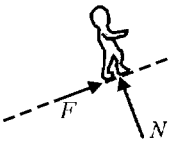
$$\Rightarrow T = m\left[g(3\cos\theta - 2\cos\theta_0) + a_0(3\cos\theta - 2\cos\theta_0)\right]$$

$$= m(g + a_0)(3\cos\theta - 2\cos\theta_0)$$

$$\theta = 0 \Rightarrow T_0 = m(g + a_0)(3 - 2\cos\theta_0)$$

$$\theta_0 = \frac{\pi}{2} \Rightarrow T_0 = 3m(g + a_0)$$

۳-۳۱۴.



برای حرکت از A تا B : مطلق : $U'_{abs} = \Delta T + \Delta V_g$

$$F(\Delta x_0 + s) = \frac{1}{2} m (v_r + u)^2 - \frac{1}{2} mu^2 + mg (\Delta x_0 + s) \sin\theta$$

$$= \frac{1}{2} mv_r^2 + mv_r\mu + mg (\Delta x_0 + s) \sin\theta$$

نسبی : $U'_{rel} = \Delta T_{rel} + \Delta V_{g_{rel}}$

$$F_s = \frac{1}{2} mv_r^2 + mgs \sin\theta$$

کار انجام شده توسط راهرو : $U'_{abs} - U'_{rel} = mv_r\mu + mg\,\Delta x_0 \sin\theta$

$mv_r\mu$ معرف کار انجام شده توسط تسمه فقط بخاطر حرکت راهرو است.

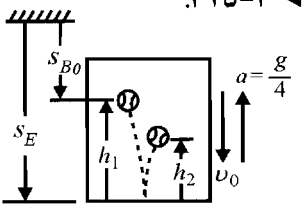
$m = 60$ kg , $v_r = 0.75$ m/s , $u = 0.6$ m/s , $\theta = 10°$, $s = 10$ m

$$\Sigma F_x = ma_{x_{rel}} \quad : \quad a_{x_{rel}} = \frac{v_r^2}{2s} = \frac{0.75^2}{2(10)} = 0.028 \text{ m/s}^2$$

$$F - (60 \times 9.81) \sin 10° = 60\,(0.028) \Rightarrow F = 103.7 \text{ N}$$

توان تولیدی مرد $P_{rel} = Fv_r = 103.7(0.75) = 77.9$ W

◀ ۳-۳۱۵.



(الف) $a = 0$ آسانسور چارچوب نیوتنی است

$$v' = ev = e\sqrt{2gh_1} = \sqrt{2gh_2} \Rightarrow h_2 = e^2h_1$$

(ب) $a = \frac{g}{4}$ بطرف بالا $E =$ آسانسور

$B =$ گوی

در برخورد : $s_B = s_E$: $s_{B_0} + v_{B_0}t + \frac{1}{2} gt^2 = s_{E_0} + v_{E_0}t - \frac{1}{2}\frac{g}{4} t^2$

$$\Rightarrow s_{B_0} + v_0t + \frac{1}{2} gt^2 = (s_{B_0} + h_1) + v_0t - \frac{1}{8} gt^2 \Rightarrow t = 2\sqrt{\frac{2h_1}{5g}}$$

$$v_{B/E} = v_B - v_E = \left(v_0 + g2\sqrt{\frac{2h_1}{5g}}\right) - \left(v_0 - \frac{g}{4} 2 \sqrt{\frac{2h_1}{5g}}\right) = \sqrt{\frac{5h_1g}{2}}$$

بعد از برخورد : $v'_{B/E_0} = -e\sqrt{\frac{5h_1g}{2}}$ بطرف بالا

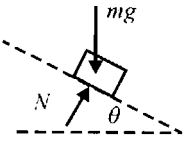
$$v'_{B/E} = v'_{B/E_0} + a_{B/E}t = -e\sqrt{\frac{5h_1g}{2}} + \frac{5}{4} gt \quad \overset{v'_{B/E}=0}{\Rightarrow} \quad t = 2e\sqrt{\frac{2h_1}{5g}}$$

$$S'_{B/E} = S'_{B/E_0} + v'_{B/E_0}t + \frac{1}{2}\frac{5}{4} gt^2 = 0 - e\sqrt{\frac{5h_1g}{2}}\, 2e\sqrt{\frac{2h_1}{5g}} + \frac{5}{8} g\, 4e^2 \frac{2h_1}{5g} = -e^2h_1$$

$$\Rightarrow h_2 = e^2h_1$$

۳-۳۱۶. ◀

$$U_{rel} = \Delta T_{rel}$$



$$mg\ l \sin\theta = \frac{1}{2} m v^2_{rel} - 0 \Rightarrow v^2_{rel} = 2gl \sin\theta$$

$$U = \Delta T : mg\ l \sin\theta + (N \sin\theta)d = \frac{1}{2} mv^2 - \frac{1}{2} mv_0$$

(d = مسافت افقی طی شده توسط قطعه)

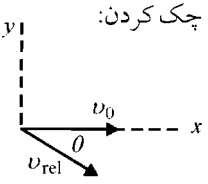
زمان لغزش از B تا C : $l = \frac{1}{2} at^2 = \frac{1}{2} g \sin\theta\, t^2 \Rightarrow t = \left(\frac{2l}{g \sin\theta}\right)^{1/2}$

$$d = v_0 t = v_0 \sqrt{\frac{2l}{g \sin\theta}}$$

$$N = mg \cos\theta$$

حل معادله کار - انرژی برای v^2 : $v_A = \left(v_0^2 + 2gl \sin\theta + 2v_0 \cos\theta \sqrt{2lg \sin\theta}\right)^{1/2}$

چک کردن:

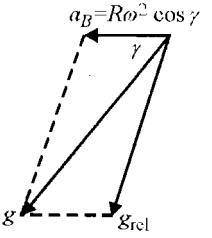


$$\mathbf{v}_A = \mathbf{v}_0 + \mathbf{v}_{rel}$$
$$= v_0\mathbf{i} + \sqrt{2gl \sin\theta}\,(\cos\theta\mathbf{i} - \sin\theta\mathbf{j})$$
$$= (v_0 + \sqrt{2gl \sin\theta} \cos\theta)\mathbf{i} - \sqrt{2gl \sin^3\theta}\,\mathbf{j}$$

$$v_A^2 = (v_0 + \sqrt{2gl \sin\theta} \cos\theta)^2 + (2gl \sin^3\theta) = v_0^2 + 2gl \sin\theta + 2v_0 \cos\theta \sqrt{2gl \sin\theta}$$ ✓

۳-۳۱۷. ◀

قانون کسینوس‌ها :



$$g^2_{rel} = g^2 + a_B^2 - 2ga_B \cos\gamma$$
$$= g^2\left(1 + \left[\frac{a_B}{g}\right]^2 - 2\frac{a_B}{g}\cos\gamma\right)$$
$$g_{rel} = g\left[1 + \frac{a_B}{g}\left(\frac{a_B}{g} - 2\cos\gamma\right)\right]^{1/2}$$

بسط دو جمله‌ای برای دو عبارت اول : $(1 + x)^n = 1 + nx + \ldots$

$$g_{rel} = g\left[1 + \frac{a_B}{g}\left(\frac{a_B}{2g} - \cos\gamma\right) + \ldots\right] = g + a_B\left(\frac{a_B}{2g} - \cos\gamma\right) + \ldots$$

$$g_{rel} = g - R\omega^2 \cos^2\gamma\left(1 - \frac{R\omega^2}{2g}\right) + \ldots$$

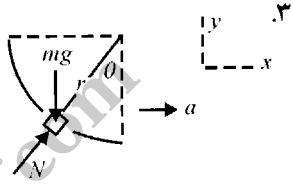
$$R\omega^2 = 6.371(10^6)(0.7292 \times 10^{-4})^2 = 0.03388\ \text{m/s}^2$$

$$g_{rel} = 9.825 - 0.03388\left(1 - \frac{0.03388}{2 \times 9.825}\right)\cos^2\gamma + \ldots = 9.825 - 0.03382 \cos^2\gamma\ \text{m/s}^2$$

◀ **۳-۳۱۸.** (الف) سرعت شاتل ثابت است بطوریکه $\dot{x}$ در راستای x هم شتاب مطلق و هم شتاب نسبی است بنابراین $F = m\ddot{x}$ معتبر است.

(ب) سرعت شاتل در موقعیت نشان داده شده در حال کاهش است بطوریکه یک مؤلفه از شتاب در جهت منفی x وجود دارد لذا شتاب مطلق در راستای x برابر است با $\ddot{x}$ منهای شتاب منفی مماسی شاتل. بنابراین $F \neq m\ddot{x}$. تنها در نقاط اوج و حضیض که $\dot{v} = 0$ است، $F = m\ddot{x}$ معتبر خواهد بود.

۳-۳۱۹.

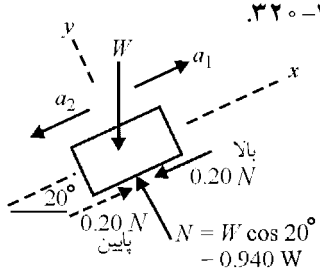


$$\Sigma F_y = 0 \;:\; N\cos\theta - mg = 0 \Rightarrow N\cos\theta = mg$$

$$\Sigma F_x = ma_x \;:\; N\sin\theta = ma$$

تقسیم دو رابطه : $\tan\theta = \dfrac{a}{g} \Rightarrow \theta = \tan^{-1}\dfrac{a}{g}$

۳-۳۲۰.



بطرف بالا : $\Sigma F_x = ma_x$

$$-0.20(0.940\ W) - W\sin 20° = \frac{W}{g}a_1$$

$$a_1 = -5.2\ \text{m/s}^2$$

$$v^2 = v_1^2 + 2as \Rightarrow 0 = 6^2 + 2(-5.2)\,s \Rightarrow s = 3.46\ \text{m}$$

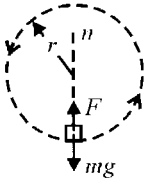
بطرف پایین : $\Sigma F_x = ma_x$

$$-W\sin 20° + 0.20\,(0.94\ W) = \frac{W}{g}(-a_2)$$

$$a_2 = 1.51\ \text{m/s}^2$$

$$v^2 = v_0^2 + 2a_2 s \Rightarrow v_2^2 = 0^2 + 2(1.51)(3.46) \Rightarrow v_2 = 3.24\ \text{m/s}$$

۳-۳۲۱. شرایط بحرانی وقتی رخ می‌دهد که وزنه در موقعیت پایین باشد.



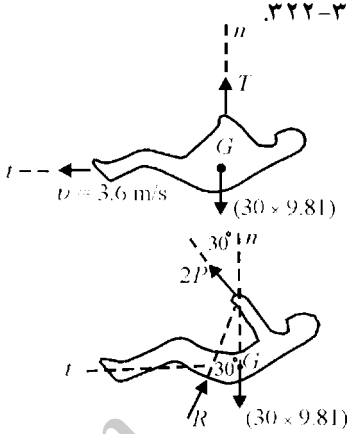
$$\Sigma F_n = ma_n \;:\; F - mg = mr\omega^2$$

$$80 - 0.030(9.81) = 0.030(0.175)\omega^2$$

$$\omega = 123.2\ \text{rad/s}$$

$$N = \omega\left(\frac{60}{2\pi}\right) = 1177\ \text{rev/min}$$

۳-۳۲۲.



کل سیستم $\Sigma F_n = m\dfrac{v^2}{r}$: $T - (30 \times 9.81) = 30\dfrac{3.6^2}{4.5}$

$$T = 380.7 \text{ N}$$

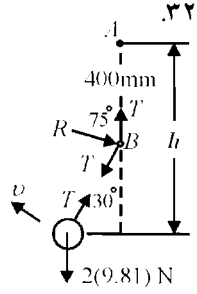
دختر بچه $\Sigma F_n = m\dfrac{v^2}{r}$

$$2P\cos 30° + R\cos 30° - (30 \times 9.81) = 30\dfrac{3.6^2}{4.5}$$

$$\Sigma F_t = 0 \;:\; 2P\sin 30° - R\sin 30° = 0$$

حل دو رابطه : $P = 109.9$ N ، $R = 220$ N

۳-۳۲۳.



$v^2 = 2gh = 2(9.81)(0.4 + 0.4\cos 30°) = 14.46 \text{ m}^2/\text{s}^2$

$\Sigma F_n = ma_n$: $T - 2(9.81)\cos 30° = 2\dfrac{14.64}{0.4}$

$$T = 90.2 \text{ N}$$

تعادل نیروها در B : $R = 2T\cos 75° = 2(90.2)(0.259) = 46.7$ N

۳-۳۲۴. احتمالات

الف - دو گوی با سرعت v_1

ب- یک گوی با سرعت $2v_1$

در هر دو حالت بقای مومنتوم برقرار است لذا $2(1mv_1) = 1(2m)v_1$

اما با $e = 1$ انرژی جنبشی نیز پایستار است.

حالت اولیه : $T = 2\left(\dfrac{1}{2}mv_1^2\right) = mv_1^2$

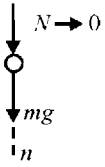
حالت نهایی :

$$\begin{cases} T_a' = 2\left(\dfrac{1}{2}mv_1^2\right) = mv_1^2 \\ T_b' = 1\left(\dfrac{1}{2}m(2v_1)^2\right) = 2mv_1^2 \end{cases}$$

بنابراین حالت ب امکان‌ناپذیر است.

۳-۳۲۵. دینامیک در B (بالای حلقه)

$$\Sigma F_n = ma_n \;:\; mg = m\frac{v_B^2}{R} \;\Rightarrow\; v_B^2 = gR$$

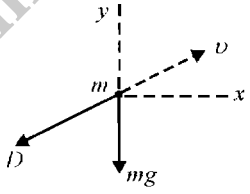


کار-انرژی جنبشی از A تا B:

$$T_A + U_{A-B} = T_B \;:\; 0 + \frac{1}{2}k\delta^2 - mg\,\mu_k R - mg\,(2R) = \frac{1}{2}m\,(gR)$$

$$\delta = \sqrt{\frac{mgR(5 + 2\mu_k)}{k}}$$

۳-۳۲۶. $\Sigma \mathbf{F} = m\mathbf{a}$



$$-C_D\frac{1}{2}\rho v^2 s\frac{\mathbf{v}}{v} - mg\,\mathbf{i} = m\,(a_x\mathbf{i} + a_y\mathbf{j})$$

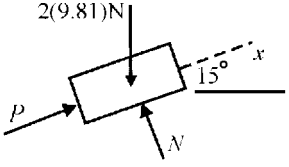
$$-C_D\frac{1}{2}\rho v s\,(v_x\mathbf{i} + v_y\mathbf{j}) - mg\,\mathbf{j} = m\,(a_x\mathbf{i} + a_y\mathbf{j})$$

$$\Rightarrow \begin{cases} a_x = -C_D\frac{1}{2}\rho s v v_x/m \\ a_y = -C_D\frac{1}{2}\rho s v v_y/m - g \end{cases} \quad (\text{که در آن } v = \sqrt{v_x^2 + v_y^2})$$

دو عبارت شتاب بواسطهٔ عبارت سرعت بهم مرتبط هستند لذا غیرخطی می‌باشند.

۳-۳۲۷.

$$\int \Sigma F_x\,dt = \Delta G_x \;:$$

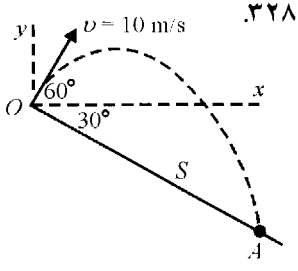


$$\int_0^2 P dt = \frac{1}{2}(0.5\times 28) + \frac{1}{2}(0.5\times 20) = 12\ \text{N.s}$$

$$\int_0^2 \Sigma F_x\,dt = 12 - 2\,(9.81)\sin 15^\circ \times 2 = 1.844\ \text{N.s}$$

$$\Delta G_x = m\,\Delta v_x \;:\; \Delta G_x = 2(v - 2)\ \text{kg.m/s} \Rightarrow 1.844 = 2(v - 2) \Rightarrow v = 2.92\ \text{m/s}$$

۳-۳۲۸.



$$x = S\cos 30^\circ = (u\cos 60^\circ)\,t$$

$$0.866\,S = 10\left(\frac{1}{2}\right)t \qquad (1)$$

$$y = -S\sin 30^\circ = (u\sin 60^\circ)t - \frac{1}{2}gt^2$$

$$-\frac{1}{2}S = 10\,(0.866)\,t - \frac{9.81}{2}t^2 \qquad (2)$$

(1) و (2) حل : $t = 2.35\ \text{s}\ ,\ S = 13.59\ \text{m}$

$v_y = u \sin 60° - gt = 10(0.866) - 9.81(2.35) = -14.4\ \text{m/s}$

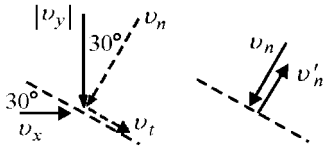
$v_x = u \cos 60° = 10\left(\frac{1}{2}\right) = 5\ \text{m/s}$

$v_n = 14.4 \cos 30° - 5 \sin 30° = 9.97\ \text{m/s}$

$v_t = 14.4 \sin 30° + 5 \cos 30° = 11.53\ \text{m/s}$

$e = 0.6 \Rightarrow \frac{v'_n}{v_n} = 0.6 \Rightarrow v'_n = 0.6(9.97) = 6\ \text{m/s}$

$\Rightarrow v = \sqrt{6^2 + 11.53^2} = 13\ \text{m/s}$

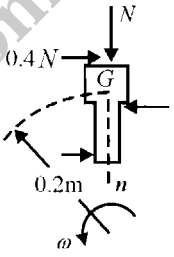


۳-۳۲۹.

$\omega = 3000\left(\frac{2\pi}{60}\right) = 314.2\ \text{rad/s}$

$\Sigma F_n = ma_n \quad : \quad N = 2(0.2)(314.2)^2 = 39.5(10^3)\text{N}$

$M = 4\mu_k Nr = 4(0.4)(39.5)(10^3)(0.3) = 18.96\ \text{kN.m}$



۳-۳۳۰.

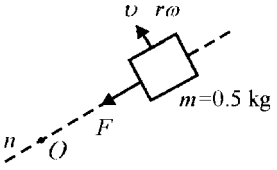
$\Sigma F_n = ma_n \ : \ a_n = r\dot{\theta}^2 = r\omega^2$

$\left.\begin{aligned} F_1 &= mr_1\omega_1^2 \\ F_2 &= mr_2\omega_2^2 \end{aligned}\right\} \Rightarrow \Delta F = m(r_2\omega_2^2 - r_1\omega_1^2)$

$\Delta F = 0.5\,(0.175 \times 400^2 - 0.150 \times 300^2)\left(\frac{2\pi}{60}\right)^2 = 79.5\ \text{N}$

$F = kx \Rightarrow \Delta F = k\Delta x = k\Delta r$

$\Rightarrow \quad k = \frac{79.5}{0.175 - 0.150} = 3180\ \text{N/m}$ یا $3.18\ \text{kN/m}$



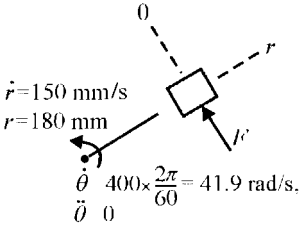
۳-۳۳۱.

$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

$F = 0.5(0 + 2 \times 0.150 \times 41.9) = 6.28\ \text{N}$

$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2) = 0 \Rightarrow \ddot{r} = r\dot{\theta}^2 = 0.180(41.9)^2$

$= 316\ \text{m/s}^2$



۳-۳۳۲.

$$\Sigma F = ma \;:\; G\frac{mm_0}{(D-s)^2} = ma$$

$$\left.\begin{aligned} s &= D - r_0 \\ a &= g_0 = 3.73 \text{ m/s}^2 \end{aligned}\right\} \; G\frac{m_0}{r_0^2} = g_0$$

$$\Rightarrow a = g_0\frac{r_0^2}{(D-s)^2}$$

$$\int_{v_1}^{v_2} v\,dv = \int_0^s a\,ds \;\Rightarrow\; \frac{1}{2}(v^2 - v_1^2) = \int_0^{D-r_0} g_0\frac{r_0^2}{(D-s)^2}\,ds = g_0 r_0^2\left(\frac{1}{D-s} - \frac{1}{D}\right)$$

$$\Rightarrow v^2 = v_1^2 + 2g_0 r_0^2\frac{s}{(D-s)D}$$

$s = 10{,}000$ km

$$v^2 = (20{,}000)^2 + 2(3.73)\frac{(3600)^2}{1000}(3400)^2\frac{10{,}000}{3400(13400)} = 645(10^6)$$

$\Rightarrow v = 25400$ km/h در برخورد

۳-۳۳۳.

$$y_A = y_{C'} + v_{yC'}t - \frac{1}{2}gt^2$$

$$0 = 2R + 0(t) - \frac{1}{2}gt^2 \;\Rightarrow\; t = 2\sqrt{\frac{R}{g}}$$

$$x_A = x_{C'} + v_{xC'}t$$

$$0 = 3R - v_{C'}\sqrt{\frac{R}{g}} \;\Rightarrow\; v_{C'} = \frac{3}{2}\sqrt{gR}$$

$$T_B + U_{B-C'} = T_{C'}$$

$$\frac{1}{2}mu^2 - mg(2R) = \frac{1}{2}m\left[\frac{3}{2}\sqrt{gR}\right]^2 \;\Rightarrow\; u = \frac{5}{2}\sqrt{gR}$$

$$\Sigma F_n = ma_n \;:\; mg = m\frac{v_{C'}^2}{R} \;\Rightarrow\; v_{C'} = \sqrt{gR}$$

$$x_A = x_{C'} + v_{xC'}t$$

$$0 = x_{\min} - \sqrt{gR}\,2\sqrt{\frac{R}{g}} \;\Rightarrow\; x_{\min} = 2R$$

۳-۳۳۴. در حالت لغزش نهایی: $U_{1\text{-}2} = \Delta T \Rightarrow -\mu_k mg\,d = 0 - \frac{1}{2}mv'^2 \;\Rightarrow\; v' = \sqrt{2\mu_k gd}$

بعد از تصادف $v_A' = \sqrt{2(0.9)(9.81)(15)} = 16.27$ m/s

بعد از تصادف $v_B' = \sqrt{2(0.9)(9.81)(30)} = 23.02$ m/s

برخورد: $m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

$$1800\,v_A + 0 = 1800(16.27) + 900(23.02) \Rightarrow v_A = 27.78 \text{ m/s}$$

در لغزش اولیه : $U_{1\text{-}2} = \Delta T$

$$-\mu_k \not{m}\, gd = \frac{1}{2}\not{m}\,(v_A^2 - v_{A_0}^2)$$

$$-(0.9)(9.81)(15) = \frac{1}{2}(27.78^2 - v_{A0}^2) \Rightarrow v_{A0} = 32.2 \text{ m/s}$$ یا 115.9 km/h بلی

۳-۳۳۵. برای سیستم مرد و طناب برای سقوط کامل:

(الف) $U'_{1\text{-}2} = 0 = \Delta V_g + \Delta V_e$: $0 = 80(9.81)(-44) + \frac{1}{2}k(44 - 20)^2$

$$\Rightarrow k = 119.9 \text{ N/m}$$

(ب) $U'_{1\text{-}2} = 0 = \Delta T + \Delta V_g + \Delta V_e$: $0 = \frac{1}{2}80v^2 - 80(9.81)(20 + y) + \frac{1}{2}119.9\, y^2$

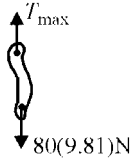
(y تغییرطول طناب)

برای ماکزیمم v^2 : $40\frac{d(v^2)}{dy} = 80(9.81) - 119.9\, y = 0$

$$\Rightarrow y = 6.55 \text{ m}$$

$$v^2_{\max} = \frac{1}{40}\left\{80(9.81)(20 + 6.55) - \frac{1}{2}119.9(6.55)^2\right\} = 457 \Rightarrow v_{\max} = 21.4 \text{ m/s}$$

(ج) حداکثر شتاب در انتها رخ می‌دهد که کشش بیشترین مقدار را دارد



$$T_{\max} = ky = 119.9(44 - 20) = 2880 \text{ N}$$

$$+\uparrow \Sigma F_y = ma_{\max} : 2880 - 80(9.81) = 80\, a_{\max}$$

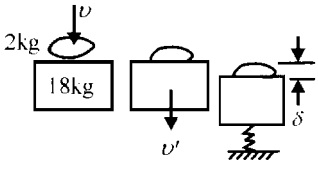
$$a_{\max} = 26.2 \text{ m/s}^2 \text{ یا } \frac{8}{3}g$$

۳-۳۳۶.

$$v = \sqrt{2gh} = \sqrt{2(9.81)(2)} = 6.26 \text{ m/s}$$

$$\Delta G = 0 : 18(6.26) + 0 = (18 + 2)v'$$

$$v' = 5.64 \text{ m/s}$$



فشردگی اولیه فنر : $\delta_0 = \frac{W}{2k} = \frac{18(9.81)}{2(1.2)10^3} = 0.0736 \text{ m}$

$$\Delta T + \Delta V_g + \Delta V_e = 0$$

$$\Delta T = 0 - \frac{1}{2}20(5.64)^2 = -317.8 \text{ J}$$

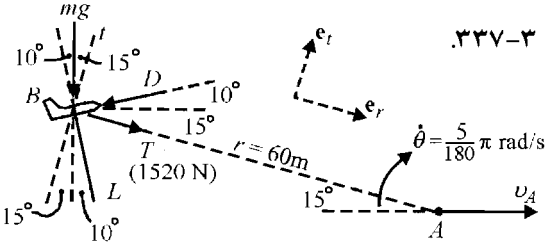
$$\Delta V_g = -20(9.81)\,\delta = -196.2\,\delta \text{ J}$$

$$\Delta V_e = \frac{1}{2}2(1.2 \times 10^3)[(0.0736 + \delta)^2 - (0.0736)^2] \text{ J}$$

$$-317.8 - 196.2 + 1.2(10^3)[(0.0736 + \delta)^2 - (0.0736)^2] = 0$$

$$\Rightarrow \delta = 0.659 \text{ m یا } 65.9 \text{ mm}$$

۳-۳۳۷.



$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} = \mathbf{0} + \dot{r}\dot{\theta}^2\mathbf{e}_r + 0\mathbf{e}_t$$

$$a_B = 60\left[\frac{5\pi}{180}\right]^2 = 0.457 \text{ m/s}^2$$

$$\Sigma F_t = ma_t \;:\; L\cos 25° - D\sin 25° - 200\,(9.81)\cos 15° = 0$$

$$\Sigma F_r = ma_r \;:\; 1520 + 200\,(9.81)\sin 15° - L\sin 25° - D\cos 25° = 200\,(0.457)$$

حل دو رابطه فوق: $D = 954$ N , $L = 2540$ N

۳-۳۳۸. مومنتوم در هر دو جهت x و y پایستار است.

$$\Delta G_x = 0 \;:\; [m_A + m_B]\,v\sin\theta - [m_A \times 0 + m_B(80\sin 60°)] = 0$$

$$\Delta G_y = 0 \;:\; [m_A + m_B]\,v\cos\theta - [m_A \times 60 + m_B(80\cos 60°)] = 0$$

$$m_A = m_B \Rightarrow \tan\theta = \frac{80\sin 60°}{60 + 80\cos 60°} = 0.693 \Rightarrow \theta = 34.7°$$

درصد اتلاف انرژی: $n = \dfrac{T_1 - T_2}{T_1}100\% = 100\%\left(1 - \dfrac{T_2}{T_1}\right)$

$$= 100\%\left(1 - \frac{\frac{1}{2}2mv^2}{\frac{1}{2}m_A{v_A}^2 + \frac{1}{2}m_B{v_B}^2}\right) = 100\%\left(1 - \frac{2(60.8)^2}{60^2 + 80^2}\right) = 74.0\%$$

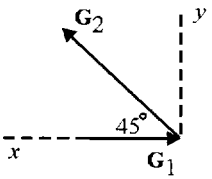
$$e = \frac{\text{سرعت نسبی جدا شدن}}{\text{سرعت نسبی نزدیک شدن}} = 0$$

۳-۳۳۹.

طبق مثال ۲-۶: $2s = \dfrac{v^2\sin 2\theta}{g} \Rightarrow 110 = \dfrac{v^2\sin 90°}{9.81} \Rightarrow v = 32.85$ m/s

$$\mathbf{G}_1 = m\mathbf{v}_1 = 0.146\left(\frac{145}{3.6}\right)(-\mathbf{i}) = -5.88\,\mathbf{i} \text{ m/s}$$

$$\mathbf{G}_2 = m\mathbf{v} = 0.146\,(32.85)\left(\frac{\mathbf{i}}{\sqrt{2}} + \frac{\mathbf{j}}{\sqrt{2}}\right) = 3.39\,(\mathbf{i} + \mathbf{j}) \text{ m/s}$$



$$\mathbf{F}_{av}\Delta t = \mathbf{G}_2 - \mathbf{G}_1 \;:\; \mathbf{F}_{av}(0.005) = 3.39(\mathbf{i} + \mathbf{j}) - (-5.88\,\mathbf{i})$$

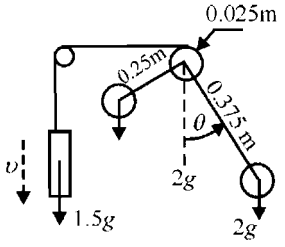
$$\Rightarrow \mathbf{F}_{av} = 1854\mathbf{i} + 678\mathbf{j} \Rightarrow F_{av} = \sqrt{1854^2 + 678^2} = 1974 \text{ N}$$

(نکته: از وزن توپ طی برخورد با چوگان صرفنظر می‌شود. اگر وزن توپ منظور شود، باز هم F_{av} حدود 1974N خواهد بود)

۳-۳۴۰.

کار انجام شده طی جابجایی θ: $U = 1.5g\,(0.025\theta) + 2g(0.25\sin\theta) - 2g(0.375)(1-\cos\theta)$

برای $\theta = 30° \Rightarrow U = 1.659$ J



$$\Delta T = \frac{1}{2}\left[1.5\,v^2 + 2\left(\frac{0.25}{0.025}\,v\right)^2 + 2\left(\frac{0.375}{0.025}\,v\right)^2\right]$$

$$= 325.8\,v^2$$

$U_{1\text{-}2} = \Delta T \Rightarrow v = 0.0714$ m/s یا 71.4 mm/s

۳-۳۴۱.

رها شدن A (حالت ① → حالت ②)

$T_1 + U_{1\text{-}2} = T_2$: $0 + m_A g\,1.8(1-\cos 60°) = \frac{1}{2} m_A v_{A_2}^2 \Rightarrow v_{A_2} = 4.20$ m/s

برخورد (② → ③)

$$\begin{cases} m_A v_{A_2} + m_B \cancel{v_{B_2}}^{0} = m_A v_{A_3} + m_B v_{B_3} & (1) \\ v_{B_3} - v_{A_3} = 0.7\,(v_{A_2} - \cancel{v_{B_2}}^{0}) & (2) \end{cases}$$

حل (1) و (2): $v_{A_3} = 2.42$ m/s ، $v_{B_3} = 5.36$ m/s

B بالا آمدن (③ → ④)

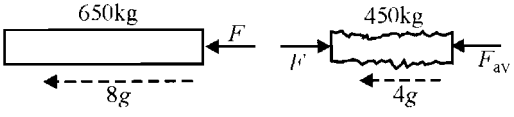
$T_3 + U_{3\text{-}4} = T_4$: $\frac{1}{2} m_B (5.36)^2 - m_B(9.81)[2.4\,(1-\cos 30°) + s\sin 30°] = 0$

$\Rightarrow s = 2.28$ m

۳-۳۴۲.

$\overset{+}{\leftarrow} \Sigma F = ma$: $F = 650(8)(9.81) = 51000$ N

$F_{av} - 51000 = 450(4)(9.81) \Rightarrow F_{av} = 68700$ N یا 68.7 kN



انرژی جنبشی اولیه $= \frac{1}{2}(1100)\left(\frac{38.9}{3.6}\right)^2 = 64200$ J یا 64.2 kJ = اتلاف انرژی ΔE

۳-۳۴۳. I. لوکوموتیو 300mm حرکت می‌کند (حالت ① → حالت ②)

$$U = \Delta T \Rightarrow Fd = \frac{1}{2} m (v_2^2 - v_1^2) \Rightarrow 200(10^3)(0.300) = \frac{1}{2} 200 (10^3)(v_2^2 - 0)$$

$$\Rightarrow v_2 = 0.775 \text{ m/s}$$

II. A برخورد با (② → ③)

$$m_L v_2 = (m_L + m_A)v_3 \Rightarrow 200(10^3)(0.775) = (200 + 100)10^3 v_3 \Rightarrow v_3 = 0.517 \text{ m/s}$$

III. (300 mm) L و A حرکت (③ → ④)

$$200(10^3)(0.300) = \frac{1}{2} (200 + 100)10^3 (v_4^2 - 0.517^2)$$

$$\Rightarrow v_4 = 0.817 \text{ m/s}$$

IV. B برخورد با (④ → ⑤)

$$(m_L + m_A)v_4 = (m_L + m_A + m_B)v_5 \Rightarrow (200 + 100)10^3(0.817) = (200 + 100 + 100)10^3 v_5$$

$$\Rightarrow v_5 = 0.613 \text{ m/s}$$

V. (300 mm) B و L و A حرکت (⑤ → ⑥)

$$200(10^3)(0.300) = \frac{1}{2} (200 + 100 + 100)10^3 (v_6^2 - 0.613^2) \Rightarrow v_6 = 0.822 \text{ m/s}$$

VI. C برخورد با (⑥ → ⑦)

$$(m_L + m_A + m_B)v_6 = (m_L + m_A + m_B + m_C)v_7$$

$$\Rightarrow (200 + 100 + 100)10^3(0.822) = (200 + 100 + 100 + 100)10^3 v_7$$

$$\Rightarrow v_7 = 0.657 \text{ m/s}$$ (جواب الف)

(ب) بدون شل بودن کوپلینگ‌ها : $U = \Delta T$

$$200(10^3)(0.900) = \frac{1}{2} 500 (10^3)(v'^2 - 0) \Rightarrow v' = 0.849 \text{ m/s}$$

۳-۳۴۴.

$$v_{1x} = -30 \cos 45° = -21.21 \text{ m/s}$$

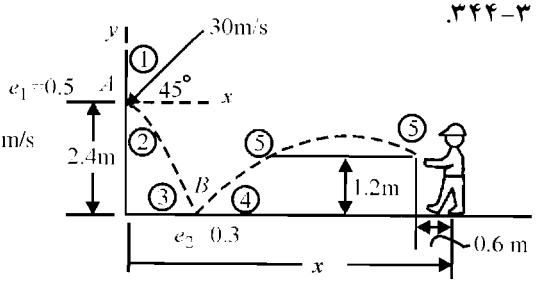
$$v_{1y} = -30 \sin 45° = -21.21 \text{ m/s}$$

$$v_{2x} = -e_1 v_{1x} = -0.5 (-21.21) = 10.6 \text{ m/s}$$

$$v_{2y} = v_{1y} = -21.21 \text{ m/s}$$

$$v_{3x} = v_{2x} = 10.6 \text{ m/s}$$

$$v_{3y} = -\sqrt{v_{2y}^2 + 2g(2.4)} = -\sqrt{21.21^2 + 2(9.81)(2.4)} = -22.29 \text{ m/s}$$



$$v_{3y} = v_{2y} - gt_3 \quad : \quad -22.29 = -21.21 - 9.81\,t_3 \Rightarrow t_3 = 0.1103 \text{ s}$$

$$v_{4x} = v_{3x} = 10.6 \text{ m/s}$$

$$v_{4y} = -e_2 v_{3y} = -0.3(-22.29) = 6.687 \text{ m/s}$$

$$y_5 = y_4 + v_{y4}\,t_5 - \frac{1}{2}gt_5^2$$

$$-1.2 = -2.4 + 6.687\,t_5 - \frac{9.81}{2}t_5^2 \Rightarrow t_5 = 1.15 \text{ s}, 0.213 \text{ s}$$

$$\left.\begin{aligned} x &= x_3 + v_{4x}\,t_5 + 0.6 \\ x_3 &= v_{2x}\,t_3 = 10.6(0.1103) = 1.17 \text{ m} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} x &= 1.17 + 10.6(1.15) + 0.6 = 13.96 \text{ m} \\ x &= 1.17 + 10.6(0.213) + 0.6 = 4.03 \text{ m} \end{aligned}\right.$$

۳-۳۴۵. برای یک مدار فرار حداقل (بیضی) $e = 1$ و $a \to \infty$ بنابراین طبق معادله ۳-۴۳:

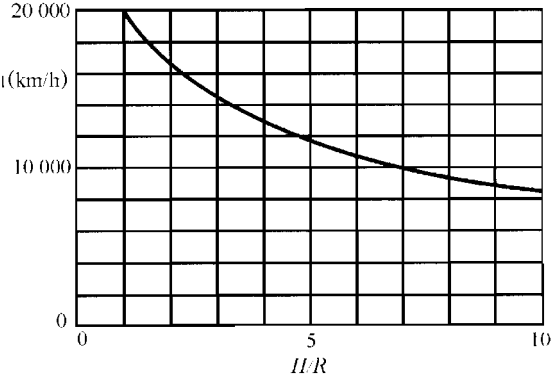
$$v_{esc} = R\sqrt{\frac{2g}{R+H}} = 6371\sqrt{\frac{2 \times 9.825 \times 10^{-3}}{6371 + 2000}} \times 3600 = 35140 \text{ km/h}$$

$$\Delta v = 35140 - 26140 = 9000 \text{ km/h}$$

۳-۳۴۶. برای مدار دایره‌ای با ارتفاع H، از معادله ۳-۴۴ خواهیم داشت:

$$v = R\sqrt{\frac{g}{R+H}} = R\sqrt{\frac{g/R}{1+H/R}} = 6371\sqrt{\frac{9.825/6371 \times 10^3}{1+H/R}} \times 3600 = \frac{28482}{\sqrt{1+H/R}} \text{ km/h}$$

H/R	$\sqrt{1+H/R}$	v_1(km/h)
1	1.4142	20140
2	1.7321	16444
3	2	14241
4	2.2361	12738
5	2.4495	11628
6	2.6458	10765
7	2.8284	10070
8	3	9494
9	3.1623	9007
10	3.3166	8588



۳-۳۴۷. نتایج مسئله ۳-۲۹۹ : $\Delta v_A = R\sqrt{\frac{g}{r_1}}\left(\frac{2r_2}{r_1 + r_2} - 1\right)$

حالت اسمی $(\Delta v_A)_n = 6371(10^3)\sqrt{\frac{9.825}{(6371 + 275)10^3}}\left(\sqrt{\frac{2(6371 + 35900)}{(6371+275) + (6371+35900)}} - 1\right)$

$= 2437.3$ m/s

حالت عملی $(\Delta v_A)_a = 6371(10^3)\sqrt{\frac{9.825}{(6371 + 275)10^3}}\left(\sqrt{\frac{2(6371 + 1125)}{(6371+275) + (6371+1125)}} - 1\right)$

$= 229.4$ m/s

$\frac{(\Delta v_A)_a}{(\Delta v_A)_n} = \frac{t'}{t} \Rightarrow t' = \frac{(\Delta v_A)_a}{(\Delta v_A)_n} \cdot t = \frac{229.4}{2437.3}(90) = 8.47$ s

۳-۳۴۸.

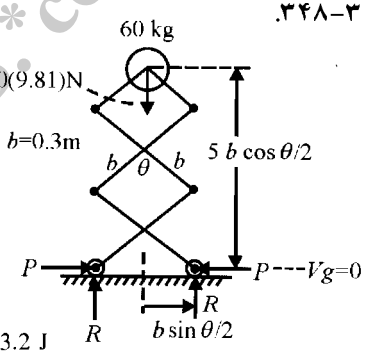
$U'_{1-2} = \Delta V_g + \Delta T$

$\theta_1 = 120°$, $v_1 = 0$

$\theta_2 = 60°$, $v_2 = 3$ m/s

$U'_{1-2} = 2Pb\left(\sin\frac{\theta_1}{2} - \sin\frac{\theta_2}{2}\right)$

$= 0.6\,P\,(\sin 60° - \sin 30°) = 0.2196$ J



$\Delta V_g = mg\,\Delta h = mg(5b)\left(\cos\frac{\theta_2}{2} - \cos\frac{\theta_1}{2}\right)$

$= 60(9.81)(1.5)(\cos 30° - \cos 60°) = 323.2$ J

$\Delta T = \frac{1}{2}60\,(3)^2 - 0 = 270$ J

$0.2196P = 323.2 + 270 \Rightarrow P = 2701$ N یا 2.70 kN

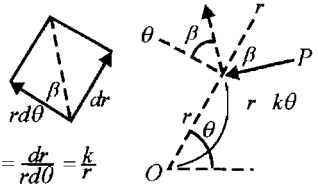
برای کل سیستم : $\Sigma F = ma$

$2R - 60(9.81) = 60(20) \Rightarrow R = 894$ N

۳-۳۴۹. ◀

معادله ۳-۲۷ $\Sigma M_O = \dot{H}_O$

$H_O = mv\cos\beta \times r = mv\frac{r^2}{\sqrt{r^2 + k^2}}$



$\tan\beta = \frac{dr}{rd\theta} = \frac{k}{r}$

$\dot{H}_O = mv\frac{\sqrt{r^2 + k^2}\,2r\dot{r} - r^2\frac{\dot{r}r}{\sqrt{r^2 + k^2}}}{r^2 + k^2} = mv\frac{(r^2 + k^2)2r\dot{r} - r^3\dot{r}}{(r^2 + k^2)^{3/2}} = \frac{r^2 + 2k^2}{(r^2 + k^2)^{3/2}}\dot{r}r\,mv$

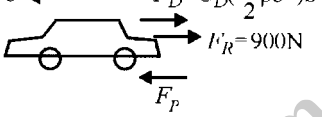
از طرفی $v \sin\beta = \dot{r} \Rightarrow \dot{r} = v\dfrac{k}{\sqrt{r^2+k^2}} \Rightarrow \dot{H}_O = \dfrac{r^2+2k^2}{(r^2+k^2)^2} rv^2 km$

$M_O = Pr\sin\beta = Pr\dfrac{k}{\sqrt{r^2+k^2}}$

$\Rightarrow Pr\dfrac{k}{\sqrt{r^2+k^2}} = \dfrac{r^2+2k^2}{(r^2+k^2)^2} rv^2 km \Rightarrow P = \dfrac{r^2+2k^2}{(r^2+k^2)^{3/2}} mv^2$

۳-۳۵۰ ◀

$\overset{+}{\leftarrow} \Sigma F = 0 \;:\; F_P = F_D + F_R$



آسیب ندیده : $F_P = 0.3\left[\dfrac{1}{2}(1.2062)\left(\dfrac{320}{3.6}\right)^2\right](2.8) + 900 = 4903\ \text{N}$

توان لازم : $P = F.v = 4903\left(\dfrac{320}{3.6}\right) = 435.8(10^3)\ \text{N.m/s}$

(توان موجود تغییر نمی‌کند) آسیب دیده :

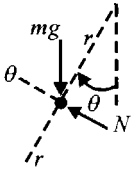
$P = F'.v' \Rightarrow 435.8(10^3) = \left[0.4\left(\dfrac{1}{2}\,1.2062\,v'\right)^2 2.8 + 900\right] v'$

حل معادله درجه ۳ : $v' = 81.4\ \text{m/s}$ یا $293\ \text{km/h}$

۳-۳۵۱ ◀

$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2)$

$mg\cos\theta = m(\ddot{r} - r\dot{\theta}^2) \Rightarrow \ddot{r} - r\dot{\theta}^2 = g\cos\theta \quad (\theta = \omega_0 t)$



حل مکمل : فرض $r_C = A\sinh\omega_0 t + B\cosh\omega_0 t \quad \ddot{r} - r\dot{\theta}^2 = 0$

حل جزیی : فرض $r_P = C\cos\omega_0 t \Rightarrow C = -\dfrac{g}{2\omega_0{}^2}$

شرایط مرزی : $r = 0$ وقتی $t = 0 \Rightarrow \dfrac{g}{2\omega_0}$

$\dot{r} = 0$ وقتی $t = 0 \Rightarrow A = 0$

$\Rightarrow r = \dfrac{g}{2\omega_0{}^2}(\cosh\theta - \cos\theta)$

۳-۳۵۲ ◀

$F_R = -k_1 v \;,\; k_1 = \dfrac{225}{90}\dfrac{\text{N-hr}}{\text{km}} = 9\dfrac{\text{N.s}}{\text{m}}$

$F_D = -k_2 v^2 \;,\; k_2 = \dfrac{225}{90^2}\dfrac{\text{N-hr}^2}{\text{km}^2} = 0.36\dfrac{\text{N.s}}{\text{m}^2}$

(الف) $P_{45} = Fv = \left[9\left(\dfrac{45}{3.6}\right) + 0.36\left(\dfrac{45}{3.6}\right)^2\right]\left(\dfrac{45}{3.6}\right) = 2109\ \text{W}$ یا $2.11\ \text{kW}$

$$\Rightarrow P_{90} = Fv = \left[9\left(\frac{90}{3.6}\right) + 0.36\left(\frac{90}{3.6}\right)^2\right]\left(\frac{90}{3.6}\right) = 11250 \text{ W} \text{ یا } 11.25 \text{ kW}$$

$$(ب) \quad -k_1 v - k_2 v^2 = m\frac{dv}{dt} \Rightarrow \int_0^t dt = -m\int_{v_1}^{v_2} \frac{dv}{v(k_1 + k_2 v)}$$

$$\Rightarrow t = -\frac{m}{k_1} \ln\left[\frac{v_2(k_1 + k_2 v_1)}{v_1(k_1 + k_2 v_2)}\right]$$

$$\Rightarrow t = -\frac{1000}{9} \ln\left[\frac{5\left(9 + 0.36\frac{90}{3.6}\right)}{90\left(9 + 0.36\frac{5}{3.6}\right)}\right] = 250 \text{ s}$$

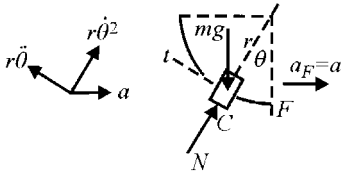
$$-k_1 v - k_2 v^2 = mv\frac{dv}{ds} \Rightarrow \int_0^s ds = -m\int_{v_1}^{v_2} \frac{dv}{k_1 + k_2 v}$$

$$\Rightarrow s = -\frac{m}{k_2} \ln(k_1 + k_2 v)\Big|_{v_1}^{v_2} = -\frac{m}{k_2} \ln\left[\frac{k_1 + k_2 v_2}{k_1 + k_2 v_1}\right] = 1775 \text{ m}$$

* ۳۵۳-۳

$$\mathbf{a}_C = \mathbf{a}_F + \mathbf{a}_{C/F}$$

$$\mathbf{a}_{C/F} = r\ddot{\theta}\mathbf{e}_t + r\dot{\theta}^2\mathbf{e}_n$$



$$\Sigma F_t = ma_t : -mg \sin\theta = m(r\ddot{\theta} - a\cos\theta)$$

$$r\ddot{\theta} = a\cos\theta - g\sin\theta \qquad (1)$$

$$\Sigma F_n = ma_n : N - mg\cos\theta = m(r\dot{\theta} + a\sin\theta)$$

$$N = m(r\dot{\theta}^2 + a\sin\theta + g\cos\theta) \qquad (2)$$

رابطه (1) را در $2\dot{\theta}$ ضرب می‌کنیم :

$$r\frac{d}{dt}(\dot{\theta}^2) = 2\frac{d\theta}{dt}(a\cos\theta - g\sin\theta) \Rightarrow rd(\dot{\theta}^2) = 2(a\cos\theta - g\sin\theta)\,d\theta$$

$$\Rightarrow r\int_0^{\dot{\theta}^2} d(\dot{\theta}^2) = 2\int_0^{\theta}(a\cos\theta - g\sin\theta)d\theta \Rightarrow r\dot{\theta}^2 = 2(a\sin\theta + g\cos\theta)_0^{\theta}$$

$$\Rightarrow \dot{\theta} = \sqrt{\frac{2}{r}(a\sin\theta + g\cos\theta - g)}$$

جای‌گذاری در رابطه (2) : $N = m(2a\sin\theta + 2g\cos\theta - 2g + a\sin\theta + g\cos\theta)$

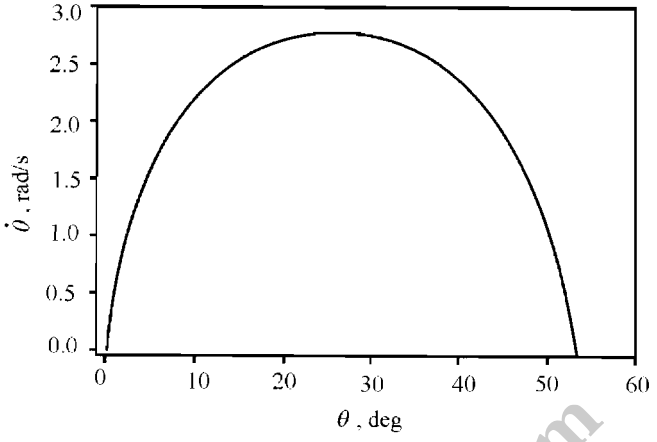
$$= m(3a\sin\theta + 3g\cos\theta - 2g)$$

ماکزیمم θ وقتی رخ می‌دهد که $\dot{\theta} = 0$ باشد لذا

$$a\sin\theta + g\cos\theta - g = 0$$

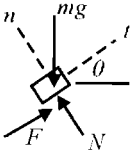
$$\Rightarrow \theta_{max} = 53.1° \quad \text{وقتی} \quad a = \frac{g}{2}$$

$$a = \frac{g}{2} = 4.905 \text{ m/s}^2 \;,\; r = 0.3 \text{ m} \Rightarrow \dot{\theta} = 8.09\sqrt{0.5\sin\theta + \cos\theta - 1} \text{ rad/s}$$



۳۵۴-۳ *

$a_n = r\Omega^2 = 0.325(7.5)^2 = 18.28 \text{ m/s}^2$

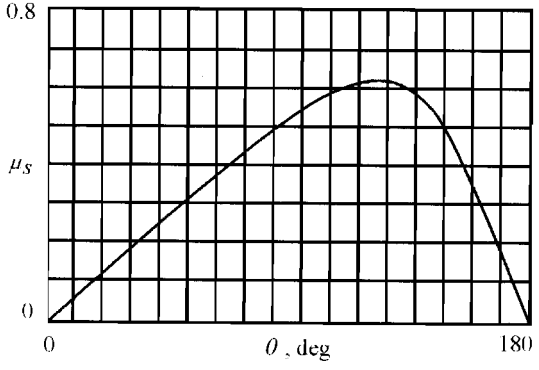


$$\begin{cases} \sum F_n = ma_n : N - mg\cos\theta = 18.28 \text{ m} \\ \sum F_t = ma_t : F - mg\sin\theta = 0 \end{cases}$$

در آستانه لغزش $F = F_{\max} = \mu_s N$

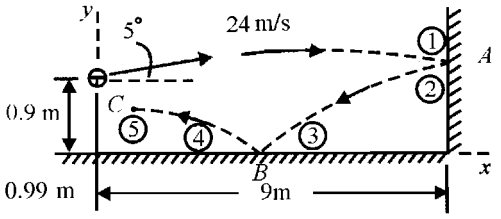
حل همزمان: $\mu_s = \dfrac{9.81\sin\theta}{18.28 + 9.81\cos\theta}$ ⇒ نمودار μ_s به ازای θ را ببینید

$\dfrac{d\mu_s}{d\theta} = 0 \Rightarrow \mu_{\min} = 0.622 \text{ @ } \theta = 121.9°$



(توجه کنید که مشاهدهٔ اینکه لغزش اول در زوایای بزرگتر از $\theta = 121.9°$ رخ دهد غیرممکن است!)

۳-۳۵۵ *



$$v_{0x} = v_0 \cos\theta = 24 \cos 5° = 23.91 \text{ m/s}$$

$$v_{0y} = v_0 \sin\theta = 24 \sin 5° = 2.09 \text{ m/s}$$

$$t_{01} = \frac{9}{23.91} = 0.376 \text{ s}$$

$$y_1 = 0.9 + 2.09(0.376) - \frac{9.81}{2}(0.376)^2 = 0.99 \text{ m}$$

$$v_{1x} = v_{0x} = 23.91 \text{ m/s}$$

$$v_{1y} = v_{0y} - gt_{01} = 2.09 - 9.81(0.376) = -1.6 \text{ m/s}$$

حالا معادلات زیر را برنامه‌نویسی کنید:

$$v_{2x} = -e\, v_{1x} \qquad (1)$$

$$v_{2y} = v_{1y} \qquad (2)$$

$$v_{3x} = v_{2x} \qquad (3)$$

$$v_{3y} = -\sqrt{v_{2y}^2 + 2gy_1} \qquad (4)$$

$$t_{23} = \frac{(v_{2y} - v_{3y})}{g} \qquad (5)$$

$$x_3 = 9 + v_{2x} t_{23} \qquad (6)$$

$$v_{4x} = v_{3x} \qquad (7)$$

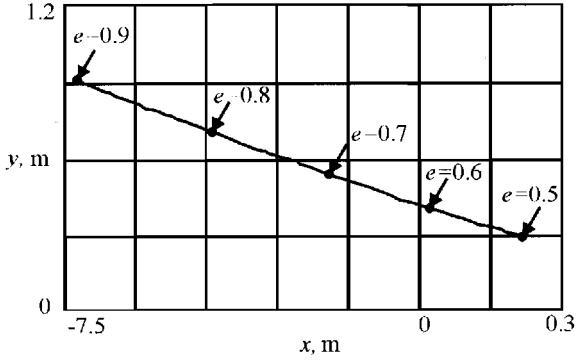
$$v_{4y} = -e\, v_{3y} \qquad (8)$$

$$t_{45} = \frac{v_{4y}}{g} \qquad (9)$$

$$x_5 = x_3 + v_{4x} t_{45} = x \qquad (10)$$

$$y_5 = v_{4y} t_{45} - \frac{1}{2} g t_{45}^2 = y \qquad (11)$$

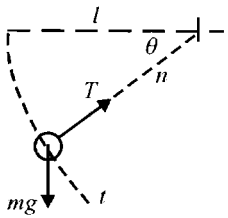
معادلات (1) تا (11) را برای $0.9 \le e \le 0.5$ حل کرده و نمودار زیر را رسم کنید.



$$x = 0 \Rightarrow e = 0.617 \ , \ y = 0.727 \text{ m}$$

۳۵۶-۳ *

$$\sum F_t = ma_t \Rightarrow mg\cos\theta = ml\ddot{\theta} \Rightarrow \ddot{\theta} = \frac{g}{l}\cos\theta$$



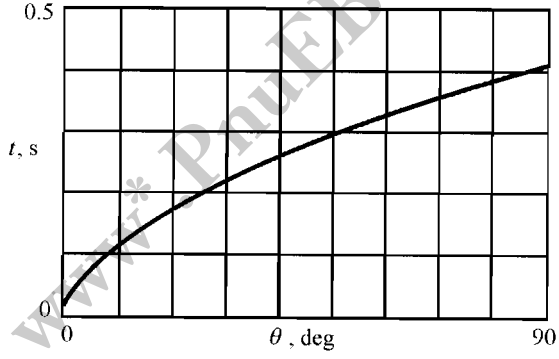
$$vdv = a_t ds \;:\; vdv = l\ddot{\theta}(ld\theta) = gl\cos\theta\, d\theta$$

$$\int_{v_0=0.1}^{v} vdv = \int_{\theta_0=0}^{\theta} gl\cos\theta\, d\theta \Rightarrow v^2 = 2gl\sin\theta + v_0^2$$

$$v = \frac{ds}{dt} = \frac{ld\theta}{dt} \;:\; \sqrt{2gl\sin\theta + v_0^2} = l\frac{d\theta}{dt} \Rightarrow \int_{t_0=0}^{t} dt = \int_{\theta_0=0}^{\theta} l\frac{d\theta}{\sqrt{2gl\sin\theta + v_0^2}}$$

$$\Rightarrow t = 0.5\int_0^{\theta} \frac{d\theta}{\sqrt{9.81\sin\theta + 0.01}}$$

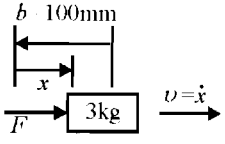
با استفاده از پیوست ج-۱۲ انتگرال‌گیری عددی انجام دهید برای $\left(0 \le \theta \le \frac{\theta}{2}\right)$



وقتی $\theta = 90°$ ⇐ $t = 0.409$ s

۳۵۷-۳ *

توان $P = Fv$



$$F = k(b - x) = 1.8(10^3)(0.1 - x)\ \text{N}$$

$$U = \Delta T \;:\; U = \int_0^{\theta} Fdx = \int_0^{\theta} 1.8(10^3)(0.1 - x)dx = 1.8(10^3)\left(0.1\,x - \frac{x^2}{2}\right)\text{J}$$

$$\Delta T = \frac{1}{2}mv^2 - 0 = \frac{1}{2}3\,v^2\ \text{J} \Rightarrow v^2 = \frac{2}{3}1.8(10^3)\left(0.1\,x - \frac{x^2}{2}\right)$$

$$P^2 = F^2v^2 \Rightarrow P^2 = (1.8)^2(10^6)(0.1 - x)^2 \times \frac{2}{3}\,1.8\,(10^3)\left(0.1x - \frac{x^2}{2}\right)$$

$$\Rightarrow P^2 = 3.89(10^9)(0.1 - x)^2\left(0.1x - \frac{x^2}{2}\right)$$

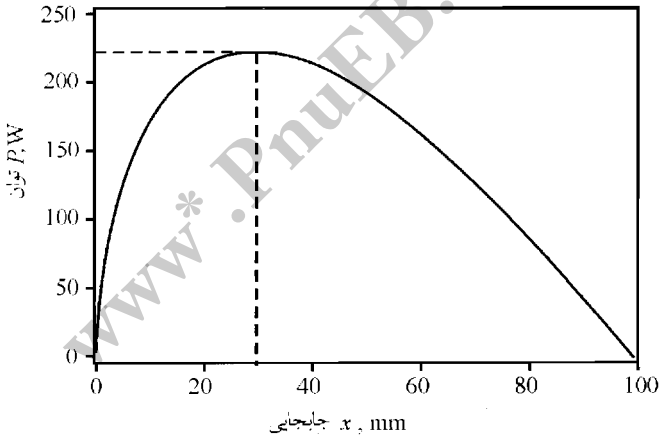
$$\frac{d(P^2)}{dx} = 3.89(10^9)\left\{2(0.1 - x)(-1)\left(0.1\,x - \frac{x^2}{2}\right) + (0.1 - x)^2(0.1 - x)\right\}$$

$$= 3.89(10^9)(-0.2\,x + x^2 + 0.01 - 0.2x + x^2)(0.1 - x)$$

$$= 3.89(10^9)(0.1 - x)(x^2 - 0.2x + 0.005) \times 2 = 0 \quad (\text{برای max یا min})$$

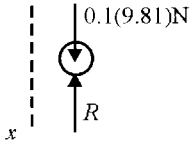
$$\Rightarrow x = 0.1 \text{ یا } x = 0.1 \pm 0.0707 = 0.1707 \text{ m یا } x = 0.0293 \text{ m}$$

$$P = \sqrt{38.9(10^8)}\,(0.1 - x)\sqrt{0.1x - 0.5x^3}\ W \overset{x=0.0293}{\Rightarrow} P_{max} = 220\ W$$



۳۵۸-۳ *

$$\Sigma F_x = ma_x$$



$$0.1(9.81) - R = 0.1\,a$$

$$(0.981 - R)\,dx = 0.1\,a\,dx$$

$$adx = vdv \Rightarrow (0.981 - R)dx = 0.1\,vdv = 0.05\,d(v^2)$$

برای بازه‌های کوچک : $(0.981 - R)\Delta x = 0.05\,\Delta(v^2) \Rightarrow \Delta(v^2) = (19.62 - 20R)\,\Delta x$

برنامه‌ای بنویسید که جدول زیر را تشکیل دهد:

x (m)	Δx (m)	R (N)	$19.62-20R$ m/s^2	Δv^2 (m/s)^2	v^2 (m/s)^2	v (m/s)
0		0	19.62		0	0
	1			19.62		
1		0.05	18.62		4.0	2.0
	1			18.62		
2		0.08	18.02		9.0	3.0
	1			18.02		
3		0.18	16.02		16.0	4.0
	1			16.02		
4		0.3	13.62		25.0	5.0
	1			13.62		
5		0.44	10.82		36.0	6.0
	1			10.82		
6		0.6	7.62		44.9	6.7
	1			7.6		
7		0.7	5.62		49.0	7.0
	1			5.6		
8		0.8	3.62		56.2	7.5
	1			3.6		
9		0.981	0		63	7.94
	1			0		
10					63	7.94

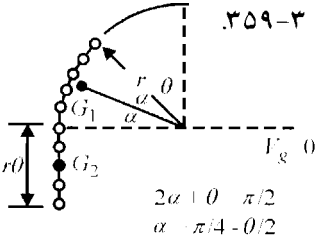
بنابراین $v = 7.94$ m/s

$$R = kv^2 \Rightarrow mg - kv^2 = ma$$

$$\int_0^x \frac{dx}{m} = \int_0^v \frac{vdv}{mg - kv^2} \Rightarrow v = \sqrt{\frac{mg}{k}\left(1 - e^{\frac{-2kx}{m}}\right)}$$

عددگذاری: $v = 8.28$ m/s

* ۳-۳۵۹.



برای $\theta \neq 0$:

$$V_g = \rho g\left(\frac{\pi r}{2} - r\theta\right)\bar{r}\sin\alpha - \rho g\, r\theta\,\frac{r\theta}{2}$$

$$\bar{r} = \frac{r\sin\alpha}{\alpha} = r\,\frac{\sin\left(\frac{\pi}{4} - \frac{\pi}{2}\right)}{\frac{\pi}{4} - \frac{\theta}{2}}$$

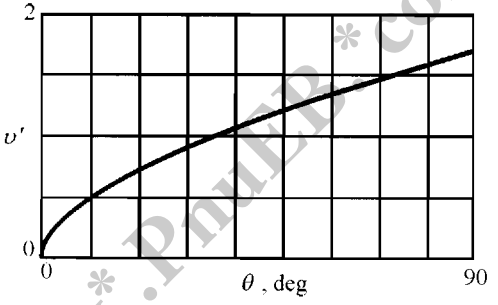
$$V_g = \rho g r^2 \left\{ \left(\frac{\pi}{2} - \theta\right) \frac{\sin^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}{\frac{\pi}{4} - \frac{\theta}{2}} - \frac{\theta^2}{2} \right\}$$

برای $\theta = 0$:

$$V_g = \rho g \frac{\pi r}{2} \frac{2r}{\pi} = \rho g r^2$$

$$\Rightarrow \Delta V_g = (V_g)_\theta - (V_g)_{\theta=0} = 3\rho g r^2 \left\{ 1 + \frac{\theta^2}{2} - \left(\frac{\pi}{2} - \theta\right) \sin^2 \frac{\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}{\frac{\pi}{4} - \frac{\theta}{2}} \right\}$$

$$\left.\begin{array}{l} U'_{1\text{-}2} = 0 = \Delta T + \Delta V_g \\ \Delta T = \frac{1}{2} \rho \frac{\pi r}{2} v^2 \end{array}\right\} \Rightarrow v' = \frac{v}{\sqrt{gr}} = 2\sqrt{\frac{1}{\pi}(2 + \theta^2) - \left(1 - \frac{2\theta}{\pi}\right) \frac{\sin^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}{\frac{\pi}{4} - \frac{\theta}{2}}}$$



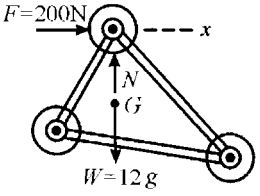
فصل چهارم

سینتیک سیستم ذرات

۴-۱.

$$\Sigma F_x = m\,\mathbf{a}_x \;:\; 200 = 12\,\bar{a}_x$$

$$\bar{a}_x = \bar{a} = 16.67\ \text{m/s}^2$$



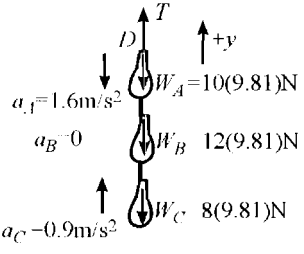
۴-۲.

برای کل مجموعه : $\Sigma F_y = \Sigma m_i a_i$

$$T - 9.81(10 + 12 + 8) = 10(-1.6) + 12(0) + 8(0.9)$$

$$T - 294 = -8.8$$

$$T = 286\ \text{N}$$



۴-۳. مرکز جرم وسط تیر میانی است لذا معادلهٔ ۴-۱ برای کل سیستم چنین خواهد بود.

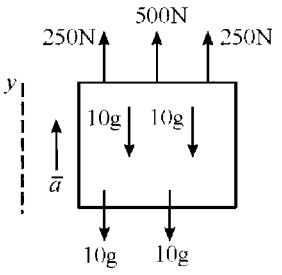
$$\Sigma F = m\,\bar{a} \;:\; 100 = 3(8)\,a \quad \Rightarrow \quad a = 4.17\ \text{m/s}^2$$

۴-۴.

$$\Sigma F_y = m\bar{a}_y$$

$$500 + 250 + 250 - 40(9.81) = 40\,\bar{a}$$

$$\bar{a} = 15.19\ \text{m/s}^2$$

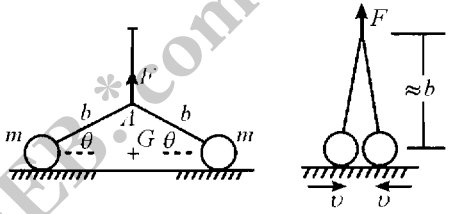


۵-۴. مادامیکه $F < 2mg$ مرکز جرم ثابت باقی می‌ماند.

برای سیستم : $U = \Delta T$

$$F(b - b\sin\theta) = 2\left(\frac{1}{2}m v^2\right)$$

$$v = \sqrt{\frac{Fb}{m}(1 - \sin\theta)}$$

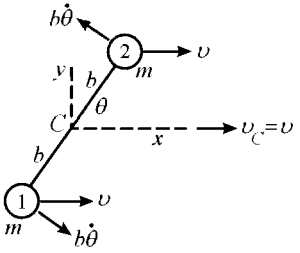


۶-۴.

گوی 1 : $\mathbf{G}_1 = m[(v + b\dot{\theta}\sin\theta)\,\mathbf{i} - (b\dot{\theta}\cos\theta)\,\mathbf{j}]$

گوی 2 : $\mathbf{G}_2 = m[(v - b\dot{\theta}\sin\theta)\,\mathbf{i} + (b\dot{\theta}\cos\theta)\,\mathbf{j}]$

$$\mathbf{G} = \mathbf{G}_1 + \mathbf{G}_2 = m[v + v]\,\mathbf{i} = 2mv\,\mathbf{i}$$



۷-۴.

برای سیستم $\Sigma M_O = \dot{H}_O$ ⇒ پایستار است H_O

$$0.060(0.25)(300) = 0.06(0.25)^2\omega + 1.5(0.5)^2\omega$$

$$\omega = 11.88\ \text{rad/s}$$

نیروی افقی بزرگی روی میله توسط تکیه‌گاه اعمال می‌شود به طوری که در جهت افقی $\Sigma F \neq 0$ است. بنابراین $\dot{G}_x \neq 0$ و مومنتوم خطی سیستم پایدار نیست.

۸-۴.

$\Sigma M_O = \dot{H}_O$ (محور دوران $O - O$)

$$M = \frac{dH_O}{dt} \Rightarrow \int_0^t M\,dt = \int_0^{H_O} dH_O = H_O \Rightarrow Mt = 4m(r\omega)r \Rightarrow t = \frac{4mr^2\omega}{M}$$

۹-۴. برای سیستم دو گوی : $U' = 0 = \Delta V_g + \Delta T$

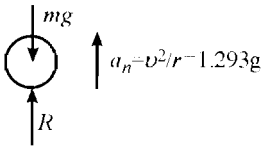
$$0 = -mgr - mgr\left(1 - \frac{1}{\sqrt{2}}\right) + \frac{1}{2}2mv^2 \Rightarrow v^2 = gr\left(2 - \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow v = 1.137\sqrt{gr}$$

گوی 1 درست قبل از رسیدن به A : $\Sigma F_y = ma_t$, $a_y = a_n = 1.293\,g$

$$R - mg = m(1.293\,g)$$

$$R = 2.29\,mg$$



۱۰-۴. $\int_0^{t'} M_z\,dt = H_{z_2} - H_{z_1}$, $H_z = \Sigma m_i r_i (r_i\dot{\theta})$

$$H_z = 2(3)(0.3)^2\dot{\theta} + 2(3)(0.5)^2\dot{\theta} = 2.04\dot{\theta}$$

$$30t = 2.04(20 - [-20]) = 81.6 \Rightarrow t = 2.72\,s$$

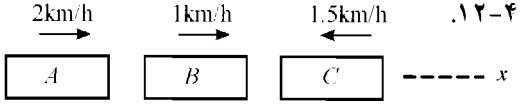
۱۱-۴. برای کل سیستم $\Delta G_x = 0$ (x افقی)

$$(300 + 400 + 100)v - (300\times 0.6 - 400\times 0.3 + 100\times 1.2\cos 30°) = 0$$

$$800v = 163.9 \Rightarrow v = 0.205\,m/s$$

صرفنظر از ترتیب اتفاقات، مومنتوم پایستار است لذا سرعت نهایی تغییر نخواهد کرد.

۱۲-۴. $\Sigma F_x = 0$ برای سیستم $\Rightarrow \Delta G_x = 0$



$$(65\times 2 + 50\times 1 - 75\times 1.5)$$

$$-(65 + 50 + 75)v = 0 \Rightarrow v = 0.355\,km/h$$

درصد اتلاف انرژی $= \dfrac{T_i - T_f}{T_i}100 = 100\left(1 - \dfrac{T_f}{T_i}\right) = n$

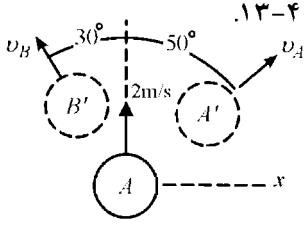
$$n = 100\left\{1 - \frac{\frac{1}{2}(65 + 50 + 75)(0.355)^2}{\frac{1}{2}(65\times 2^2 + 50\times 1^2 + 75\times 1.5^2)}\right\} = 95\%$$

۱۳-۴. برای کل سیستم $\Sigma F_x = \Sigma F_y = 0$

$$\Rightarrow \Delta G_x = 0 \;:\; -mv_B\sin 30° + mv_B\sin 50° - 0 = 0$$

$$\Delta G_y = 0 \;:\; mv_B\cos 30° + mv_B\cos 50° - m(2) = 0$$

حل دو معادله : $v_A = 1.015\,m/s$, $v_B = 1.556\,m/s$



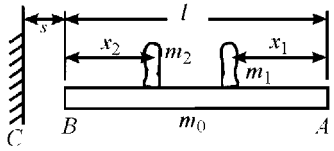
۴-۱۴. ثابت $= \Sigma m_i x_i$ نسبت به C

$$m_1 l + m_2(0) + m_0 \frac{l}{2} = m_1(s + l - x_1) + m_2(s + x_2) + m_0\left(s + \frac{l}{2}\right)$$

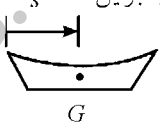
$$\Rightarrow \quad s = \frac{m_1 x_1 - m_2 x_2}{m_0 + m_1 + m_2}$$

وقتی بهم می‌رسند که : $x_2 + x_1 = l$

$$\Rightarrow \quad s = \frac{(m_1 + m_2) x_1 - m_2 l}{m_0 + m_1 + m_2}$$



۴-۱۵. با صرفنظر کردن از نیروهای هیدرولیک، مومنتوم خطی پایستار بوده و سرعت $v_2 = v_1 = 1$ knot مرکز جرم نسبت به محورهای مرجع که با سرعت ثابت 1 knot حرکت می‌کند تغییر مکان نمی‌دهد. بنابراین:



$$(\Sigma m_i x_i)_1 = (\Sigma m_i x_i)_2$$

$$60(0.6) + 80(2.4) + 84(4.8) + 150(2.4) =$$

$$= 60(4.2 + x) + 90(1.2 + x) + 80(30 + x) + 150(2.4 + x)$$

$$996 = 960 + 380x \quad \Rightarrow \quad x = 0.0947 \text{ m}$$

زمان یا ترتیب جابجا شدن افراد روی نتیجه نهایی اثر ندارد زیرا تمام نیروها داخلی هستند.

۴-۱۶.

$$\mathbf{H}_O = \mathbf{H}_G + \bar{\rho} \times 2m\bar{\mathbf{v}}$$

$$\mathbf{H}_G = \Sigma \rho_i \times m_i \dot{\rho}_i = 2r \times m \times r\omega \mathbf{k} = 2mr^2\omega \mathbf{k}$$

$$\bar{\rho} \times 2m\bar{\mathbf{v}} = (x\mathbf{i} + y\mathbf{j}) \times 2mv\mathbf{i} = -2mvy\mathbf{k}$$

$$\mathbf{H}_O = 2mr^2\omega \mathbf{k} - 2mvy\mathbf{k} \quad \Rightarrow \quad \mathbf{H}_O = 2m(r^2\omega - vy)\mathbf{k}$$

۴-۱۷.

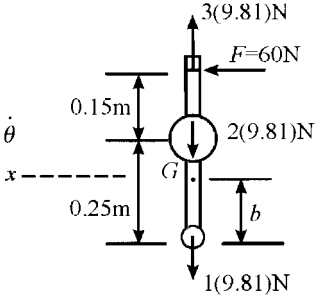
$$\Sigma F_x = m\bar{a}_x : \quad 60 = (2 + 1)\bar{a} \quad \Rightarrow \quad a = 20 \text{ m/s}$$

$$2(0.25 - b) = 1b \quad \Rightarrow \quad b = 0.167 \text{ m}$$

$$H_G = \Sigma m r^2 \dot{\theta} = \left[2(0.083)^2 + 1(0.167)^2\right]\dot{\theta} = 0.0417\dot{\theta}$$

$$\Sigma M_G = \dot{H}_G \quad \Rightarrow \quad 60(0.15 + 0.083) = 0.0417\ddot{\theta}$$

$$\Rightarrow \quad \ddot{\theta} = 336 \text{ rad/s}^2$$



۴-۱۸.

$$\int \Sigma F_x\,dt = 0 \quad \Rightarrow \quad \Delta G_x = 0$$

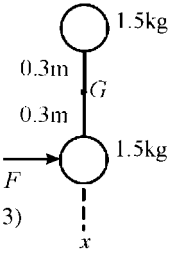
$$\int \Sigma F_y\,dt = \Delta G_y \;:\; 10 = 2(1.5)\,v_y$$

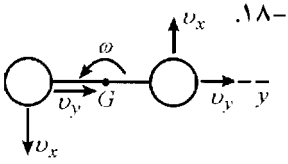
$$\Rightarrow \quad v_y = 3.33\ \text{m/s}$$

$$\int \Sigma M_G\,dt = \Delta H_G \;:\; 10(0.3) = 2(1.5)\,v_x(0.3)$$

$$\Rightarrow \quad v_x = 3.33\ \text{m/s}$$

$$v = 3.33\sqrt{2} = 4.71\ \text{m/s}$$ هر دو کره





۴-۱۹.

$$U'_{1\text{-}2} = \Delta T + \Delta V_g$$

$$= 3\left(\frac{1}{2}\times 2.75\times 1.560^2\right) - 0 - 2.75\times 9.81\,(0.360 + 0.1054)$$

$$= 10.04 - 12.56 = -2.52\,\text{J}$$

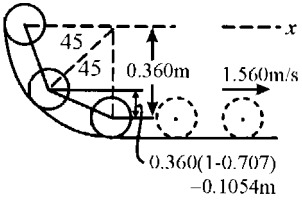
اتلاف انرژی $\Delta Q = 2.52\,\text{J}$

$$I_x = \int \Sigma F_x\,dt = \Delta G_x = G_2 - G_1$$

$$G_2 = 3\,m\,v = 3(2.75)(1.560)$$

$$G_1 = 0$$

$$\Rightarrow \quad I_x = 12.87\ \text{N.s}$$



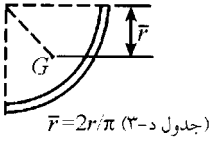
۴-۲۰.

$$\Delta T = \Delta V_e = 0 \quad \Rightarrow \quad U' = \Delta V_g = -\Delta Q$$

$$\Delta V_g = -m\,g\,\bar{r}$$

$$|\Delta V_g| = \frac{\pi r\rho}{2}\,g\,\frac{2r}{\pi} = \rho\,g\,r^2 = \Delta Q$$

انرژی تلف شده به صورت حرارت و صدا در اثر برخورد طناب با سطح در می‌آید.

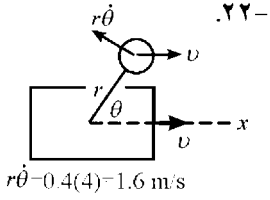


۴-۲۱.

(الف) $$\Sigma F_x = m\,\bar{a}_x \;:\; F = 2m\,\bar{a} \quad \Rightarrow \quad \bar{a} = \frac{F}{2m}$$

(ب) $$H_G = 2m\left(\frac{L}{2}\right)^2\dot{\theta} \quad \Rightarrow \quad \dot{H}_G = mL^2\,\frac{\ddot{\theta}}{2}$$

$$\Sigma M_G = \dot{H}_G \quad \Rightarrow \quad Fb = \frac{mL^2\ddot{\theta}}{2} \quad \Rightarrow \quad \ddot{\theta} = \frac{2Fb}{mL^2}$$

۴-۲۲. 

برای سیستم $\Sigma F_x = 0 \Rightarrow \Delta G_x = 0$

$(G_x)_{\theta=0} = (20+5)(0.6) = 15.0$ N.s

$(G_x)_{\theta=60^\circ} = (20+5)v - 5(1.6)\sin 60^\circ = 25v - 6.93$ N.s

$\Rightarrow 15.0 = 25v - 6.93 \Rightarrow v = \dfrac{21.9}{25} = 0.88$ m/s

۴-۲۳. افت عمودی گوی‌ها : $h_1 = 0.5$ m , $h_2 = 0.375$ m , $h_3 = 0.25$ m

برای سیستم $\Delta V_g = -mg(h_1 + h_2 + h_3)$

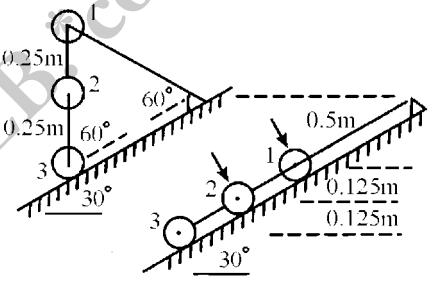
$$\Delta T = \frac{1}{2}m(v_1^2 + v_2^2 + 0) = \frac{1}{2}m\left(v^2 + \left[\frac{v}{2}\right]^2\right) = \frac{5}{8}mv^2$$

$U' = \Delta T + \Delta V$

$0 = \dfrac{5}{8}mv^2 - mg(h_1 + h_2 + h_3)$

$v^2 = \dfrac{8}{5}(9.81)(0.5 + 0.375 + 0.25) = 17.66$

$v = 4.2$ m/s

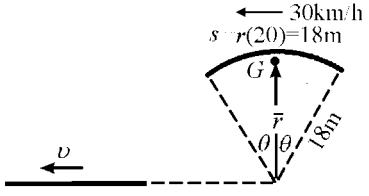


اتلاف انرژی پتانسیل تبدیل به افت انرژی برخورد می‌شود.

۴-۲۴. m = جرم کل واگنها

$\theta = \dfrac{s}{2r} = \dfrac{18}{2(18)} = \dfrac{1}{2}\text{rad} = \dfrac{1}{2}\dfrac{180}{\pi} = 28.65^\circ$

$\bar{r} = \dfrac{r\sin\theta}{\theta} \dfrac{18\sin 28.65^\circ}{\frac{1}{2}} = 17.26$ m



برای سیستم $\Delta T + \Delta V_g = 0$

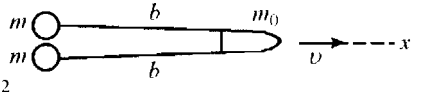
$\dfrac{1}{2}m\left[v^2 - \left[\dfrac{30}{3.6}\right]^2\right] - mg(17.26) = 0 \Rightarrow v^2 = 408 \Rightarrow v = 20.2$ m/s یا $v = 72.7$ km/h

۴-۲۵. برای سیستم $\Delta G_x = 0$: $(m_0 v + 2mv) - m_0 v_0 = 0$

$$v = \frac{m_0}{m_0 + 2m} v_0$$

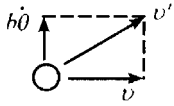
$$U = \Delta T \;:\; 0 = \frac{1}{2} m_0 v^2 + 2\left[\frac{1}{2} m (v^2 + b^2 \dot{\theta}^2)\right] - \frac{1}{2} m_0 v_0^2$$

$$(m_0 + 2m) v^2 + 2 m b^2 \dot{\theta}^2 = m_0 v_0^2$$



v جایگزینی : $\dfrac{m_0 v_0^2}{m_0 + 2m} + 2 m b^2 \dot{\theta}^2 = m_0 v_0^2$

$$\Rightarrow \dot{\theta} = \frac{v_0}{b}\sqrt{\frac{m_0}{m_0 + 2m}}$$



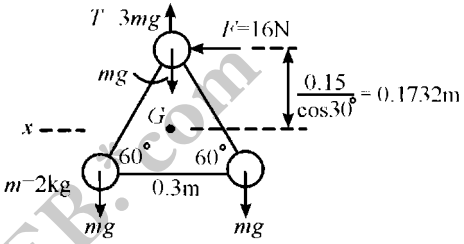
۲۶-۴. برای سیستم $\Sigma F_x = m \bar{a}_x$:

$$16 = 3(2)\,\bar{a} \Rightarrow \bar{a} = 2.67 \text{ m/s}$$

$$\Sigma M_G = \dot{H}_G \;:$$

$$16(0.1732) = \frac{d}{dt}(3 \times 2 \times 0.1732^2 \dot{\theta})$$

$$\Rightarrow \ddot{\theta} = \frac{16(0.1732)}{6(0.1732)^2} = 15.4 \text{ rad/s}^2$$



برای گوی بالایی $a = \bar{a} + r\ddot{\theta} = 2.67 + 0.1732(15.4) = 5.33 \text{ m/s}^2$

۲۷-۴.

$$\Delta H_z = 0 \;:\; \Sigma M v r = \Sigma m r^2 \omega = H_z$$

$$\Rightarrow 4 m \omega (0.8^2 + 0.4^2) = 4 m \omega' (0.8^2 + 0.65^2)$$

$$\frac{\omega'}{\omega} = \frac{N'}{N} = \frac{(0.8^2 + 0.4^2)}{(0.8^2 + 0.65^2)} = 0.753$$

$$N' = 0.753 \times 120 = 90.4 \text{ rpm}$$

$$T = \Sigma \frac{1}{2} m v^2 = \Sigma \frac{1}{2} m (r\omega)^2$$

$$|\Delta T| = 4 \times \frac{1}{2} \times 2\,[0.8^2 + 0.4^2]\left[\frac{120 \times 2\pi}{60}\right]^2 - 4 \times \frac{1}{2} \times 2\,[0.8^2 + 0.65^2]\left[\frac{90.4 \times 2\pi}{60}\right]^2$$

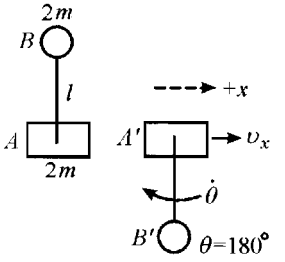
$$|\Delta T| = 505 - 380 = 125 \text{ J} \qquad \text{اتلاف}$$

ΔT ناشی از اتلاف انرژی برخورد بواسطه برخورد گویهای داخلی با خارجی

۲۸-۴. برای کل سیستم $\int \Sigma F_x dt = \Delta G_x = G_{A'} + G_{B'} - 0$

$$0 = 2 m v_x + 2 m (v_x - l\dot{\theta})$$

$$2 v_x = l\dot{\theta} \qquad (1)$$



برای کل سیستم $U' = \Delta T + \Delta V_g$

$$\Delta T = \frac{1}{2}(2\,\text{m})\,v_x^2 + \frac{1}{2}(2m)(v_x - l\dot{\theta})^2 - 0$$

$$= m(2v_x^2 - 2l\dot{\theta}v_x + l^2\dot{\theta}^2)$$

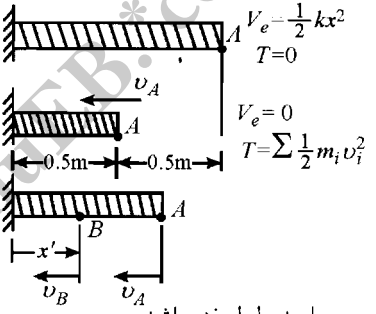
$$\Delta V_g = -2mg(2l) = -4mgl$$

$$U'_{1\text{-}2} = 0 \Rightarrow 0 = m(2v_x^2 - 2l\dot{\theta}v_x + l^2\dot{\theta}^2) - 4mgl \Rightarrow 2v_x^2 - 2l\dot{\theta}v_x + l^2\dot{\theta}^2 = 4gl \quad (2)$$

ترکیب (1) و (2) : $2v_x^2 - 4v_x^2 + 4v_x^2 = 4gl \Rightarrow v_x^2 = 2gl \Rightarrow v_x = \sqrt{2gl}$

$$\dot{\theta} = \frac{2v_x}{l} = 2\frac{\sqrt{2gl}}{l} = 2\sqrt{\frac{2g}{l}}$$

◀ ۲۹-۴.



انرژی پتانسیل فنر $V_e = \frac{1}{2}kx^2$

$$= \frac{1}{2}\left(\frac{900}{0.5}\right)(0.5)^2 = 225\,\text{J}$$

طی جمع شدن فنر $x \to 0$

$$v_B = \frac{x'}{0.5}v_A$$

$dm = \rho\,dx'$ فرض کنید ρ جرم واحد طول فنر باشد

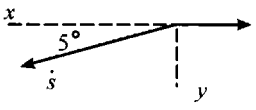
$$T = \int \frac{1}{2}v_B^2\,dm = \frac{1}{2}\int_0^{0.5}\frac{x'^2}{(0.5)^2}v_A^2\rho\,dx' = \frac{1}{6}\rho(0.5)v_A^2$$

$$\Rightarrow T = \frac{1}{6}mv_B^2 = \frac{1}{6}(1.5)v_A^2 = 0.25v_A^2 \qquad \text{که در آن } \rho(0.5) = m$$

$$V_e = T \Rightarrow 225 = 0.25v_A^2 \Rightarrow v_A = 30\,\text{m/s}$$

(انرژی جنبشی به حرارت، صدا و تغییر طول تبدیل می‌شود)

◀ ۳۰-۴.



سیستم پایدار است $\Rightarrow \Delta T + \Delta V_g = 0$

واگن : $\Delta T = \frac{1}{2}mv^2 - 0 = \frac{1}{2}25000v^2$

$\Delta V_g = 0$

وسیله نقلیه : $\Delta T = \frac{1}{2}mv^2 - 0 = \frac{1}{2}7500\left[(\dot{s}\cos 5° - v)^2 + (\dot{s}\sin 5°)^2\right] - 0$

$$\Delta V_g = -W\Delta h = -7500(9.81)(12\sin 5°) = -76950\,\text{J}$$

$$\Rightarrow \quad 12500\,v^2 + 3750\left[(\dot{s}\cos 5^\circ - v)^2 + (\dot{s}\sin 5^\circ)^2\right] - 76950 = 0 \qquad (1)$$

برای سیستم $\Sigma F_x = 0 \quad \Rightarrow \quad \Delta G_x = 0$

$$7500\,(\dot{s}\cos 5^\circ - v) - 25000\,v = 0 \quad \Rightarrow \quad \dot{s}\cos 5^\circ - v = 3.33\,v$$

$$\Rightarrow \quad \dot{s}\sin 5^\circ = 4.33\,v\tan 5^\circ = 0.379\,v$$

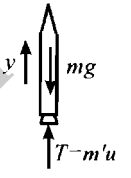
جای‌گذاری در (1) : $12500\,v^2 + 3750\left[(3.33\,v)^2 + (0.379\,v)^2\right] - 76950 = 0$

$$\Rightarrow \quad v^2 = 1.409 \quad \Rightarrow \quad v = 1.186\ \text{m/s}$$

۴-۳۱. $\Sigma F_y = m\,a_y$: $T = m'\,u = 13(10^3)(2400) = 31.2(10^6)\ \text{N}$

$mg = 2.7(10^6)(9.81) = 26.5(10^6)\ \text{N}$

$$\Rightarrow \quad 31.2(10^6) - 26.5(10^6) = 2.7(10^6)\,a \quad \Rightarrow \quad a = 1.746\ \text{m/s}^2$$



۴-۳۲. برای حداکثر سرعت، شتاب $a = 0$ بنابراین مقاومت $= 1000\ \text{N} = T$

$T = m'\,u$: $1000 = 1.6\,u \quad \Rightarrow \quad u = 625\ \text{m/s}$

۴-۳۳. $F = m'\,\Delta v$: $(30 - 20) = \dfrac{4.5}{60}(v - 0) \quad \Rightarrow \quad v = 133.3\ \text{m/s}$

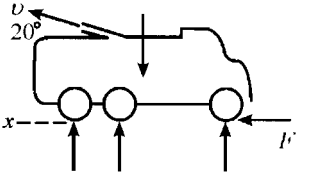
۴-۳۴. $F = m'\,\Delta v_x$, $\Delta v_x = v\cos 20^\circ$

$Q = A\,v$: $5.3 \times \dfrac{1}{60} = \dfrac{\pi\,(0.050)^2}{4}\,v \quad \Rightarrow \quad v = 45\ \text{m/s}$

$\Delta v_x = v\cos 20^\circ = 42.27\ \text{m/s}$

$m' = \rho\,Q = 1000\,\dfrac{5.3}{60} = 88.3\ \text{kg/s}$

$F = 88.3(42.27) = 3733\ \text{N}$



۴-۳۵. نیروی مقاومت آب برابر است با کل نیروی پیشران خالص T

$T = m'(u - v)$

سرعت شیپوره $u = \dfrac{Q}{A} = \dfrac{0.082}{\dfrac{\pi\,(0.050)^2}{4}} = 41.8\ \text{m/s}$

چگالی آب دریا طبق جدول د-۱ : $\rho = 1030\ \text{kg/m}^3$

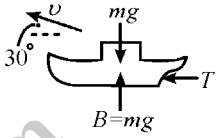
$$m' = \rho Q = 1030(0.082) = 84.5\,\text{kg/s}$$

$$v = \frac{70}{3.6} = 19.44\,\text{m/s}$$

$$R = T = 84.5(41.87 - 19.44) = 1885\,\text{N}$$

۳۶-۴.

$$v = 40\,\text{m/s}$$



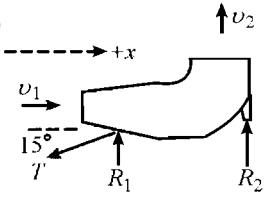
$$m' = \rho Q = 1030(0.080) = 82.4\,\text{kg/s}$$

$$\Sigma F = m' \Delta v$$

$$T = 82.4(40\cos 30° - 0) = 2850\,\text{N} \quad \text{یا} \quad T = 2.85\,\text{kN}$$

۳۷-۴.

$$\Sigma F_x = m' \Delta v_x$$

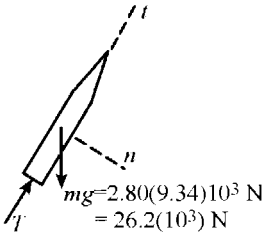


$$-T\cos 15° = (43 + 0.8)(0 - 720)$$

$$T = 32600\,\text{N} \quad \text{یا} \quad T = 32.6\,\text{kN}$$

۳۸-۴.

$$T = m' u = 120(640) = 76.8(10^3)\,\text{N}$$



$$\Sigma F_t = m a_t \;:\; 76.8(10^3) - 26.2(10^3)\cos 30° = 2.80(10^3) a_t$$

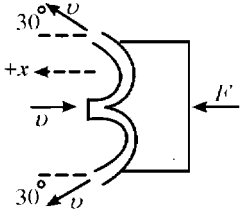
$$a_t = 19.34\,\text{m/s}^2$$

$$\Sigma F_n = m a_n \;:\; 26.2(10^3)\sin 30° = 2.80(10^3) a_n$$

$$a_n = 4.67\,\text{m/s}^2$$

۳۹-۴.

$$\Sigma F_x = m' \Delta v_x$$



$$m' = \rho A v = 1000\left(\frac{\pi(0.020)^2}{4}\right)40 = 12.57\,\text{kg/s}$$

$$\Delta v_x = v\cos 30° - (-v) = v(1 + \cos 30°)$$

$$= 40(1 + 0.866) = 74.64\,\text{m/s}$$

$$F = 12.57 \times 74.64 = 938\,\text{N}$$

۴-۴۰.

$$Q = Av \quad : \quad \frac{30}{60} = \frac{2(\pi \times 0.1^2)}{4} v_2 \Rightarrow v_2 = 31.8 \text{ m/s}$$

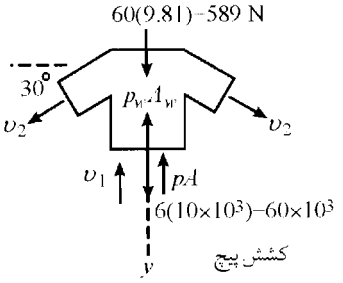
$$v_1 = 2v_2 \frac{A_2}{A_1} = 2(31.8)\left(\frac{0.1}{0.25}\right)^2 = 10.19 \text{ m/s}$$

$$m' = \rho Q = 1030 \times \frac{30}{60} = 515 \text{ kg/s}$$

$$\Sigma F_y = m' \Delta v_y$$

$$60 \times 10^3 + 589 - 0.0240p - 550(10^3)\frac{\pi \times 0.25^2}{4} = 515(31.8 \sin 30° - [-10.19])$$

$$\Rightarrow p = 840(10^3) \text{ Pa} \quad \text{یا} \quad p = 840 \text{ kPa}$$



۴-۴۱.

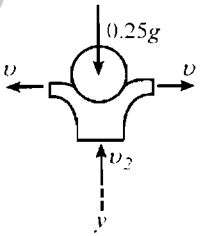
$\Sigma F_y = m' \Delta v_y$ توپ و جریان درست زیر آن

$m' = \rho A v$ در شیپوره m' در توپ

$$= 1000 \frac{n(0.012)^2}{4}(10) = 1.131 \text{ kg/s}$$

$$0.25(9.81) = 1.131(0 - [-v_2])$$

$$\Rightarrow v_2 = 2.17 \text{ m/s}$$



برای جریان آب $\Delta V_g + \Delta T = 0 \Rightarrow mgh + \frac{1}{2} m (v_2^2 - v_1^2) = 0$

$$\Rightarrow h = \frac{1}{2 \times 9.81}(10^2 - 2.17^2) = 4.86 \text{ m}$$

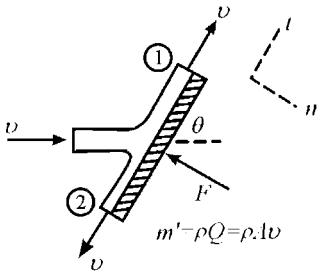
۴-۴۲. سیستم شامل ناودانی و سیال نشان داده شده می‌باشد. Q دبی جریان می‌باشد.

برای Q_1 :

$$\begin{cases} \Delta v_n = 0 - v \sin\theta \\ \Delta v_t = v(1 - \cos\theta) \end{cases}$$

برای Q_2 :

$$\begin{cases} \Delta v_n = 0 - v \sin\theta \\ \Delta v_t = -v - v\cos\theta = -v(1 + \cos\theta) \end{cases}$$



برای سیستم $\Sigma F_n = m' \Delta v_n \; : \; -F = \rho Q(0) - (\rho Q v \sin\theta) \quad (1)$

$$\Sigma F_t = m' \Delta v_t \; : \; 0 = \rho Q_1 v - \rho Q_1 v \cos\theta - \rho Q_2 v - \rho Q v \cos\theta \quad (2)$$

(1) : $F = \rho Q v \sin\theta$ یا $F = \rho A v^2 \sin\theta$

(2) : $0 = Q_1(1 - \cos\theta) - Q_2(1 + \cos\theta)$

$$Q = Q_1 + Q_2 \quad : \quad \begin{cases} Q_1 = \dfrac{Q}{2}(1 + \cos\theta) \\ Q_2 = \dfrac{Q}{2}(1 - \cos\theta) \end{cases}$$

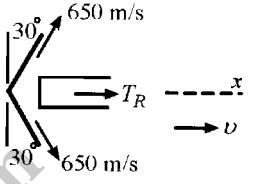
۴-۴۳.

$$\Sigma F = \Sigma m' u$$

با پره‌ها $T_R = m'_g u \sin 30° + m'_a v$

$$= (50 + 0.65)(650)\sin 30° + 50(55.6 - 0)$$

$$= 19240\ \text{N}$$



بدون پره‌ها $T = m'_g u - m'_a u$

$$T = (50 + 0.65)\,650 - 50(55.6) = 30100\ \text{N}$$

$$n = \frac{19240}{30100} = 0.638$$

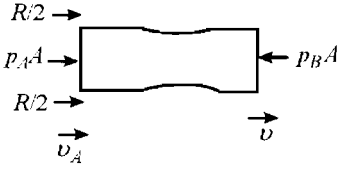
۴-۴۴.

پیوستگی : $\rho_A A v_A = \rho_B A v \quad \Rightarrow \quad v_A = v\dfrac{\rho_B}{\rho_A}$

در B : $m' = \rho A v = \rho_B A v$

$$\Sigma F = m'\Delta v \quad \Rightarrow \quad R + p_A A - p_B A = \rho_B A v(v - v_A)$$

$$= \rho_B A v^2\left(1 - \frac{\rho_B}{\rho_A}\right)$$



$$A = \frac{\pi d_2}{4} \quad \Rightarrow \quad R = \rho_B \frac{\pi d^2}{4} v^2\left(1 - \frac{\rho_B}{\rho_A}\right) + (p_B - p_A)\frac{\pi d_2}{4}$$

$$R = \frac{\pi d^2}{4}\left[\rho_B\left(1 - \frac{\rho_B}{\rho_A}\right)v^2 + (p_B - p_A)\right]$$

۴-۴۵.

$$Q = A v$$

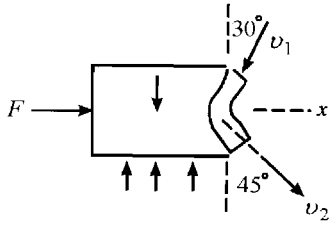
$$\frac{2\times 10^{-3}}{60} = \frac{(0.25\times 10^{-3})^2\pi}{4} v_1 \quad \Rightarrow \quad v_1 = 679\ \text{m/s}$$

$$v_2 = 0.6 v_1 = 0.6(679) = 407\ \text{m/s}$$

$$\Sigma F_x = m'\Delta v_x$$

$$m' = \frac{2\times 10^{-3}}{60}\times 1100 = 0.0367\ \text{kg/s}$$

$$F = 0.0367\,(407\sin 45° - [-679\sin 30°]) = 23.0\ \text{N}$$

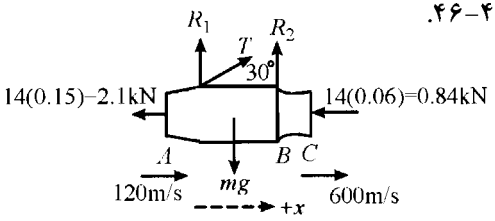


۴-۴۶.

$$\Sigma F_x = m' \Delta v_x$$

$$-2.1 - 0.84 + T\cos 30° = [31.6(600) - 30(120)]\,10^{-3}$$

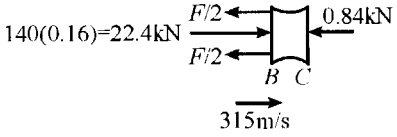
$$\Rightarrow T = 21.8\,\text{kN}$$



$$\Sigma F_x = m' \Delta v_x$$

$$22.4 - 0.84 - F = 31.6(600 - 315)10^{-3}$$

$$\Rightarrow F = 12.55\,\text{kN}$$



۴-۴۷.

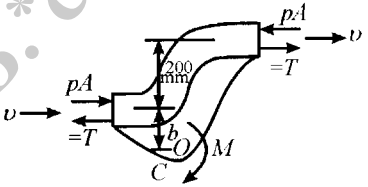
$$Q = A v$$

$$v = \frac{Q}{A} = \frac{\frac{20}{60}}{\frac{\pi (0.1)^2}{4}} = 42.44\,\text{m/s}$$

$$\Sigma M_O = \dot{H}_O = m' v_2 d_2 - m' v_1 d_1$$

$$m' = \frac{20}{60} \times 1000 = 333.3\,\text{kg/s}$$

$$M = 333.3(42.44)0.2 = 2830\,\text{N/m}$$



۴-۴۸.

$$\Sigma F_x = m' \Delta v_x$$

$$R - m_0 g - m_w g = \rho Q (v\cos 45° - v_0)$$

$$m_0 = 310\,\text{kg}$$

جرم آب $m_w = \rho V = 1000 \frac{\pi}{4} (0.2)^2(6) = 188.5\,\text{kg}$

$$Q = 0.125\,\text{m}^3/\text{s}$$

$$A = \frac{\pi}{4}(0.1)^2 = 0.00785\,\text{m}^2$$

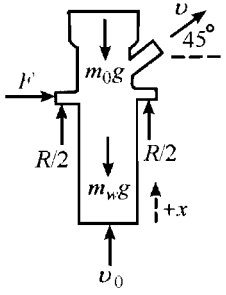
$$A_0 = \frac{\pi}{4}(0.25)^2 = 0.0491\,\text{m}^2$$

$$v = \frac{Q}{A} = \frac{0.125}{0.00785} = 15.92\,\text{m/s}$$

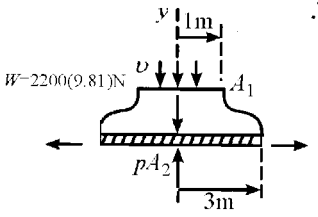
$$v_0 = \frac{Q}{A_0} = \frac{0.125}{0.0491} = 2.55\,\text{m/s}$$

$$R - (310 + 188.5)9.81 = 1000(0.125)(15.92\cos 45° - 2.55) = 1088$$

$$\Rightarrow R = 5980\,\text{N}$$



۴-۴۹.



$$\Sigma F_y = m' \Delta v_y \quad , \quad \Delta v_y = 0 - (-v) = v$$

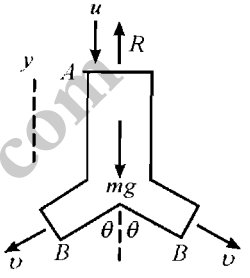
$$m' = \rho A_1 v$$

$$\Rightarrow \; pA_2 - W = \rho A_1 v^2$$

$$p(\pi \times 3^2) - 2200(9.81) = 1.206\,(\pi \times 1^2)\,(45)^2$$

$$\Rightarrow \; p = 1035\ \text{Pa} \quad \text{یا} \quad p = 1.035\ \text{kPa}$$

۴-۵۰.



$$m' = \rho A v_A = \rho \frac{\pi d^2}{4} u$$

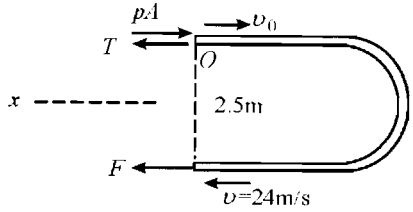
$$\Delta v_y = -v\cos\theta - (-u) = u - v\cos\theta$$

$$\Sigma F_y = m' \Delta v_y$$

$$R - mg = \rho \frac{\pi d^2}{4} u\,(u - v\cos\theta)$$

$$R = mg + \pi \frac{d^2}{4} u(u - v\cos\theta)$$

۴-۵۱.



سرعت در O :

$$Q = A v_0 \;\Rightarrow\; \frac{3}{60} = \frac{\pi (0.080)^2}{4} v_0$$

$$\Rightarrow \; v_0 = 9.95\ \text{m/s}$$

$$m' = \rho A v = \rho Q = \frac{3}{60}(1000) = 50\ \text{m/s}$$

$$\Sigma M_O = \dot{H}_O = m' v d - 0$$

$$2.5F = 50(24)2.5 \quad \Rightarrow \quad F = 1200\ \text{N}$$

$$\Sigma F_x = \dot{G}_x = m' \Delta v_x$$

$$T - \left[800(10^3)\,\frac{\pi (0.080)^2}{4}\right] + 1200 = 50[24 - (-9.95)]$$

$$\Rightarrow \; T = 4518\ \text{N} \quad \text{یا} \quad T = 4.52\ \text{kN}$$

۴-۵۲.

برای کامیون و برف روب به‌عنوان یک مجموعه :

$$\Sigma F_x = m' \Delta v_x$$

$$P = \frac{60000}{60}\left[\frac{20}{3.6} - 0\right] = 55600\ \text{N} \quad \text{یا} \quad P = 5.56\ \text{kN}$$

$$\Sigma F_y = m' \Delta v_y$$

$$R = \frac{60000}{60}[12\cos 45° - 0] = 8490\ \text{N} \quad \text{یا} \quad R = 8.49\ \text{kN}$$

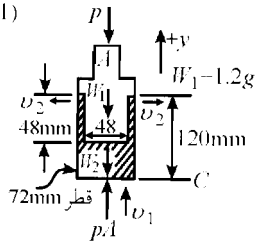
۴-۵۳.

$$\Sigma F_y = m' \Delta v_y$$

$$Q = A v_1 \Rightarrow \frac{2.3}{60} = \frac{\pi (0.072)^2}{4} v_1 \Rightarrow v_1 = 9.415 \text{ m/s}$$

وزن آب $W_2 = \left[\frac{\pi(0.072)^2}{4}(0.120) - \frac{\pi(0.048)^2}{4}(0.048)\right]1000(9.81)$

$= 3.95 \text{ N}$



$$pA = 80(10^3)\frac{\pi(0.072)^2}{4} = 325.7 \text{ N}$$

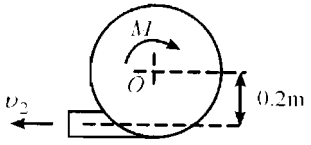
$$m' = \rho Q = 1000\left(\frac{2.3}{60}\right) = 38.3 \text{ kg/s}$$

$$\Rightarrow \quad P + (1.2 \times 9.81) + 3.95 - 325.7 = 38.3(0 - [-9.415]) \Rightarrow p = 671 \text{ N}$$

۴-۵۴.

$$M = M_O = m'(v_2 d_2 - 0)$$

$$v_2 = \frac{Q}{A} = \frac{\frac{16}{60}}{\frac{\pi(0.150)^2}{4}} = 15.09 \text{ m/s}$$



چگالی هوا $\rho = 1.206 \text{ kg/m}^3$: طبق جدول د-۱

$$m' = \rho Q = 1.206\left(\frac{16}{60}\right) = 0.322 \text{ kg/s}$$

$$M_O = 0.322(15.09 \times 0.2 - 0) = 0.971 \text{ N.m}$$

$$P = 0.32 + M_O \frac{\omega}{100} = 0.32 + 0.971\left(\frac{3450 \times \frac{2\pi}{60}}{1000}\right) = 0.671 \text{ kW}$$

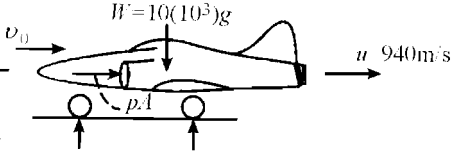
۴-۵۵.

$$m'_{air} = 48 \text{ kg/s}$$

$$m'_{fuel} = \frac{48}{18} = 2.67 \text{ kg/s}$$

سرعت ورود هوا $v_0 = \frac{m'_{air}}{\rho A}$

$$= \frac{48}{1.206 \times 1.160} = 34.3 \text{ m/s}$$



گازهای خروجی $m'_{exh.} = 48 + 2.67 = 50.67 \text{ kg/s}$

$$pA = -2.0(10^3)(1.160) = -2320 \text{ N}$$

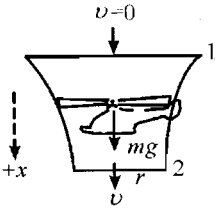
خالص $T = m'_{exh.} u - m'_{air} v_0 - pA$

$$= 50.67(940) - 48(34.3) - (-2320) = 48303 \text{ N}$$

$$\Sigma F_x = m a_x : \quad 48303 = 10000 a \Rightarrow a = 4.83 \text{ m/s}^2$$

۴-۵۶. mg = وزن هیلکوپتر = نیرو روی مجموعهٔ جریان هوا و هلیکوپتر

برای سیستم بین مقاطع ۱ و 2 $\Sigma F_x = m' \Delta v_x$



$$mg = \rho \pi r^2 v (v - 0)$$

$$v = \frac{1}{r}\sqrt{\frac{mg}{\pi \rho}}$$

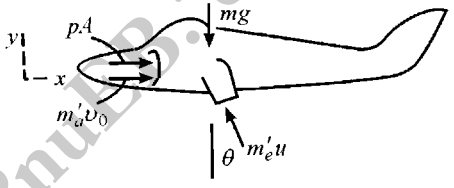
آهنگ افزایش انرژی جنبشی = توان

$$P = \frac{1}{2} m' (v_2^2 - v_1^2) = \frac{1}{2} m' v^2 = m' v \frac{v}{2} = mg \frac{v}{2}$$

$$\Rightarrow P = \frac{mg}{2r}\sqrt{\frac{mg}{\pi \rho}}$$

۴-۵۷.

$mg = 8600(9.81) = 84.4(10^3)$ N



هوا $m'_a = 90$ kg/s

سوخت $m'_f = \frac{90}{18} = 5$ kg/s

گاز خروجی $m'_e = 90 + 5 = 95$ kg/s , $m'_e u = 95(1020) = 96900$ N

$pA = -2(10^3)(1.10) = -2200$ N

$v_0 = \frac{m'_a}{\rho A} = \frac{90}{1.206(1.10)} = 67.8$ m/s $\Rightarrow$ $m'_a v_0 = 90(67.8) = 6110$ N

برای برخاستن قائم $\Sigma F_x = 0$: $6110 - 2200 - 96900 \sin\theta = 0$

$$\Rightarrow \sin\theta = 0.0403 \Rightarrow \theta = 2.31°$$

$\Sigma F_y = m a_y$: $96900 \cos 2.31° - 84400 = 8600\, a_y$ $\Rightarrow$ $a_y = 1.448$ m/s^2

۴-۵۸.

برای کل سیستم $\Sigma M = m' (v_2 d_2 - v_1 d_1)$

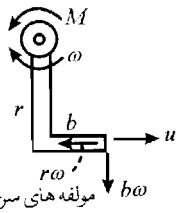
u = سرعت آب نسبت به نازل $= \frac{Q}{4A}$

$m' = \rho Q$

$$-M = \rho Q \left(r^2 \omega + b^2 \omega - \frac{Q}{4A} r - 0\right)$$

$$M = \rho Q \left(\frac{Qr}{4A} - [r^2 + b^2]\omega\right)$$

$M = 0$ برای $\Rightarrow$ $\omega = \omega_0 = \frac{Qr}{4A(r^2 + b^2)}$



مولفه های سرعت مطلق آب در خروجی

۴-۵۹.

$$F = m' \Delta v$$

$$m' = \rho A v = 1.206 \frac{\pi (0.040)^2}{4} 240 = 0.364 \text{ kg/s}$$

$$F = 0.364 \times 480 \sin \frac{\theta}{2} = 174.6 \sin \frac{\theta}{2} N$$

برای پره $\Sigma M_O = 0$

$$174.6 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \left(\frac{0.100}{\sin \theta}\right) - 6(9.81)(0.240 \sin \theta) = 0$$

$$87.3 \times 0.100 = 6(9.81)(0.240 \sin \theta) \Rightarrow \sin \theta = 0.617 \Rightarrow \theta = 38.2°$$

۴-۶۰ ◀

$$T_1 = p_1 A \quad , \quad T_2 = p_2 A$$

توان $P = M \omega$

$$M = \frac{40(10^3)}{900 \left(\frac{2\pi}{60}\right)} = 424 \text{ N.m}$$

$$\Sigma M_O = m' (v_2 d_2 - v_1 d_1)$$

$$424 + 0.3 \Delta F = \frac{20}{60}(1000)[18(0.2) - (18)(-0.075)]$$

$$\Delta F = \frac{1650 - 424}{0.3} = 4090 \text{ N}$$

$C = 250 + 4090 = 4340$ N به طرف بالا

$D = 4090 - 250 = 3840$ N به طرف پائین

۴-۶۱ ◀

سطح ورودی هوا $= \frac{\pi}{4}[(0.412)^2 - (0.375)^2] = 0.02287 \text{ m}^2$

سطح خروجی $= \frac{\pi}{4}(0.350)^2 = 0.0962 \text{ m}^2$

$$pA = -(-30.7(10^3))(0.0962) = 2954 \text{ N}$$

هوا $m'_a = \frac{16000}{3600} = 4.44$ kg/s

گندم $m'_y = \frac{135000}{3600} = 37.5$ kg/s

$$v_1 = \frac{m'_a}{\rho A} = \frac{4.44}{1.206(0.02287)} = 161 \text{ m/s}$$

$$\Sigma F_y = m' \Delta v_y$$

$$-C + 2954 - 30(9.81) = 4.44(40 - (-161)) + 37.5(40 - 0) \Rightarrow C = 267 \text{ N}$$

۴-۶۲ ◀

$$Q = \frac{1.3}{60} = 0.02167\ \text{m}^3/\text{s}$$

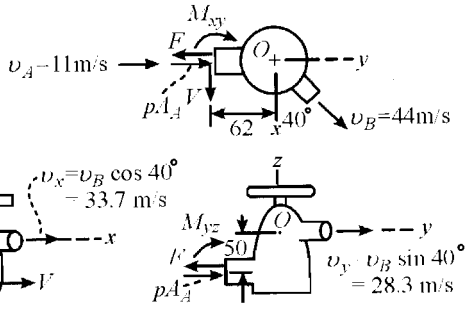
$$m' = \rho Q = 1000(0.02167) = 21.67\ \text{kg/s}$$

$$A_A = \frac{\pi (0.050)^2}{4} = 1.96(10^{-3})\ \text{m}^2$$

$$A_B = \frac{\pi (0.025)^2}{4} = 0.49(10^{-3})\ \text{m}^2$$

$$v_A = \frac{Q}{A_A} = \frac{0.02167}{1.96(10^{-3})} = 11\ \text{m/s}$$

$$v_B = \frac{Q}{A_B} = \frac{0.02167}{0.49(10^{-3})} = 44\ \text{m/s}$$



$$(x-y)\quad \Sigma F_x = m'\Delta v_x \ :\ 1000(10^3)\frac{\pi (0.050)^2}{4} - F = 21.67(28.3 - 11) \Rightarrow F = 1588\ \text{N}$$

$$\Sigma F_y = m'\Delta v_y \ :\ V = 21.67(33.7 - 0) \Rightarrow V = 730\ \text{N}$$

$$\Sigma M_{A-A} = m'\Delta (vd) \ :\ M_{xy} = 21.67(33.7 \times 0.062) = 35.4\ \text{N.m}$$

$$(y-z)\quad \Sigma M_{A-A} = m'\Delta (vd) \ :\ M_{yz} = 21.67(28.3 \times 0.050) = 30.6\ \text{N.m}$$

$$M = \sqrt{M_{xy}^2 + M_{yz}^2} = 54.8\ \text{N.m}$$

$$(x-z)\quad \Sigma M_O = 0 \ :\ T - Vd = 0 \Rightarrow T = 733(0.050) = 36.6\ \text{N}$$

۴-۶۳ ◀

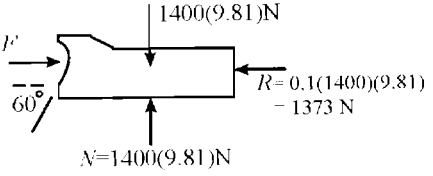
طبق قسمت ب مثال ۴-۵ $m' = \rho A (v - u) = 1000\dfrac{\pi (0.140)^2}{4}(150 - u)$

$$\Rightarrow m' = 15.39(150 - u)\ \text{kg/s}$$

$$F = \rho A (v-u)^2(1 - \cos 120°)$$

$$= 15.39(150 - u)^2(1 - (-0.5))$$

$$= 23.1(150 - u)^2$$



$$\Sigma F = m\dot{u} \Rightarrow 23.1(150 - u)^2 - 1373 = 1400\,\dot{u}$$

$$\int_0^4 \frac{du}{0.01649(150 - u)^2 - 0.981} = \int_0^3 dt$$

برای انتگرال‌گیری، فرض کنید $\omega = 150 - u$

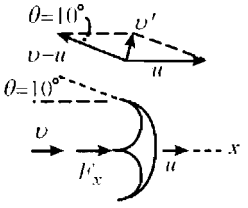
$$\int_{u=0}^{u=u} \frac{d\omega}{0.981 - 0.01649\omega^2} = 3 \Rightarrow \frac{1}{2\sqrt{0.01649}\sqrt{0.981}} \ln \left[\frac{0.990 + 0.1284(150 - u)}{0.990 - 0.1284(150 - u)}\right]_0^u = 3$$

$$\Rightarrow 3.93 \ln \frac{1 - 0.00634u}{1 - 0.00703u} = 3 \Rightarrow \frac{1 - 0.00634u}{1 - 0.00703u} = 2.145 \Rightarrow u = 131.0\ \text{m/s}$$

۴-۶۴ ◀

$$v = \sqrt{2gh} = \sqrt{2(9.81)(300)} = 76.7 \text{ m/s}$$

$$u = 0.47v = 36.1 \text{ m/s}$$



$$\Delta v_x = [u - (v - u)\cos\theta] - v$$

$$= -(v - u)(1 + \cos\theta)$$

$$= -(76.7 - 36.1)(1 + 0.985) = -80.7 \text{ m/s}$$

نیروی متوسط مماسی هر جت آب F_x

$$\Sigma F'_x = m' \Delta v_x \Rightarrow 1000(76.7A) - 80.7| = 6.19(10^6)A \text{ N}$$

(که در آن A سطح مقطع جت آب است)

توان تئوریک $P = 6F_x u = 6(6.19)(10^6)A(36.1) = 1.340A(10^9)$ W

$$\frac{\text{توان عملی}}{\text{توان تئوریک}} = \frac{22(10^6)}{1.340A(10^9)} = 0.90(0.85) \Rightarrow A = 0.0215 \text{ m}^2$$

$$\frac{\pi d^2}{4} = 0.0215 \Rightarrow d = 0.1653 \text{ m} \quad \text{یا} \quad d = 165.3 \text{ mm}$$

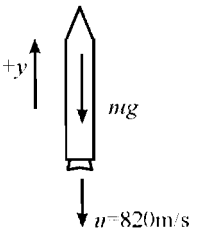
$$u = \frac{D}{2}\omega \Rightarrow D = \frac{2u}{\omega} = \frac{2(36.1)}{270\left(\frac{2\pi}{60}\right)} = 2.55 \text{ m}$$

۴-۶۵

$$\Sigma F_y = ma + \dot{m}u$$

$$= 6.80m - 220(820)$$

$$\Rightarrow m = 10.86(10^3) \text{ kg} \quad \text{یا} \quad m = 10.86 \text{ تن}$$



۴-۶۶

$$mg = 2.04(10^6)(9.81) = 20.0(10^6) \text{ N}$$

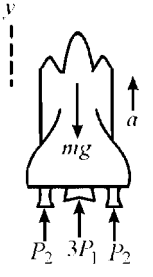
$$3P_1 = 3(2.00)(10^6) = 6.00(10^6) \text{ N}$$

$$2P_1 = 2(11.80)(10^6) = 23.6(10^6) \text{ N}$$

ضربه ویژه $I = \frac{u}{g} = 455 \text{ s} \Rightarrow u = 455(9.81) = 4460 \text{ m/s}$

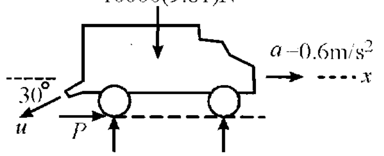
$$\Sigma F_y = ma_y : (6.00)10^6 + (23.6)10^6 - 20.0(10^6) = 2.04(10^6)a$$

$$\Rightarrow a = 4.70 \text{ m/s}^2$$



$$P_1 = m'u \Rightarrow 2.00(10^6) = m'(4460) \Rightarrow m' = 448 \text{ kg/s}$$

.۶۷-۴



$\Sigma F_x = m\dot{v} + \dot{m}u$

$u = 20 \text{ m/s}$

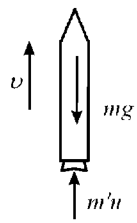
$\dot{m} = -40 \text{ kg/s}$

$m = 10000 \text{ kg}$

(الف) آبپاش کار می‌کند $P = 10000(0.6) - 40(20)\cos 30° \Rightarrow P = 5307 \text{ N}$ یا $P = 5.31 \text{ kN}$

(ب) آبپاش کار نمی‌کند $\dot{m} = 0 \Rightarrow P = 6000 \text{ N}$ یا $P = 6.00 \text{ kN}$

.۶۸-۴



$m = m_0 - m't$

$\Sigma F = ma \; : \; m'u - (m_0 - m't)g = (m_0 - m't)a$

$$a = \frac{dv}{dt} = \frac{m'u}{m_0 - m't} - g$$

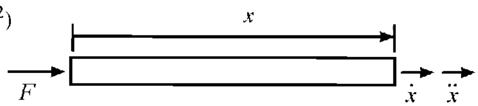
$$\int_0^v dv = \int_0^t \frac{m'u}{m_0 - m't}dt - \int_0^t g\,dt \Rightarrow v = -u\ln(m_0 - m't)\Big|_0^t - gt\Big|_0^t$$

$$\Rightarrow v = u\ln\left(\frac{m_0}{m_0 - m't}\right) - gt$$

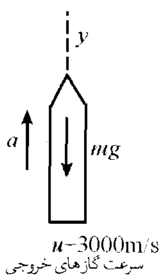
.۶۹-۴

$\Sigma F = m\dot{v} + \dot{m}u \quad (m = \rho x \; , \; v = \dot{x} \; , \; \dot{m} = \rho\dot{x} \; , \; u = \dot{x})$

$\Rightarrow F = \rho x\ddot{x} + \rho\dot{x}\dot{x} \Rightarrow F = \rho(x\ddot{x} + \dot{x}^2)$



.۷۰-۴



$\dot{m} = -m' = -5.2 \text{ kg/s}$

$m = 200 + 1200 - 5.2t = 1400 - 5.2t \text{ kg}$

$\Sigma F = m\dot{v} + \dot{m}u \quad : \quad -mg = ma - 5.2(3000)$

$(1400 - 5.2t)(a + 8.70) = 15600$

$$a = \frac{15600}{1400 - 5.2t} - 8.70 \text{ m/s}^2$$

وقتی $t = 60\text{ s}$: $a = \dfrac{15600}{1400 - 5.2(60)} - 8.70 = 5.64 \text{ m/s}^2$

حداکثر شتاب وقتی رخ می‌دهد که $5.2t = 1200$ یا $t = 231 \text{ s}$ باشد.

$$a_{max} = \frac{15600}{1400 - 5.2(231)} - 8.70 = 69.3 \text{ m/s}^2$$

۴-۷۱.

$$\Sigma F_x = m\dot{v} + \dot{m}u$$

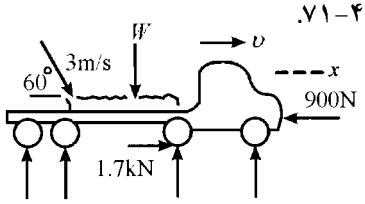
$$\Sigma F_x = 1.7(10^3) - 900\,\text{N}$$

$$m = 5400 + 4(100) = 5800\,\text{kg} \qquad (t = 4\,\text{s}\ \text{در})$$

$$\dot{m} = 100\,\text{kg/s}$$

$$u = \frac{2.5}{3.6} - 3\cos 60° = -0.8\,\text{m/s}$$

$$800 = 5800\dot{v} + 100(-0.8) \quad \Rightarrow \quad a = \dot{v} = 0.1518\,\text{m/s}^2$$



۴-۷۲.

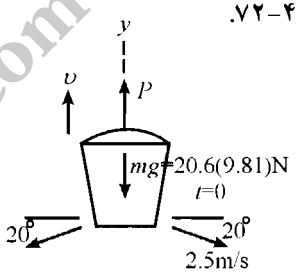
$$\Sigma F_y = m\dot{v} + \dot{m}u$$

$$\Sigma F_y = P - mg = P - 20.6(9.81) = P - 202\,\text{N}$$

$$\dot{m} = -\rho A v = -2(1000)\frac{\pi \times 0.030^2}{4}(2.5) = -3.53\,\text{kg/s}$$

$$u = 2.5\sin 20° - 0 = 0.855\,\text{m/s}$$

$$P - 202 = 20.6(0.5) - 3.53(0.855) \quad \Rightarrow \quad P = 209\,\text{N}$$



۴-۷۳.

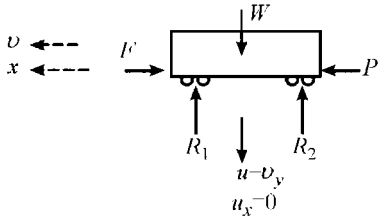
$$\Sigma F_x = m\dot{v}_x + \dot{m}u_x$$

$$F = 20\,\frac{25000 + \frac{1}{2}(90000)}{1000} = 1400\,\text{N}$$

$u_x = 0$ (سرعت نسبت به واگن در راستای x)

$$P - 1400 = \left(25000 + \frac{1}{2}(90000)\right)(0.045)$$

$$\Rightarrow \quad P = 4550\,\text{N} \quad \text{یا} \quad P = 4.55\,\text{kN}$$



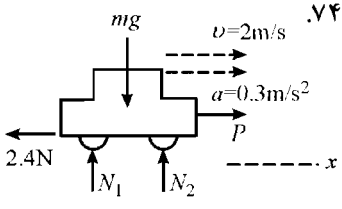
۴-۷۴.

$$m = 40 + 30(1.2) = 76\,\text{kg}$$

$$\dot{m} = -\rho v = -1.2(2) = -2.4\,\text{kg/s}$$

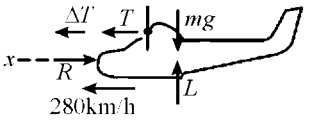
$$\Sigma F_x = m\dot{v} + \dot{m}u$$

$$P - 2.4 = 76(0.3) - 2.4(2) \quad \Rightarrow \quad P = 20.4\,\text{N}$$



۴-۷۵. برای سرعت اولیهٔ ثابت: مقاومت $T = R$ پیشران پروانه

توان اضافی $= \Delta T . v$



$$\Delta T \times \frac{280}{3.6} = 223.8(10^3) \quad \Rightarrow \quad \Delta T = 2880\,\text{N}$$

$$\Sigma F_x = m\dot{v} + \dot{m}u \qquad \left(m = 4.5 \times \frac{1000}{12} = 375\,\text{kg/s}\right)$$

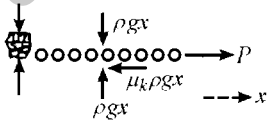
$$2880 = 16.4(10^3)\dot{v} + 375\left(\frac{280}{3.6}\right) \quad \Rightarrow \quad \dot{v} = a = -1.1603\,\text{m/s}^2$$

۴-۷۶.

حل ۱ : کل زنجیر

$$\Sigma F_x = \dot{G}_x : \quad P - \mu_k \rho g x = \frac{d}{dt}(\rho x \dot{x})$$

$$= \rho(\dot{x}^2 + x\ddot{x})$$



$$a = \ddot{x} = \frac{P}{\rho x} - \mu_k g - \frac{\dot{x}^2}{x}$$

حل ۲ : معادله ۴-۲۰ برای قسمت متحرک

$$\Sigma F = m\dot{v} + \dot{m}u$$

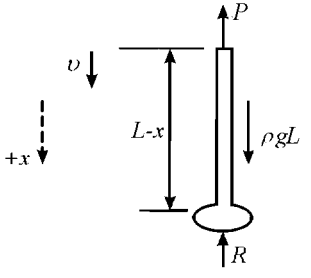
$$P - \mu_k \rho g x = \rho x\ddot{x} + \rho \dot{x}\dot{x} \qquad (u = \dot{x})$$

$$\Rightarrow \quad a = \ddot{x} = \frac{P}{\rho x} - \mu_k g - \frac{\dot{x}^2}{x}$$

۴-۷۷.

$\dot{x} = v =$ ثابت

$P =$ وزن حلقه‌های در حال بالا رفتن $= \rho g(L - x)$



$$\Sigma F_x = \frac{dG_x}{dt}$$

$$\rho g L - \rho g(L - x) = \frac{d}{dt}(\rho [L - x] v)$$

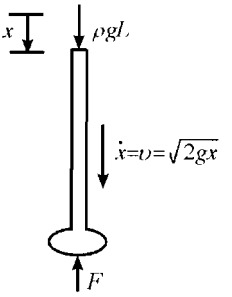
$$\rho g x - R = -\rho v\dot{x} = -\rho v^2 \quad \Rightarrow \quad R = \rho g x + \rho v^2$$

۴-۷۸.

روش اول :

$$G_x = \rho(L - x)\sqrt{2gx} = \rho\sqrt{2g}\left(Lx^{\frac{1}{2}} - x^{\frac{3}{2}}\right)$$

$\Sigma F_x = \dot{G}_x$ برای کل سیستم



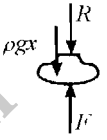
$$\rho g L - F = \rho\sqrt{2g}\left(\frac{1}{2}Lx^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}\right)\dot{x}$$

$$= \rho\sqrt{2g}\left(\frac{1}{2}L\sqrt{2g} - \frac{3}{2}\sqrt{2g}\,x\right)$$

$$= \rho g L - 3\rho g x$$

$$\Rightarrow F = 3\rho g x$$

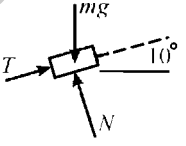
روش دوم : $R = m'\Delta v = \rho\dot{x}(\dot{x}) = \rho\dot{x}^2 = \rho(2gx) = 2\rho g x$



$\Sigma F_x = 0$: $2\rho g x + \rho g x - F = 0 \Rightarrow F = 3\rho g x$

۷۹-۴.

$\Sigma F_x = m a_x$: $T - mg\sin\theta = m a_x$



$T = m' u = 1(120) = 120\,\text{N}$ ثابت

$m = m_0 - m' t = 60 - 1t$

زمان پیش راندن $t = \frac{10}{1} = 10\,\text{s}$

$$m' u - (m_0 - m' t) g\sin\theta = (m_0 - m' t)\frac{dv}{dt}$$

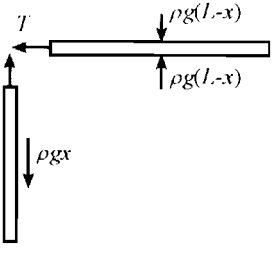
$$\int_0^t \left[\frac{m' a}{m_0 - m' t} - g\sin\theta\right] dt = \int_0^v dv \Rightarrow v = u\ln\left(\frac{m_0}{m_0 - m' t}\right) - g t\sin\theta$$

$$t = 10\,\text{s} \Rightarrow v = 120\ln\left(\frac{60}{60 - 1(10)}\right) - 9.81(10)\sin 10^\circ = 4.84\,\text{m/s}$$

۸۰-۴.

سیستم پایستار است $\Rightarrow \Delta V_g + \Delta T = 0$

$$-\rho g x\frac{x}{2} + \frac{1}{2}\rho L\dot{x}^2 = 0 \Rightarrow \frac{g}{L}x^2 = \dot{x}^2 \Rightarrow \dot{x} = \sqrt{\frac{g}{L}}x$$



(الف) $a = \ddot{x} = \sqrt{\frac{g}{L}}\dot{x} = \sqrt{\frac{g}{L}}\sqrt{\frac{g}{L}}x \Rightarrow a = \frac{g}{L}x$

(ب) $\Sigma F = m a$: $T = \rho(L - x)\frac{g}{L}x \Rightarrow T = \rho g x\left(1 - \frac{x}{L}\right)$

چک کردن روی قسمت عمودی

$$\rho g x - T = \rho x\frac{g}{L}x \Rightarrow T = \rho g x\left(1 - \frac{x}{L}\right) \quad ok$$

(ج) $v\,dv = a_x dx$: $\int_0^v v\,dv = \frac{g}{L}\int_0^L x\,dx \Rightarrow \frac{v^2}{2} = \frac{g}{L}\frac{L^2}{2} \Rightarrow v = \sqrt{gL}$

۴-۸۱

v = ثابت $\Rightarrow$ $\dot{v}$ = شتاب = 0

$\Rightarrow \Sigma F_y = 0$ برای تمام قسمت‌ها

$P + \rho g y - T = 0$ (1)

$\rho v^2 + \rho g h - T = 0$ (2)

T حذف $\Rightarrow$ $P = \rho v^2 + \rho g (h - y)$

قسمت سمت چپ (جرم ثابت، انتهای بالای متحرک) :

$\Sigma F_x = \dot{G}_x$

$T + R - \rho g h - \rho g (L - h - y) = \dfrac{d}{dt}(\rho h v)$

$\rho v^2 + \rho g h + R - \rho g h - \rho g (L - h - y)$

$= \dfrac{d}{dt}(\rho h v) = \rho v^2 \qquad (\dot{h} = v)$

$\rho v^2 + R - \rho g (L - b - y) = \rho v^2$

$R = \rho g (L - h - y)$ = وزن تودهٔ زنجیر

۴-۸۲

m_0 = جرم اولیه واگن = $25(10^3)$ kg

$\dot{m} = 4(10^3)$ kg/s

واگن جرمی دریافت می‌کند که سرعت اولیهٔ افقی آن صفر است، بنابراین برای راستای افقی x

$\Sigma F_x = \dfrac{d}{dt}(m v) \Rightarrow 0 = \dfrac{d}{dt}(m + \dot{m} t) v \Rightarrow (m_0 + \dot{m} t) a + \dot{m} v = 0$

$a = \dfrac{dv}{dt} = -\dfrac{\dot{m} v}{m_0 + \dot{m} t}$

$\displaystyle\int_{v_0}^{v} \frac{dv}{v} = -\int_0^t \frac{\dot{m}}{m_0 + \dot{m} t}\, dt \Rightarrow v = \frac{dx}{dt} = \frac{m_0 v_0}{m_0 + \dot{m} t}$

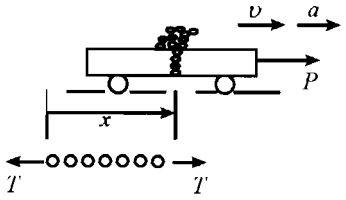
$\displaystyle\Rightarrow \int_0^x dx = \int_0^t \frac{m_0 v_0}{m_0 + \dot{x} t}\, dt \Rightarrow x = \frac{m_0 v_0}{\dot{m}} \ln\left(\frac{m_0 + \dot{m} t}{m_0}\right)$

$\displaystyle t = \frac{32}{4} = 8\,\text{s} \Rightarrow x = \frac{25(10^3)(1.2)}{4(10^3)} \ln\left(\frac{25 + 4(8)}{25}\right) \Rightarrow x = 6.18\,\text{m}$

۸۳-۴

$m = m_0 - \rho x$

به گاری منتقل نمی‌شود $T \Rightarrow \Sigma F = ma$



$$P = (m_0 - \rho x)\, a$$

$$a = \frac{P}{m_0 - \rho x}$$

$$v\,dv = a\,dx \Rightarrow \int_{v_0}^{v} v\,dv = \int_0^x \frac{P\,dx}{m_0 - \rho x}$$

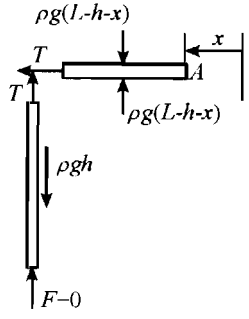
$$\frac{v^2}{2}\Big]_{v_0}^{v} = -\frac{P}{\rho}\ln(m_0 - \rho x)\Big]_0^x = \frac{P}{\rho}\ln\frac{m_0}{m_0 - \rho x}$$

$$\Rightarrow v = \sqrt{v_0^2 + \frac{2P}{\rho}\ln\frac{m_0}{m_0 - \rho x}}$$

$$T = m'\Delta v = \rho . v(v) \Rightarrow T = \rho v^2$$

هیچ عکس‌العملی بین حلقه‌های جداشونده و گاری وجود ندارد لذا $\dot{m}\,u$ صفر بوده و $\Sigma F = m\,a$

۸۴-۴. فرض کنید x جابجایی زنجیر و T کشش در زنجیر در انتها باشد.



قسمت افقی $\Sigma F_x = m\,a_x$

$$T = \rho(L - h - x)\ddot{x}$$

قسمت عمودی $\Sigma F_v = m\,a_v$

$$\rho g h - T = \rho h \ddot{x}$$

حذف T : $\ddot{x} = \frac{gh}{L - x}$

$$\dot{x}\,d\dot{x} = \ddot{x}\,dx \;:\; \int_0^{v_1^2}\frac{1}{2}d(\dot{x})^2 = \int_0^{L-h}\frac{gh}{L-x}\,dx$$

$$\frac{\dot{x}}{2}\Big]_{\dot{x}=0}^{v_1} = -gh\ln(L-x)\Big|_0^{L-h} \Rightarrow \frac{v_1^2}{2} = gl\ln(L-x)\Big|_{L-h}^{0} = gh\ln\frac{L}{h}$$

(الف) $v_1 = \sqrt{2gh\ln\left(\frac{L}{h}\right)}$

(ب) سقوط آزاد انتهای A : $v_2^2 = v_1^2 + 2gh = 2gh\ln\frac{L}{h} + 2gh$

$$v_2 = \sqrt{2gh\left(1 + \ln\left[\frac{L}{h}\right]\right)}$$

(ج) $Q =$ اتلاف انرژی پتانسیل ($\Delta T = 0$ زیرا)

$$Q = \rho g h\frac{h}{2} + \rho g(L-h)h \Rightarrow Q = \rho g h\left(L - \frac{h}{2}\right)$$

۸۵-۴. برای هواپیما بعلاوهٔ بخش متحرک زنجیرها

$$\Sigma F = 0 = m\dot{v} + \dot{m}u$$

$$= \left(m + 2\rho\frac{x}{2}\right)\dot{v} + \left[2\frac{d}{dt}\left(\rho\frac{x}{2}\right)\right]v$$

$$-(m+\rho x)\frac{dv}{dt} = \rho v\frac{dx}{dt} \Rightarrow \frac{dv}{dt} = -\frac{\rho\,dx}{m+\rho x}$$

$$\int_{v_0}^{v}\frac{dv}{v} = -\int_0^x\frac{\rho\,dx}{m+\rho x} \Rightarrow \ln\frac{v}{v_0} = -\ln\frac{m+\rho x}{m} \Rightarrow \frac{v}{v_0} = \frac{m}{m+\rho x}$$

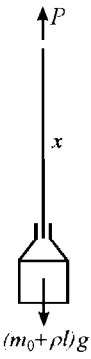
$$\Rightarrow v = \frac{v_0}{1+\frac{\rho x}{m}} \Rightarrow x = 2L \text{ برای } : v = \frac{v_0}{1+\frac{2\rho L}{m}}$$

$$v = \frac{dx}{dt} \Rightarrow \int_0^x\left(1+\frac{\rho x}{m}\right)dx = \int_0^t v_0\,dt \Rightarrow x + \frac{\rho x^2}{2m} = v_0 t$$

$$\Rightarrow x^2 + \frac{2m}{\rho}x - \frac{2mv_0 t}{\rho} = 0$$

$$\Rightarrow x = -\frac{m}{\rho} \pm \frac{1}{2}\sqrt{\frac{4m^2}{\rho^2} + \frac{8mv_0 t}{\rho}} \Rightarrow x = \frac{m}{\rho}\left[\sqrt{1+\frac{2v_0 t\rho}{m}} - 1\right]$$ برای ریشهٔ مثبت

۸۶-۴ ◀



$$U = \Delta T + \Delta V_g$$

$$U = -Fx$$

$$\Delta T = \frac{1}{2}(m_0 + \rho[L-x])v^2$$

$$\Delta V_g = -\left[\{m_0+\rho(L-x)\}x + \rho x\frac{x}{2}\right]g$$

$$-Fx = \frac{1}{2}(m_0+\rho[L-x])v^2 - \left(m_0+\rho\left[L-\frac{x}{2}\right]\right)gx$$

دیفرانسیل نسبت به زمان :

$$-Fv = (m_0+\rho[L-x])va - \frac{1}{2}\rho v^3 - \left(m_0+\rho\left[L-\frac{x}{2}\right]\right)gv + \rho gv\frac{x}{2}$$

یا $[m_0+\rho(L-x)]a = [m_0+\rho(L-x)]g + \frac{\rho v^2}{2} - F$

$$\Rightarrow a = g + \frac{\frac{\rho v^2}{2} - F}{m_0+\rho(L-x)}$$

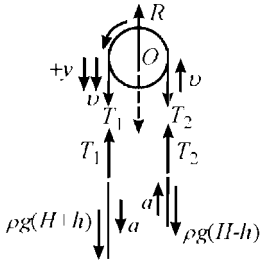
برای کل سیستم $\Sigma F_x = \dot{G}_x$

$$(m_0+\rho L)g - P = \frac{d}{dt}[m_0+\rho(L-x)]v = [m_0+\rho(L-x)]a - \rho v^2$$

جای‌گذاری a : $P = \rho gx + \frac{\rho v^2}{2} + F$

◀ ۸۷-۴. از جرم قرقره و وزن قسمت کوچکی از زنجیر که با قرقره در تماس است صرفنظر می‌شود.

$\Sigma M_O \approx 0 \Rightarrow T_1 = T_2 = T$



برای زنجیرها $\Sigma F = ma$

$\rho g(H + h) - T = \rho (H + h)a \qquad (1)$

$T - \rho g(H - h) = \rho (H - h)\, a \qquad (2)$

حذف T : $a = \dfrac{h}{H} g \qquad (3)$

$T = \rho (H - h)\dfrac{h}{H} g + \rho g(H - h) \Rightarrow T = \rho g\left(H - \dfrac{h^2}{H}\right) \qquad (4)$

جریان یکنواخت : قرقره و زنجیر روی آن

$\Sigma F_y = m' \Delta v_y : 2T - R = \rho v(v - [-v]) \Rightarrow R = 2T - 2\rho v^2 \qquad (5)$

از طرفی $\int_0^v v\, dv = \int_0^h \dfrac{g}{H} h\, dh \Rightarrow v^2 = \dfrac{g}{H} h^2 \Rightarrow v = \sqrt{\dfrac{g}{H}}\, h \qquad (6)$

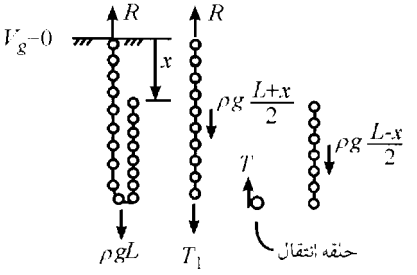
جای‌گذاری (4) و (6) در (3) : $R = 2\rho g\left(H - \dfrac{h^2}{H}\right) - 2\rho \dfrac{g}{H} h^2 \Rightarrow R = 2\rho g\left(H - \dfrac{2h^2}{H}\right)$

◀ ۸۸-۴. هیچ نیرویی روی قسمت متحرک نیست مگر وزن بنابراین (ثابت) $\ddot{x} = g$ و لذا $v^2 = 2gx$

$T_1 = m' \Delta v \Rightarrow T_1 = \left[\dfrac{d}{dt}\left(\rho \dfrac{L - x}{2}\right)\right] \times [0 - v]$

$= \dfrac{1}{2}\rho v^2$

$= \dfrac{1}{2}\rho (2gx) = \rho gx$



تعادل حلقه‌های باقیمانده $\Sigma F_x = 0$:

$T_1 + \rho g \dfrac{L + x}{2} - R = 0$

$R = \rho gx + \rho g \dfrac{L + x}{2}$

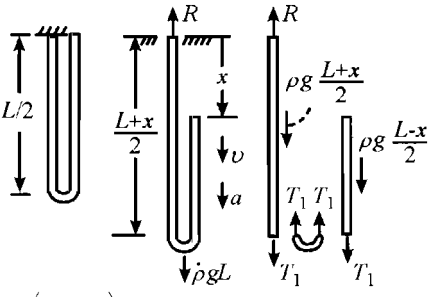
$R = \dfrac{1}{2}\rho g (L + 3x)$

اتلاف انرژی $Q = |v_{g_1} - v_{g_2}| = \left|\rho gL\left(-\dfrac{L}{4}\right) - \rho gL\left(-\dfrac{L}{2}\right)\right| \Rightarrow Q = \dfrac{1}{4}\rho gL^2$

◀ ۸۹-۴.

$\Delta V_g = \rho g \dfrac{L - x}{2}(-x) + \rho g \dfrac{x}{2}\left(-\dfrac{x}{2}\right) = -\dfrac{1}{2}\rho gx\left(L - \dfrac{x}{2}\right)$

$\Delta T = \dfrac{1}{2}\rho \dfrac{L - x}{2} v^2$



$$\Delta V_g + \Delta T = 0$$

$$\frac{1}{4}\rho(L-x)\,v^2 = \frac{1}{2}\rho g x\left(L - \frac{x}{2}\right)$$

$$v^2 = 2gx\,\frac{L-\frac{x}{2}}{L-x} \qquad (1)$$

$$v\,dv = a\,dx \Rightarrow a = \frac{1}{2}\frac{dv^2}{dx} = g\,\frac{(L-x)(L-x) - x\left(L-\frac{x}{2}\right)(-1)}{(L-x)^2}$$

$$a = g\left(1 + \frac{x\left(L-\frac{x}{2}\right)}{(L-x)^2}\right) \qquad (2)$$

برای کل طناب $\Sigma F_x = \dot{G}_x$

$$\rho gL - R = \frac{d}{dt}\left(\rho\,\frac{L-x}{2}\,v\right) = \frac{\rho}{2}\left[(L-x)a - v^2\right]$$

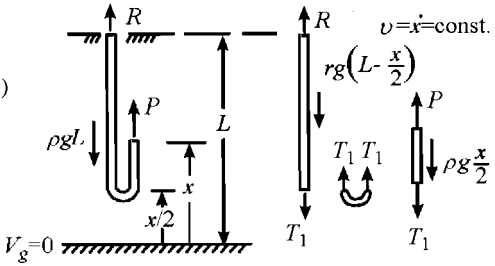
جای‌گذاری (1) و (2) $R = \frac{1}{2}\rho g\left[(L+x) + \frac{x\left(L-\frac{x}{2}\right)}{L-x}\right]$

تعادل قسمت ثابت : $T_1 = \frac{1}{2}\rho g x\,\frac{\left(L-\frac{x}{2}\right)}{(L-x)}$

◄ ۴-۹۰.

برای کل طناب $V_g = \rho g\left(L-\frac{x}{2}\right)\frac{L+\frac{x}{2}}{2} + \rho g\,\frac{x}{2}\,\frac{3x}{4}$

$$= \frac{\rho g}{2}\left(L^2 + \frac{x^2}{2}\right)$$



کل طناب $\Sigma F_x = \dot{G}_x$

$$R + P - \rho gL = \frac{d}{dt}\left(\rho\,\frac{x}{2}\,v\right) = \frac{1}{2}\rho v^2 \qquad (1)$$

کار - انرژی : $dU' = dT + dV_g$

$$Pdx = d\left(\frac{1}{2}\rho\,\frac{x}{2}\,v^2\right) + d\left\{\frac{\rho g}{2}\left(L^2 + \frac{x^2}{2}\right)\right\}$$

$$= \frac{1}{4}\rho v^2\,dx + \frac{1}{2}\rho g x\,dx$$

$$P = \frac{1}{4}\rho v^2 + \frac{1}{2}\rho g x \qquad (2)$$

جای‌گذاری (2) و (1) : $R = \frac{1}{4}\rho v^2 + \rho g\left(L - \frac{x}{2}\right)$

تعادل قسمت ثابت $\Sigma F_y = 0$: $R - \rho g\left(L - \frac{x}{2}\right) - T_1 = 0 \Rightarrow T_1 = \frac{1}{4}\rho v^2$

۹۱-۴ ◀ برای قسمت سقوط کننده $\Sigma F = m\dot{v} + \dot{m}u$

$$\Sigma F = \rho g x$$

$$m = \rho x$$

$$\dot{m} = \rho v$$

$$u = v = \dot{x}$$

$$\Rightarrow \rho g x = \rho x\dot{v} + \rho v\dot{x} \Rightarrow g x dt = x dv + v dx$$

$$g x dt = d(xv)$$

$$g x^2 v\, dt = xv\, d(xv)$$

$$g x^2 dx = \frac{1}{2} d[(xv^2)] \Rightarrow g\int_0^x x^2\, dx = \frac{1}{2}\int_0^{(xv)^2} d[(xv)^2]$$

$$\Rightarrow g\frac{x^3}{3} = \frac{1}{2}(xv)^2 \Rightarrow v = \sqrt{\frac{2gx}{3}}$$

$$a = \dot{v} = \sqrt{\frac{2g}{3}}\,\frac{1}{2}\,x^{-\frac{1}{2}}\,\dot{x} = \sqrt{\frac{2g}{3}}\,\frac{1}{2\sqrt{x}}\sqrt{\frac{2gx}{3}} \Rightarrow a = \frac{g}{3}$$ ثابت

$$Q = -\Delta V_g - \Delta T = +\frac{\rho g L^2}{2} - \frac{\rho L}{2} v^2_{x=L} = \frac{\rho g L^2}{2} - \frac{\rho g L^2}{3} = \frac{\rho g L^2}{6}$$

فصل ششم

سینتیک صفحه‌ای اجسام صلب

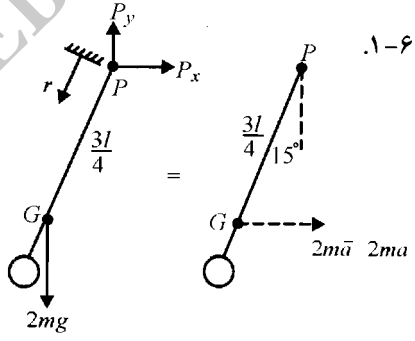
۶-۱.

$$\bar{r} = \frac{\Sigma m_i r_i}{\Sigma m_i} = \frac{m\left(\frac{l}{2}\right) + m(l)}{m + m} = \frac{3}{4}l$$

$$\curvearrowleft + \Sigma M_P = \bar{I}\alpha + m\bar{a}d \quad (\alpha = 0)$$

$$2mg\left(\frac{3l}{4}\sin 15°\right) = 2ma\left(\frac{3l}{4}\cos 15°\right)$$

$$\Rightarrow a = g\tan 15° = 0.268g$$



۶-۲.

$$\Sigma F_x = ma \;:\; 0.5N_B = 50a$$

$$\Sigma F_y = 0 \;:$$

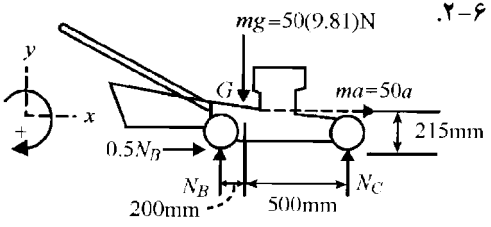
$$N_B + N_C - 50(9.81) = 0$$

$$\Sigma M_B = mad \;:$$

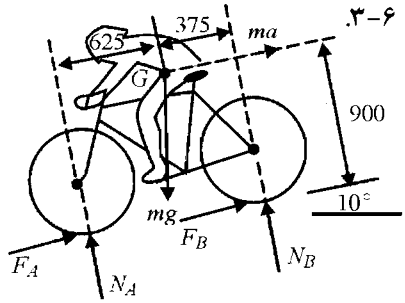
$$50(9.81)(0.2) - N_C(0.7) = 50a(0.215)$$

حل همزمان :

$$\begin{cases} N_B = 414\,\text{N} \\ N_C = 76.6\,\text{N} \\ a = 4.14\,\text{m/s}^2 \end{cases}$$



۶-۳.



واژگونی حول چرخ جلو : $N_B ،\ F_B \longrightarrow 0$

$$\curvearrowright_{+} \Sigma M_A = mad$$

$$mg\,(625 \cos 10^\circ - 900 \sin 10^\circ) = ma\,(900)$$

$$\Rightarrow\ a = 0.510\,g$$

۶-۴.

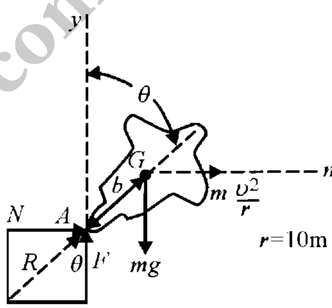
$$\Sigma \curvearrowleft_{+} M_A = m\bar{a}d\ :\ mg\,b \sin\theta = m\frac{v^2}{r} b \cos\theta \ \Rightarrow\ v^2 = gr\tan\theta$$

از طرفی $\tan\theta = \dfrac{N}{F} = \dfrac{1}{\mu}$

$$v^2 = \frac{gr}{\mu} \ \Rightarrow\ v = \sqrt{\frac{9.81 \times 10}{0.70}} = 11.84 \text{ m/s}$$

یا $v = 42.6$ km/h

$$\theta = \tan^{-1}\frac{v^2}{gr} = \tan^{-1}\frac{11.84^2}{9.81 \times 10} = 55.0^\circ$$



۶-۵.

$$mg = 1650(9.81) = 16.19(10^3) \text{ N}$$

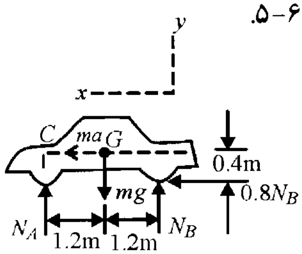
$$\curvearrowleft_{+} \Sigma M_C = mad = 0\ :$$

$$N_B(2.4) - 0.8 N_B(0.4) - 16.19(10^3)1.2 = 0$$

$N_B = 9.34(10^3)$N یا $N_B = 9.34$ kN

$$\Sigma F_y = 0\ :\ N_A + 9.34(10^3) - 16.19(10^3) = 0$$

$N_A = 6.85(10^3)$ N یا $N_A = 6.85$ kN



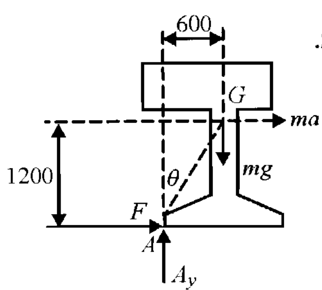
۶-۶.

$$\tan\theta = \frac{600}{1200} = 0.5 < (\mu_s = 0.60)$$

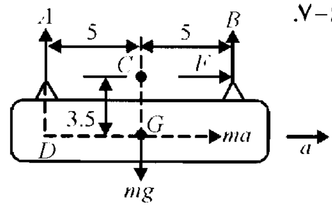
بنابراین قبل از لغزیدن واژگون می‌شود

$$\Sigma M_A = mad$$

$$mg\,(600) = ma\,(1200) \ \Rightarrow\ a = \frac{1}{2}g = 4.91 \text{ m/s}^2$$



۶-۷.



$B > A$ (زیرا ΣM_C باید CCW باشد)

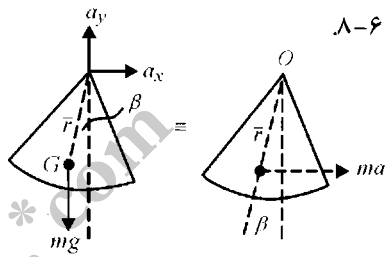
$$F_{max} = \mu B = 0.25 B$$

$$\Sigma M_D = 0 \ : \ 5mg + 0.25B(3.5) - 10B = 0$$

$$\Rightarrow \ B = 0.548\, mg$$

$$\Sigma F = ma \ : \ 0.25B = ma \ \Rightarrow \ a = \frac{0.25(0.548)\, mg}{m} = 1.344 \text{ m/s}^2$$

۶-۸.

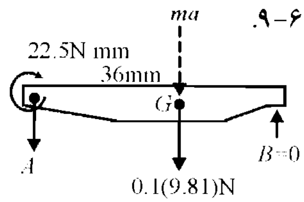


$$\Sigma M_C = m\, a\, d \ \Rightarrow \ m\, g\, \bar{r} \sin\beta = m\, a\, \bar{r} \cos\beta$$

$$\Rightarrow \ a = g \tan\beta$$

از طرفی $\theta = \frac{10}{20}\beta = 6\beta \ \Rightarrow \ a = g \tan\frac{\theta}{6}$

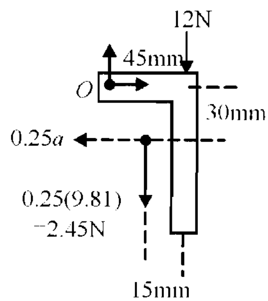
۶-۹.



$$\Sigma M_A = mad$$

$$225 + 0.1(9.81)(36) = 0.1\, a\, (36) \ \Rightarrow \ a = 72.3 \text{ m/s}^2$$

۶-۱۰.

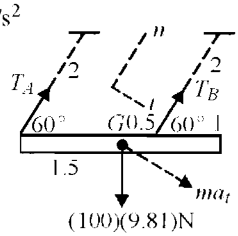


$$\Sigma M_O = mad_O$$

$$12(45) + 0.24(9.81)(30) = 0.25\, a\, (30)$$

$$a = 81.8 \text{ m/s}^2 = 8.34\, g$$

۶-۱۱.



$$\Sigma F_t = m\, a_t \ : \ 100(9.81)\cos 60° = 100\, a_t \ \Rightarrow \ a_t = 4.905 \text{ m/s}^2$$

$$\alpha = \frac{a_t}{r} = \frac{4.905}{2} = 2.45 \text{ rad/s}^2$$

$$\circlearrowleft + \Sigma M_G = 0 \ :$$

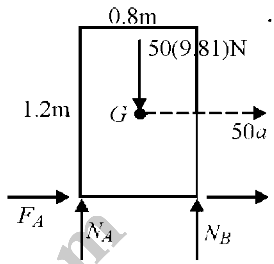
$$T_B \sin 60° \times 0.5 - T_A \sin 60° \times 1.5 = 0 \ \Rightarrow \ T_A = \frac{1}{3} T_B$$

$$\Sigma F_n = m\, a_n = 0 \ :$$

$$T_A + T_B - 100(9.81)\sin 60° = 0 \quad \Rightarrow \quad T_A + T_B = 849.6\,\text{N}$$

$$\Rightarrow \quad T_A = 212\,\text{N} \qquad T_B = 637\,\text{N}$$

۶-۱۲.



در آستانهٔ واژگونی : $N_B\,,\ F_B \rightarrow 0$

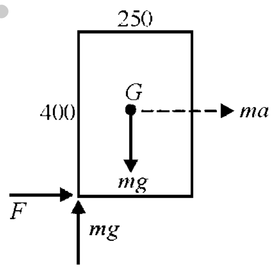
$$\overset{+}{\curvearrowright} \Sigma M_A = mad \ : \ mg\,(0.4) = ma\,(0.6)$$

$$\Rightarrow \ a = \frac{2}{3}g = 6.54\,\text{m/s}^2$$

$$\overset{+}{\rightarrow} \Sigma F = ma \ : \ P = 60(6.54) = 392\,\text{N}$$

$$\mu_s > \frac{a}{g} = \frac{2}{3}$$

۶-۱۳.



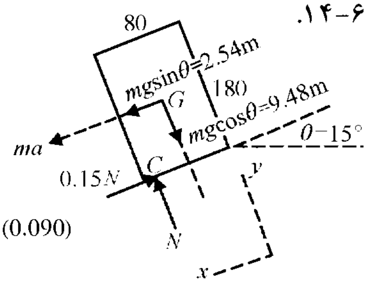
$$v = 1.2 + 0.9t^2\,\text{m/s}$$

$$a = \dot{v} = 1.8t\,\text{m/s}^2$$

$$\overset{+}{\curvearrowright} \Sigma M_A = mad = mg\,\frac{250}{2} = m\,(1.8\,t)\left(\frac{400}{2}\right)$$

$$t = 3.41\,\text{s}$$

۶-۱۴.



مجهولات : $a\,,\ N\,,\ b$

$$\Sigma F_y = 0 \ : \ N - 9.48\,m = 0 \quad \Rightarrow \quad N = 9.48\,m$$

$$\Sigma F_x = m\,a_x \ : \ 2.54\,m - 0.15(9.48\,m) = ma$$

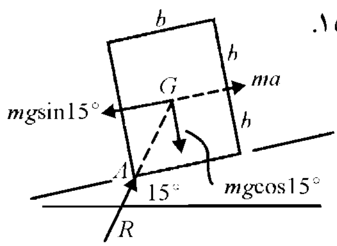
$$\Rightarrow \ a = 1.118\,\text{m/s}^2$$

$$\Sigma M_C = mad \ : \ 2.54\,m\,(0.090) - 9.48\,m\,(b) = 1.118\,m\,(0.090)$$

$b = 0.0135\,\text{m}$ یا $13.5\,\text{mm} < \dfrac{80}{2}$

بنابراین استوانه واژگون نخواهد شد.

۶-۱۵.



$$\Sigma M_A = mad$$

$$(mg\cos 15°)\,\frac{b}{2} - (mg\sin 15°)\,b = m\,a\,b$$

$$g\left(\frac{0.966}{2} - 0.259\right) = a \quad \Rightarrow \quad a = 0.224\,g$$

۶-۱۶.

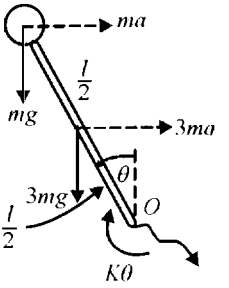
$\Sigma \curvearrowright_{+} M_O = \Sigma m\,ad$:

$K\theta - 3\,m\,g\left(\frac{l}{2}\sin\theta\right) - mg\,(l\sin\theta) = 3\,ma\left(\frac{l}{2}\cos\theta\right) + ma\,(l\cos\theta)$

$\Rightarrow \quad K\theta - \frac{5}{2}m\,g\,l\sin\theta = \frac{5}{2}m\,a\,l\cos\theta$

$m = 0.5\,\text{kg}\ ,\ l = 0.6\,\text{m}\ ,\ a = 2\,\text{g}\ ,\ \theta = 20°$

$\Rightarrow \quad K = 46.8\,\text{N.m/rad}$

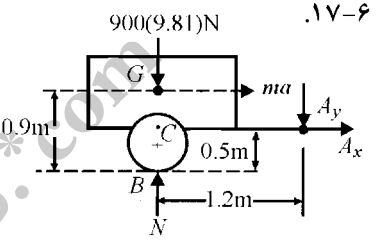


۶-۱۷.

$v^2 = 2as \ \Rightarrow \ a = \frac{v^2}{2s} = \frac{\left(\frac{60}{3.6}\right)^2}{2(30)} = 4.63\,\text{m/s}^2$

$\Sigma M_C = mad$:

$1.2A_y = 900(4.63)(0.9 - 0.5) \ \Rightarrow \ A_y = 1389\,\text{N}$

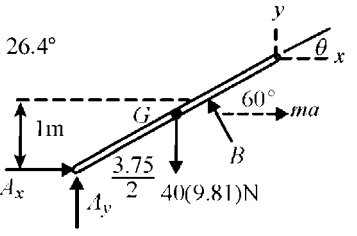


۶-۱۸.

$\Sigma M_A = mad$

$40(9.81) \times \frac{3.75}{2} \times \cos 6.4° - 2.25\,B = 40 \times 4 \times \frac{3.75}{2}\sin 26.4°$

$\Rightarrow \quad B = 234\,\text{N}$



$\theta = \sin^{-1}\frac{1}{2.25} = 24.4°$

۶-۱۹.

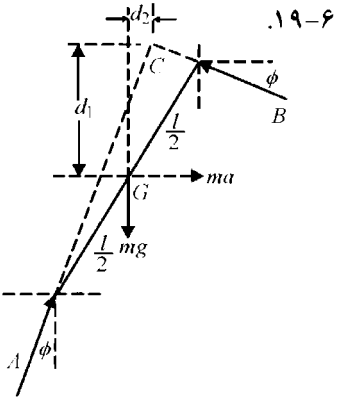
$\phi = \tan^{-1}\mu = \tan^{-1}0.4 = 21.8°$

اندازه‌گیری بازوهای گشتاور : $d_1 = 0.486\,l$

$d_2 = 0.118\,l$

$\Sigma M_C = mad$: $mgd_2 = mad_1$

$\Rightarrow \quad a = g\frac{d_2}{d_1} = 9.81\,\frac{0.118\,l}{0.486\,l} = 2.38\,\text{m/s}^2$



۶-۲۰.

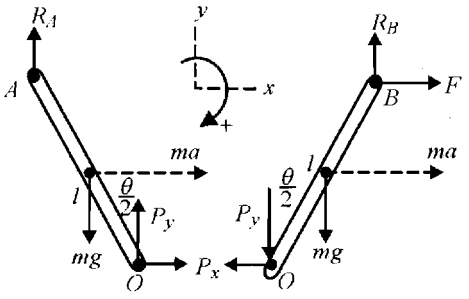
Ⓐ $\Sigma M_O = mad$:

$$R_A\, l \sin\frac{\theta}{2} - m\,g\frac{l}{2}\sin\frac{\theta}{2} = ma\frac{l}{2}\cos\frac{\theta}{2}$$

Ⓑ $\Sigma M_O = mad$:

$$Fl\cos\frac{\theta}{2} + mg\frac{l}{2}\sin\frac{\theta}{2} - R_B\, l\sin\frac{\theta}{2} -$$

$$- R_B\, l\sin\frac{\theta}{2} = ma\frac{l}{2}\cos\frac{\theta}{2}$$



دو میله با هم : $\Sigma F_y = 0$: $R_A + R_B - 2mg = 0$

معادله Ⓐ را از Ⓑ کم کرده و با رابطه فوق ترکیب می‌کنیم:

$$\theta = 2\tan^{-1}\frac{F}{mg}$$

دو میله با هم : $\Sigma F_x = m\,a_x \Rightarrow F = 2ma \Rightarrow a = \frac{g}{2}\tan\frac{\theta}{2}$

Ⓑ از رابطه : $mg\tan\frac{\theta}{2}\,l\cos\frac{\theta}{2} + mg\frac{l}{2}\sin\frac{\theta}{2} - R_B\, l\sin\frac{\theta}{2} = m\left(\frac{g}{2}\tan\frac{\theta}{2}\right)\frac{l}{2}\cos\frac{\theta}{2} \Rightarrow R_B = \frac{5}{4}mg$

از معادله F_y : $R_A = \frac{3}{4}mg$

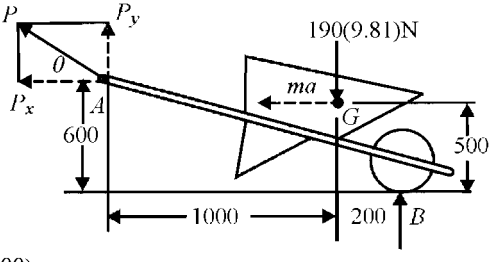
۶-۲۱.

تعادل استاتیکی : $P_x = ma = 0$

$+\circlearrowright \Sigma M_A = 0$:

$$190(9.81)(1000) - B(1200) = 0$$

$$\Rightarrow B_{st} = 1553\,\text{N}$$



دینامیک : $+\circlearrowright \Sigma M_A = mad$:

$$190(9.81)(1.000) - B(1.200) = 190(1.5)(0.100)$$

$$\Rightarrow B = 1530\,\text{N}$$

$$\left.\begin{aligned} \overset{+}{\leftarrow}\ \Sigma F_x = ma &: P_x = 190(1.5) = 285\,\text{N} \\ +\uparrow\ \Sigma F_y = 0 &: B - 190(9.81) + P_y = 0 \Rightarrow P_y = 334\,\text{N} \end{aligned}\right\} \Rightarrow P = 439\,\text{N}$$

$$\theta = \tan^{-1}\frac{334}{285} = 49.6^\circ$$

۶-۲۲.

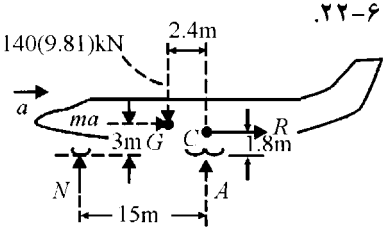
$$v^2 = v_0^2 + 2as$$

$$a = \frac{1}{2(425)}[(200)^2 - (60)^2]\frac{1}{3.6^2} = 3.30\ \text{m/s}^2$$

$\Sigma M_C = mad$:

$$15N - 140(9.81)(2.4) = 140(3.30)(3 - 1.8)$$

$$N = 257\ \text{kN}$$



۶-۲۳.

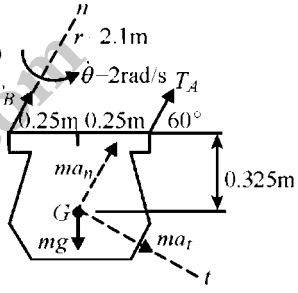
$$\curvearrowleft + \Sigma M_G = 0 : (T_A \sin 60°)(0.25) - (T_B \sin 60°)(0.25)$$

$$- (T_A \cos 60°)(0.325) - (T_B \cos 60°)(0.325) = 0$$

$$\Rightarrow 0.0540\ T_A = 0.379\ T_B \qquad (1)$$

$$\Sigma F_n = ma_n : T_A + T_B - 10(9.81)\sin 60° = 10(2.1)(2^2) \qquad (2)$$

حل (1) و (2) : $T_A = 147.9\ \text{N}$, $T_B = 21.1\ \text{N}$



۶-۲۴.

$$a_E = a_A = a_C$$

$$(a_E)_n = r\omega^2 = 0.8(3)^2 = 7.2\ \text{m/s}^2$$

$$m\,a_n = 8(7.2) = 57.6\ \text{N}$$

$$(a_E)_t = r\alpha = 0.8(6) = 4.8\ \text{m/s}^2$$

$$m\,a_t = 8(4.8) = 38.4\ \text{N}$$

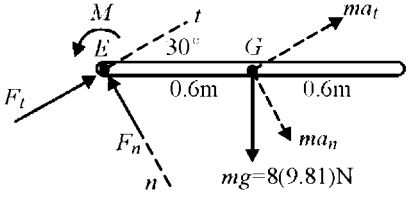
$$\curvearrowleft + \Sigma M_E = \Sigma mad$$

$$M - 8(9.81)(0.6) = 38.4(0.6\sin 30°) - 57.6(0.6\cos 30°) \Rightarrow M = 28.7\ \text{N.m CCW}$$

$$\Sigma F_t = ma_t : F_t - 8(9.81)\sin 30° = 38.4 \Rightarrow F_t = 77.6\ \text{N}$$

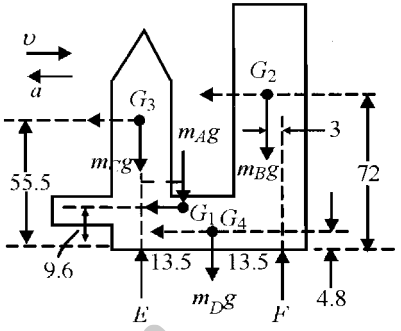
$$\Sigma F_n = ma_n : -F_n + 8(9.81)\cos 30° = 57.6 \Rightarrow F_n = 10.37\ \text{N}$$

$$F = \sqrt{F_t^2 + F_n^2} = 78.3\ \text{N}$$



۶-۲۵. $m_A = 3\,\text{Gg}$, $m_B = 3.5\,\text{Gg}$, $m_C = 0.23\,\text{Gg}$, $m_D = 3\,\text{Gg}$

$$v^2 = 2as \quad \Rightarrow \quad a = \frac{v^2}{2s} = \frac{\left(\frac{1.5}{3.6}\right)^2}{2(0.1)} = 0.868\,\text{m/s}^2$$



$$\Sigma M_E = \Sigma mad$$

$$27F - [3(6) + 3.3(27 - 3) + 3(13.5)]9.81$$

$$= [3(9.6) + 3.3(72) + 0.23(55.5) + 3(4.8)]0.868$$

$$\Rightarrow \quad 27F = 1350.8 + 254.8 \quad \Rightarrow \quad F = 59.5\,\text{MN}$$

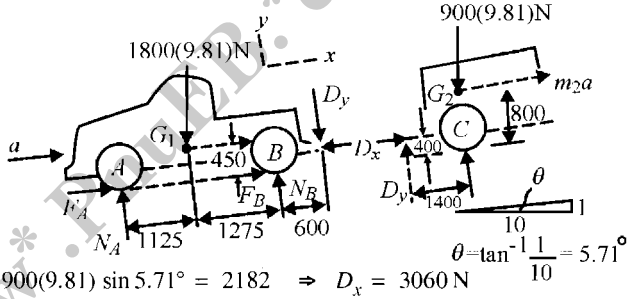
۶-۲۶. برای شتاب ثابت $v^2 = v_0^2 + 2as$

$$\left(\frac{48}{3.6}\right)^2 = \left(\frac{96}{3.6}\right)^2 - 2a(110)$$

$$\Rightarrow \quad a = 2.42\,\text{m/s}^2 \quad \text{شتاب منفی}$$

$$m_1 a = 1800(2.42) = 4364\,\text{N}$$

$$m_2 a = 900(2.42) = 2182\,\text{N}$$



تریلر : $\Sigma F_x = m a_x$: $D_x - 900(9.81)\sin 5.71° = 2182 \quad \Rightarrow \quad D_x = 3060\,\text{N}$

$$\Sigma M_C = mad \quad : \quad 1.4D_y + 3060(0.4) - 900(9.81)\sin 5.71°(0.8) = 2182(0.8)$$

$$\Rightarrow \quad D_y = 874.5\,\text{N}$$

$$\Sigma F_y = 0 \quad : \quad N_C - 900(9.81)\cos 5.71° + 874.5 = 0 \quad \Rightarrow \quad N_C = 7910\,\text{N}$$

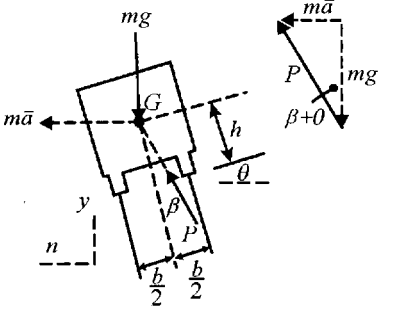
وانت : $\Sigma M_A = mad$: $1800(9.81)\cos 5.71°(1.125) - 1800(9.81)\sin 5.71°(0.450)\,2.4N_B$

$$+ 874.5(3) - 3060(0.4) = 4364(0.450)$$

$$\Rightarrow \quad N_B = 7672\,\text{N}$$

۶-۲۷. برای عدم تمایل به لغزش $\beta = 0$ (الف)

طبق دیاگرام $\tan\theta = \frac{m\bar{a}}{mg} = \frac{\frac{v^2}{r}}{g} \quad \Rightarrow \quad \theta = \tan^{-1}\frac{v^2}{gr}$



(ب) $\tan(\beta + \theta) = \frac{m\bar{a}}{mg} = \frac{\frac{v^2}{r}}{g}$

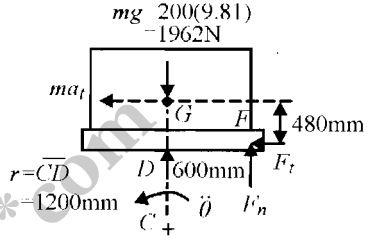
$$v^2 = gr\tan(\beta + \theta) = gr\frac{\tan\beta + \tan\theta}{1 - \tan\beta\tan\theta}$$

اگر $\mu < \dfrac{\frac{b}{2}}{h}$: ابتدا می‌لغزد $\Rightarrow \mu = \tan\beta \Rightarrow v^2 = gr\dfrac{\mu + \tan\theta}{1 - \mu\tan\theta}$

اگر $\mu > \dfrac{\frac{b}{2}}{h}$: ابتدا واژگون می شود $\Rightarrow \tan\beta = \dfrac{b}{2h} \Rightarrow v^2 = gr\dfrac{\frac{b}{2h} + \tan\theta}{1 - \frac{b}{2h}\tan\theta}$

۲۸-۶.

$$\theta = \frac{\pi}{6}\left(1 - \cos\frac{\pi t}{2}\right),\ \dot\theta = \frac{\pi^2}{12}\sin\frac{\pi t}{2},\ \ddot\theta = \frac{\pi^3}{24}\cos\frac{\pi t}{2}$$



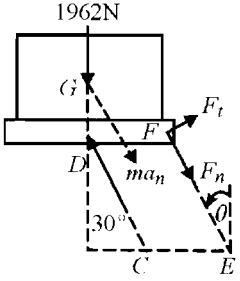
(الف) $\theta = 0,\ t = 0 \Rightarrow \ddot\theta = \dfrac{\pi^3}{24}\ \text{rad/s}^2$

$$ma_t = mr\ddot\theta = 200(1.2)\left(\frac{\pi^3}{24}\right) = 310\ \text{N}$$

$$\Sigma M_F = ma_t d : (1962 - D)0.6 = 310(0.48)$$

$$\Rightarrow D = 1714\ \text{N} \quad (\text{فشاری})$$

(ب) $t = 1\ \text{s} \Rightarrow \theta = \dfrac{\pi}{6},\ \dot\theta = \dfrac{\pi^2}{12}\ \text{rad/s},\ \ddot\theta = 0$



$$ma_n = mr\dot\theta^2 = 200(1.2)\left(\frac{\pi^4}{144}\right) = 162.4\ \text{N}$$

$$\Sigma M_F = ma_n d : 1962(0.6) - (0.6\cos 30°)D$$
$$= 162.4(0.6\cos 30° - 0.48\sin 30°)$$
$$\Rightarrow D = 2178\ \text{N} \quad (\text{فشاری})$$

۲۹-۶.

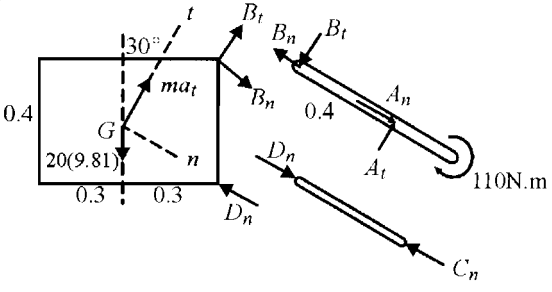
AB : $\Sigma M_A \approx 0$

$$0.400\,B_t - 100 = 0 \Rightarrow B_t = 275\ \text{N}$$

صفحه : $\Sigma F_t = ma_t$

$$275 - 20(9.81)\cos 30° = 20\,a_t$$

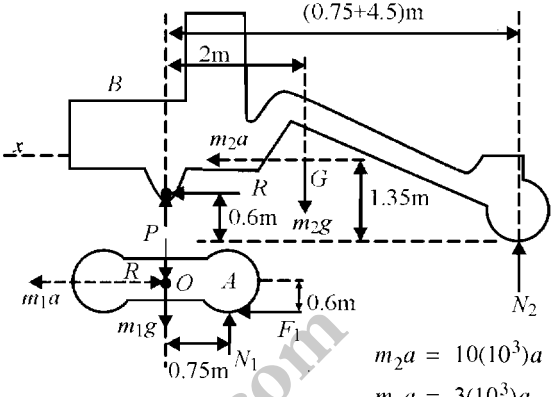
$$\Rightarrow a_t = 5.25\ \text{m/s}^2$$



$\Sigma M_B = mad$

$$D_n(0.4\cos 30°) - 20(9.81)(0.3) = 20(5.25)[0.3\cos 30° - 0.2\sin 30°] \Rightarrow D_n = 218\ \text{N}$$

۶-۳۰. برای جدا شدن چرخ‌های عقب قسمت A از زمین:



Ⓐ $\Sigma M_{N_1} = m_1 a d_1$:

$[P + 3(10^3)(9.81)](0.750)$

$- 0.6R = 3(10^3)a(0.6)$

Ⓑ $\Sigma M_{N_2} = m_2 a d_2$:

$10(10^3)(9.81)(4.5 + 0.75 - 2)$

$- P(4.5 + 0.75) + 0.6R$

$= 10(10^3)a(1.35)$

$\Sigma F_x = m a_x$: $R = 10(10^3)a$

$m_2a = 10(10^3)a$

$m_1a = 3(10^3)a$

حل سه معادله قبل : $R = 76.2\,\text{kN}$, $P = 49.8\,\text{kN}$, $a = 7.62\,\text{m/s}^2$

برای شتاب ثابت : $S = \dfrac{v^2}{2a} = \dfrac{\left(\frac{40}{3.6}\right)^2}{2(7.62)} = 8.10\,\text{m}$

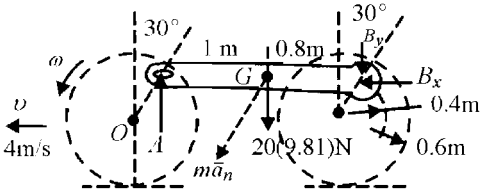
۶-۳۱.

$\omega = \dfrac{v}{r} = \dfrac{4}{0.6} = 6.67\,\text{rad/s}$

$\alpha = 0$, $\bar{a}_b = 0$, $a_O = 0$

$\bar{a}_n = \dfrac{a_A}{a_O} = r\omega^2 = 0.4(6.67)^2 = 17.78\,\text{m/s}^2$

$m\bar{a}_n = 20(17.78) = 356\,\text{N}$

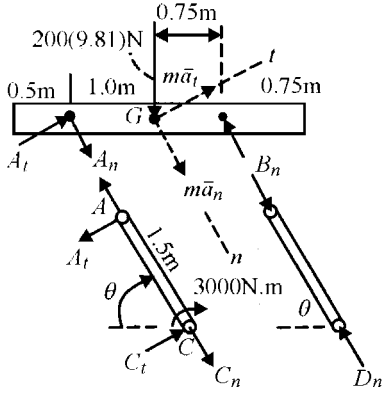


$\Sigma M_A = m\bar{a}d$: $1.8B_y + 20(9.81)(1.0) = 356\cos 30°(1.0) \Rightarrow B_y = 62.1\,\text{N}$

$\Sigma F_x = m a_x$: $B_x = 356\sin 30° = 177.8\,\text{N}$

$B = \sqrt{B_x^2 + B_y^2} = 188.3\,\text{N}$

۶-۳۲.



CA میله : $\Sigma M_A \approx 0$

$1.5A_t - 3000 = 0 \Rightarrow A_t = 2000\,\text{N}$

تیر : $\Sigma F_t = m\bar{a}_t$

$2000 - 200(9.81)\cos\theta = 200\,\bar{a}_t$

$\bar{a}_t = 10 - 9.81\cos\theta$

$\int_0^v v\,dv = \int_0^{\frac{\pi}{3}} a_t\,r\,d\theta \Rightarrow$

$$\frac{v^2}{2} = 1.5\left(10\theta - 9.81\sin\theta\right)_0^{\frac{\pi}{3}} \Rightarrow v^2 = 3\left(\frac{10\pi}{3} - 9.81\frac{\sqrt{3}}{2}\right) \Rightarrow v = 2.43\,\text{m/s}$$

$$m\,\overline{a}_n = m\,\frac{v^2}{r} = 200\,\frac{(2.43)^2}{1.5} = 791\,\text{N}$$

$$m\,\overline{a}_t = 200(10 - 9.81\cos 60°) = 1019\,\text{N}$$

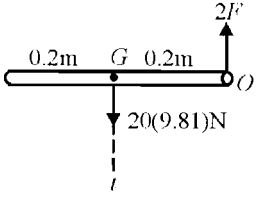
$$\left.\begin{aligned}\Sigma M_G = 0 &: \ 2000(1.0)\cos 60° - A_n(1.0)\sin 60° - B_n(0.75)\sin 60° = 0\\ \Sigma F_n = m\,\overline{a}_n &: \ 200(9.81)\sin 60° + A_n - B_n = 791\end{aligned}\right\} \Rightarrow \begin{aligned}A_n &= 270\,\text{N}\\ B_n &= 1179\,\text{N}\end{aligned}$$

$$A = \sqrt{(270)^2 + (2000)^2} = 2018\,\text{N} \quad \text{یا} \quad A = 2.02\,\text{kN}$$

۳۳-۶.

$$\Sigma M_O = I_O\alpha : \ 20(9.81)(0.2) = \frac{1}{3}20(0.4)^2\alpha$$

$$\alpha = 36.8\,\text{rad/s}^2$$



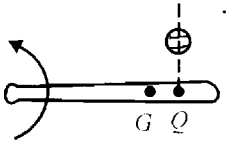
$$\overline{a} = \overline{r}\alpha = 0.2(36.8) = 7.36\,\text{m/s}^2$$

$$\Sigma F_t = m\,\overline{a}_t : \ 20(9.81) - 2F = 20(7.36)$$

$$\Rightarrow 2F = 49.0 \Rightarrow F_A = F_B = F = 24.5\,\text{N}$$

۳۴-۶.

با توجه به اینکه چوب حول دستهاش دروان می‌کند، نیروی وارد بر دسته موقعی حداقل است که برخورد در مرکز ضربه (Q) اتفاق بیفتد.



۳۵-۶. نیروی شتاب دهنده روی چرخهای عقب : $F = ma = 2800(0.6\,g) = 16481\,\text{N}$

$$\alpha \text{ استوانه} = \frac{a_t}{r} = \frac{0.6(9.81)}{0.9} = 6.54\,\text{rad/s}^2$$

$$\Sigma M_O = I_O\alpha \Rightarrow I_O = \frac{16481(0.9)}{6.54} = 2268\,\text{kg.m}^2 = 2.27\,\text{Mg.m}^2$$

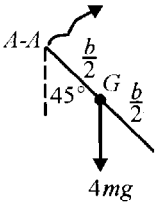
۳۶-۶.

$$I_{A-A} = 2\left(\frac{1}{3}m\,b^2\right) + m\,b^2 = \frac{5}{3}m\,b^2 \quad (\text{جرم هر سمت} = m)$$

$$I_{B-B} = \frac{1}{2}m\,b^2 + 2\left[\frac{1}{12}m\,b^2 + m\left(\left[\frac{b}{2}\right]^2 + \left[\frac{b}{2}\right]^2\right)\right] + \left[\frac{1}{12}mb^2 + mb^2\right] = \frac{7}{3}m\,b^2$$

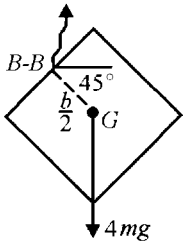
$$\circlearrowright + \Sigma M_{A-A} = I_{A-A}\,\alpha$$

$$4mg\frac{b}{2}\frac{\sqrt{2}}{2} = \frac{5}{3}mb^2\alpha \quad \Rightarrow \quad \alpha = \frac{3\sqrt{2}}{5}\frac{g}{b}$$



$$\circlearrowright + \Sigma M_{B-B} = I_{B-B}\,\alpha$$

$$4mg\frac{b}{2}\frac{\sqrt{2}}{2} = \frac{7}{3}mb^2 \quad \Rightarrow \quad \alpha = \frac{3\sqrt{2}}{7}\frac{g}{b}$$



۶-۳۷.

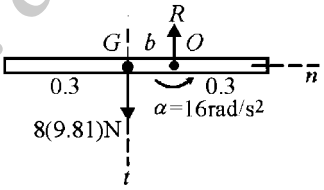
$$I_O = \frac{1}{12}mL^2 + mb^2 = 8\left(\frac{0.6^2}{12} + b^2\right) = 0.24 + 8b^2$$

$$\Sigma M_O = I_O\alpha \;:\; 8(9.81)\,b = (0.24 + 8b^2)16$$

$$\Rightarrow \quad 16b^2 - 9.81b + 0.48 = 0$$

$$\Rightarrow \quad b = 0.0536\,\text{m} \quad \text{یا} \quad b = 53.6\,\text{mm}$$

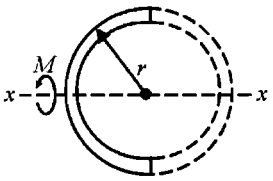
$$\Sigma F_t = m\bar{r}\alpha \;:\; 8(9.81) - R = 8(0.0536)(16) \quad \Rightarrow \quad R = 71.6\,\text{N}$$



۶-۳۸.

برای حلقه کامل به جرم $2m$ ممان اینرسی حول قطر $x-x$ برابر است با $\frac{1}{2}(2mr^2)$. بنابراین ممان اینرسی نیم حلقه حول $x-x$ برابر است با $\frac{1}{2}mr^2$

$$\Sigma M_x = I_x\alpha \;:\; M = \frac{1}{2}mr^2\alpha \quad \Rightarrow \quad \alpha = \frac{2M}{mr^2}$$



۶-۳۹.

$$\alpha = \frac{\omega}{\tau} \qquad (H = \text{توپی}\ ,\ B = \text{زائده‌ها})$$

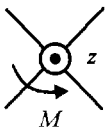
$$I_{zz} = \frac{1}{2}m_H r^2 + 4\left[\frac{1}{12}m_B l^2 + m_B\left(r + \frac{l}{2}\right)^2\right]$$

$$= \frac{1}{2}(\rho\pi r^2 d)r^2 + 4(\rho l d t)\left[\frac{1}{12}l^2 + r^2 + rl + \frac{l^2}{4}\right]$$

$$= \frac{1}{2}\rho\pi d r^4 + 4\rho l d t\left[\frac{1}{3}l^2 + rl + r^2\right]$$

$$= \rho d\left[\frac{1}{2}\pi r^4 + 4lt + \left(\frac{1}{3}l^2 + rl + r^2\right)\right]$$

$$\Sigma M_z = I_{zz}\alpha \;:\; M = \frac{\omega\rho d}{\tau}\left[\frac{1}{2}\pi r^4 + 4lt\left(\frac{1}{3}l^2 + rl + r^2\right)\right]$$



۶-۴۰.

$I_O = I_G + md_2$

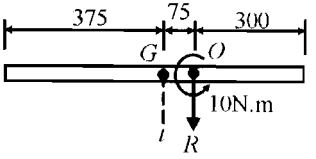
$I_G = \frac{1}{12} ml^2 = \frac{1}{12} 9(0.75)^2 = 0.422\,\text{kg.m}^2$

$I_O = 0.422 + 9(0.075)^2 = 0.4725\,\text{kg.m}^2$

$\Sigma M_O = I_O \alpha \;:\; 10 = 0.4725\alpha \;\Rightarrow\; \alpha = 21.16\,\text{rad/s}^2$

$\bar{a}_t = \bar{r}\alpha \;:\; \bar{a}_t = 0.075(21.16) = 1.587\,\text{m/s}^2$

$\Sigma F_t = m\bar{a}_t \;\Rightarrow\; R = 9(1.587) = 14.29\,\text{N}$



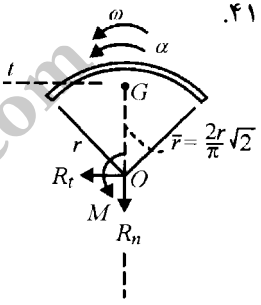
۶-۴۱.

$\Sigma M_O = I_O \alpha \;:\; M = mr^2\alpha$

$\Sigma F_t = m\bar{a}_t \;:\; R_t = m\bar{r}\alpha \;\Rightarrow\; R_t = \frac{2\sqrt{2}\, mr\alpha}{\pi}$

$\Sigma F_n = m\bar{a}_n \;:\; R_n = m\bar{r}\omega^2 = \frac{2\sqrt{2}\, mr\omega^2}{\pi}$

$R = \sqrt{R_t^2 + R_n^2} = \frac{2\sqrt{2}\, mr}{\pi}\sqrt{\alpha^2 + \omega^2}$



۶-۴۲.

برای میله باریک :

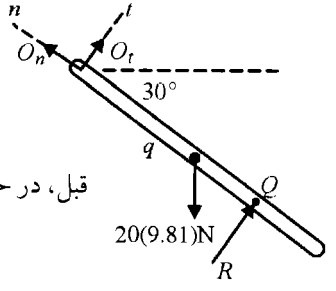
$q = \frac{k_e^2}{\bar{r}} = \frac{\frac{1}{3}L^2}{\frac{L}{2}} = \frac{2}{3}(1.8) = 1.2\,\text{m}$

دوران با محورهای ثابت :

$\Sigma M_Q = 0$ (برای تمام لحظات) قبل، در حین و بلافاصله پس از برخورد

$20(9.81)(0.3)\cos 30° - 1.2\,O_t = 0 \;\Rightarrow\; O_t = 42.5\,\text{N}$

(برای تمام حالات)



۶-۴۳.

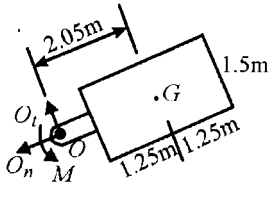
طبق جدول د-۴ : $I_G = \frac{1}{12}(6000)[(1.5)^2 + (2.5)^2] = 4250\,\text{kg.m}^2$

$I_O = I_G + md^2 = 4250 + 6000(2.05)^2 = 29465\,\text{kg.m}^2$

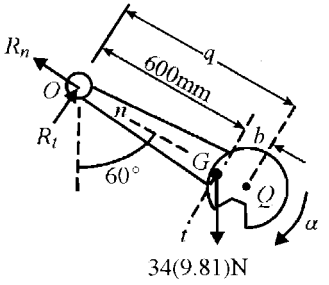
$\Sigma M_O = I_O \alpha \;:\; 30 = 29465\alpha \;\Rightarrow\; \alpha = 1.018(10^{-3})\,\text{rad/s}^2$

$\theta = \frac{1}{2}\alpha t^2 \;:\; \frac{\pi}{4} = \frac{1}{2}1.018(10^{-3})\,t_1^2 \;\Rightarrow\; t_1 = 39.28\,\text{s}$

کل زمان $t = 2t_1 = 78.6\,\text{s}$



۶-۴۴.



در موقعیت عمودی $R_t = 0$

اگر برخورد در مرکز محل ضربه رخ دهد : $q = \dfrac{k_O^2}{\bar{r}}$

$$\Rightarrow \quad 0.600 + b = \frac{(0.620)^2}{0.600}$$

$$\Rightarrow \quad b = 0.0407\,\text{m} \quad \text{یا} \quad b = 40.7\,\text{mm}$$

$$\Sigma M_Q = 0 \; : \; 34(9.81)(0.0407\sin 60°) - (0.6407)R_t = 0$$

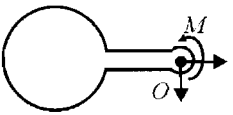
$$R_t = 18.35\,\text{N}$$

$$\Sigma F_n = m\bar{r}\omega^2 = 0$$

$$R_n - 34(9.81)\cos 60° = 0 \quad \Rightarrow \quad R_n = 166.8\,\text{N}$$

$$R = \sqrt{(166.8)^2 + (18.35)^2} = 167.8\,\text{N}$$

۶-۴۵.

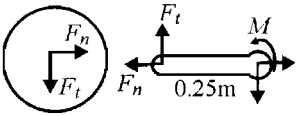


(الف) $\Sigma M_O = I_O\alpha$

$$30 = 43\left(\frac{1}{2}[0.2]^2 + [0.25]^2\right)\alpha$$

$$\alpha = 8.46\,\text{rad/s}^2$$

(ب) قاب : $F_t \approx \dfrac{M}{r} = \dfrac{30}{0.25} = 120\,\text{N}$



استوانه : $\Sigma F_t = m a_t$

$$120 = 43(0.25\alpha)$$

$$\alpha = 11.16\,\text{rad/s}^2$$

۶-۴۶.

$\curvearrowright + \Sigma M_A = I_A\alpha_A$:

$$12 - F(0.24) = 20(0.15)^2\alpha_A \quad (1)$$

$\curvearrowleft + \Sigma M_B = I_B\alpha_B$:

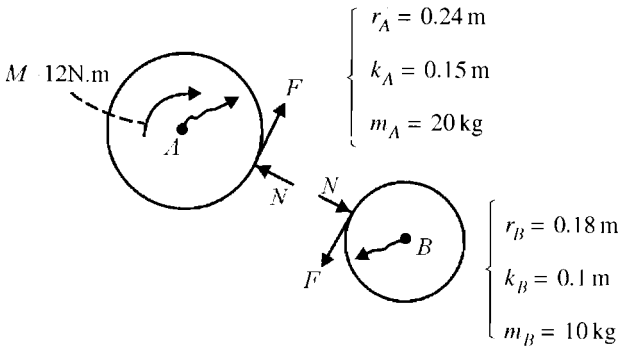
$$F(0.18) = 10(0.1)^2\alpha_B \quad (2)$$

یکسان بودن شتاب مماسی :

$$r_A\alpha_A = r_B\alpha_B \quad \Rightarrow \quad 0.24\alpha_A = 0.18\alpha_B \quad (3)$$

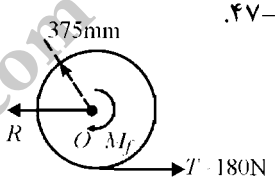
حل (1) تا (3) : $F = 14.16\,\text{N}$, $\alpha_A = 19.12\,\text{rad/s}^2$ CW

$$\alpha_B = 25.5\,\text{rad/s}^2\,\text{CCW}$$



۶-۴۷.

$M_f = 90(0.375) = 33.75\,N.m$



$(I_O)_{\text{قرقره}} = m k^2 = 28(0.3)^2 = 2.52\,kg.m^2$

$(I_O)_{\text{کابل}} = m r^2 = 60(0.65)(0.375)^2 = 5.48\,kg.m^2$

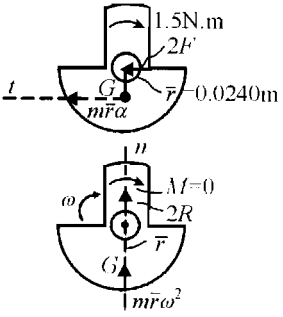
I_O کل $= 2.52 + 5.48 = 8\,kg.m^2$

$\Sigma M_O = I_O \alpha$: $180(0.375) - 33.75 = 8\alpha \Rightarrow \alpha = 4.22\,rad/s^2$

۶-۴۸.

$\omega^2 = \omega_0^2 + 2\bar{\alpha}\theta$:

$\left(\frac{1200 \times 2\pi}{60}\right)^2 = 0 + 2\bar{\alpha}(18 \times 2\pi) \Rightarrow \bar{\alpha} = 69.8\,rad/s^2$



تست استاتیک : $\Sigma M = 0$:

$0.660 - 2.8(9.81)\bar{r} = 0 \Rightarrow \bar{r} = 0.0240\,m$

(الف) $\Sigma M = I\alpha$:

$1.5 = 2.8k^2 \times 69.8 \Rightarrow k = 0.0876\,m$ یا $k = 87.6\,mm$

(ب) $\Sigma F_t = m\bar{r}\alpha$:

$2F = 2.8(0.0240)69.8 \Rightarrow F = 2.35\,N$

(ج) $\Sigma F_n = m\bar{r}\omega^2$:

$2R = 2.8(0.0240)(125.7^2) \Rightarrow R = 531\,N$

$\omega = \frac{1200 \times 2\pi}{60} = 125.7\,rad/s$

۶-۴۹.

$$\Sigma M_O = I_O \alpha$$

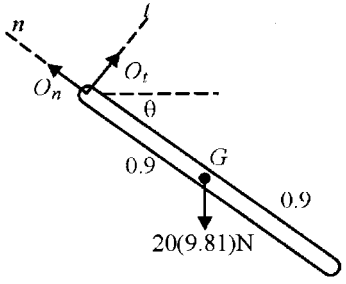
$$20(9.81)(0.9)\cos\theta = \frac{1}{3}20(1.8)^2\alpha \Rightarrow \alpha = 8.175\cos\theta \text{ rad/s}^2$$

$$\int_0^{\omega} \omega\, d\omega = \int_0^{\theta} \alpha\, d\theta : \frac{\omega^2}{2} = 8.175\sin\theta$$

$$\omega^2_{\theta=30} = 2(8.175)0.5 = 8.175 \text{ (rad/s}^2\text{)}$$

$$\Sigma F_n = m\,\bar{a}_n :$$

$$O_n - 20(9.81)\sin 30° = 20(0.9)(8.175) \Rightarrow O_n = 245 \text{ N}$$



۶-۵۰.

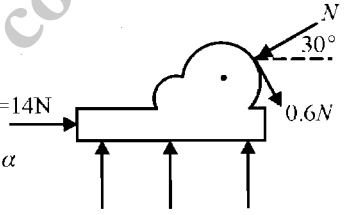
مجموعهٔ C :

$$\xrightarrow{+} \Sigma F = 0 : 14 - N\cos 30° + 0.6N\sin 30° = 0$$

$$N = 24.7 \text{ N}$$

چرخ B : $\curvearrowright_{+} \Sigma M_B = I_B\alpha : 0.6(24.7)(0.25) = 22(0.2)^2\alpha$

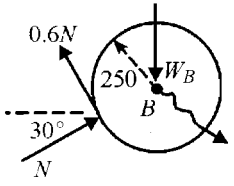
$$\alpha = 4.22 \text{ rad/s}^2$$



سرعت یکنواخت : $r_A\omega_A = r_B\omega_B$

$$\omega_B = \frac{r_A\omega_A}{r_B} = \frac{200\left[1600\frac{2\pi}{60}\right]}{250} = 134 \text{ rad/s}$$

$$\omega_B = \omega_{\cancel{0}}^{0} + \alpha t : t = \frac{\omega_B}{\alpha} = \frac{134}{4.22} = 31.8 \text{ s}$$



۶-۵۱.

$$I_B = \frac{1}{3}4(0.350)^2 = 0.1633 \text{ kg.m}^2$$

$$I_A = \frac{1}{3}(25)(0.7)^2 + \left[\frac{1}{12}4(0.350)^2 + 4(0.7^2 + 0.175^2)\right]$$

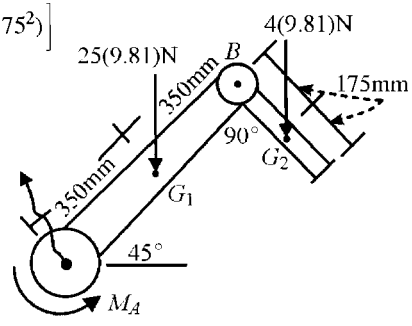
$$= 6.21 \text{ kg.m}^2$$

$$\curvearrowleft_{+} \Sigma M_A = I_A\alpha :$$

$$M_A - 25(9.81)(0.350\cos 45°)$$

$$- 4(9.81)(0.700 + 0.175)\cos 45° = 6.21(4)$$

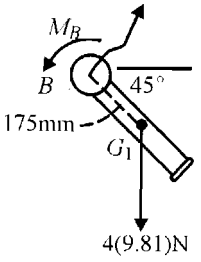
$$M_A = 109.8 \text{ N.m}$$



$\curvearrowleft + \Sigma M_B = I_B \alpha$

$M_B - 4(9.81)(0.175\cos 45°) = 0.1633(4)$

$M_B = 5.51\ \text{N.m}$



۶-۵۲.

$I_A = \frac{1}{2}(2mr^2 + 2mr^2) = 2mr^2$

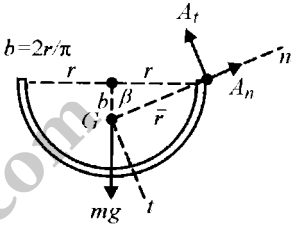
$\Sigma M_A = I_A \alpha \ : \ mgr = 2mr^2\alpha \ \Rightarrow \ \alpha = \frac{3}{2r}$

$\Sigma F_n = m\bar{a}_n = 0 \ : \ A_n = mg\sin\beta$

$\Sigma F_t = m\bar{a}_t \ : \ mg\cos\beta - A_t = m\bar{r}\alpha$

یا $A_n = mg\frac{b}{\bar{r}}$, $A_t = mg\left(\frac{r}{\bar{r}} - \frac{\bar{r}}{2r}\right)$

$\Rightarrow A = mg\sqrt{\frac{b^2}{\bar{r}^2} + \frac{r^2}{\bar{r}^2} - 1 + \frac{\bar{r}^2}{4r^2}} = mg\frac{\bar{r}}{2r} = \frac{mg}{2}\sqrt{1 + \frac{4}{\pi^2}}$ یا $A = 0.593mg$



۶-۵۳.

برای کل مجموعه $I_{zz} = 0.60 + (0.080 + 12(0.2)^2) = 1.160\ \text{kg.m}^2$

$\Sigma M_z = I_{zz}\alpha \ : \ 16 = 1.160\alpha \ \Rightarrow \ \alpha = 13.79\ \text{rad/s}^2$

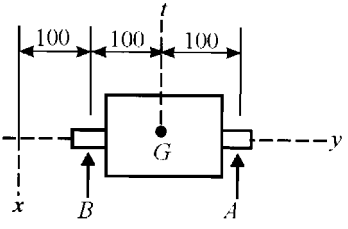
برای استوانه : $\Sigma F_t = m a_t$:

$A + B = 12(0.2)(13.79) = 33.1\ \text{N}$

$\Sigma M_O = I_{zz}\alpha$:

$0.3A + 0.1B = [0.080 + 12(0.2)^2]13.79$

حل همزمان : $A = 22.1\ \text{N}$, $B = 11.03\ \text{N}$



۶-۵۴.

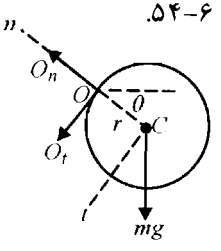
$\Sigma M_O = I_O\alpha \ : \ mgr\cos\theta = 2mr^2\alpha$

$\alpha = \frac{g}{2r}\cos\theta$

$\omega\, d\omega = \alpha\, d\theta \ : \ \int_0^\omega \omega\, d\omega = \frac{g}{2r}\int_0^\theta \cos\theta\, d\theta \ \Rightarrow \ \omega^2 = \frac{g}{r}\sin\theta$

$\Sigma F_t = m\bar{r}\alpha \ : \ mg\cos\theta + O_t = mr\left(\frac{g}{2r}\cos\theta\right) \ \Rightarrow \ O_t = -\frac{mg}{2}\cos\theta$

$\Sigma F_n = m\bar{r}\omega^2 \ : \ O_n - mg\sin\theta = mr\left(\frac{g}{r}\sin\theta\right) \ \Rightarrow \ O_n = 2mg\sin\theta$



.۵۵-۶

$$\Sigma M_O = I_O\alpha \ : \ 8(9.81)(0.450\cos 30°) = \frac{1}{3}8(0.900)^2\alpha$$

$$\alpha = 14.16\ \text{rad/s}^2$$

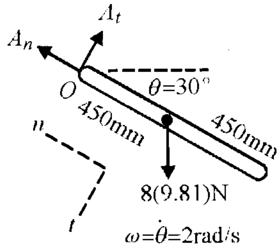
$$\Sigma F_t = m\bar{r}\alpha \ : \ 8(9.81)\cos 30° - A_t = 8(0.450)(14.16)$$

$$A_t = 16.99\ \text{N}$$

$$\Sigma F_n = m\bar{r}\omega^2 \ : \ A_n - 8(9.81)\sin 30° = 8(0.450)2^2$$

$$A_n = 53.64\ \text{N}$$

$$A = \sqrt{(16.99)^2 + (53.64)^2} = 56.3\ \text{N}$$



.۵۶-۶

طبق جدول د-۴ : $I_G = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$

$$= \frac{1}{4}100\left(0.125^2 + \frac{1}{3}0.3^2\right)$$

$$= 1.141\ \text{kg.m}^2$$

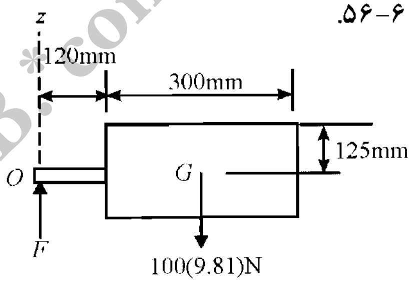
$$I_O = I_G + md_2 = 1.141 + 100(0.120 + 0.150)^2$$

$$= 8.43\ \text{kg.m}^2$$

$$\Sigma M_O = I_O\alpha \ : \ 981(0.120 + 0.150) = 8.43\alpha$$

$$\alpha = 31.4\ \text{rad/s}$$

$$\Sigma F_z = m\bar{a}_z \ : \ F - 981 = 100(-0.27)(31.4) \quad \Rightarrow \quad F = 132.7\ \text{N}$$



.۵۷-۶

$$d = \sqrt{b^2 + \left(\frac{b}{2}\right)^2} = \frac{b\sqrt{5}}{2}$$

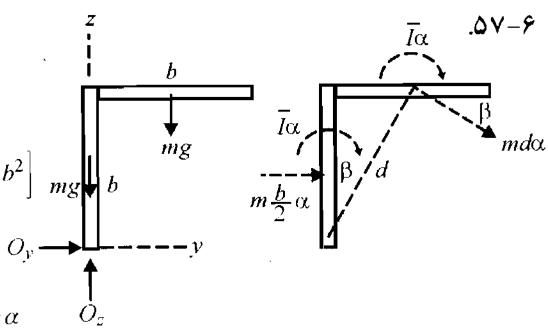
$$m = \rho bc$$

$$I_O = \frac{1}{3}mb^2 + \frac{1}{12}mb^2 + m\left[\left(\frac{b}{2}\right)^2 + b^2\right]$$

$$= \frac{5}{3}mb^2 = \frac{5}{3}\rho b^3 c$$

$$\Sigma M_O = I_O\alpha \ : \ g\rho bc\left(\frac{b}{2}\right) = \frac{5}{3}\rho b^3 c\alpha$$

$$\Rightarrow \quad \alpha = \frac{3g}{10b}$$



برای هر ورق : $\bar{I} = \frac{1}{12}mb^2 = \frac{1}{12}\rho b^3 c$

$$\cos\beta = \frac{b}{\frac{b}{2}\sqrt{5}} = \frac{2}{\sqrt{5}} \quad , \quad \sin\beta = \frac{\frac{b}{2}}{\frac{b}{2}\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$\Sigma F_y = m\,\bar{a}_y \;:\; O_y = m\frac{b}{2}\alpha + m\,\frac{b}{2}\sqrt{5}\,\alpha\,\frac{2}{\sqrt{5}} = \frac{3}{2}\,m\,b\,\alpha = \frac{9}{20}\rho\,b\,c\,g$$

$$\Sigma F_z = \Sigma m\,\bar{a}_z \;:\; O_z - 2\,m\,g = -\,m\,d\,\alpha\sin\beta$$

$$\Rightarrow \quad O_z = 2\rho\,g\,b\,c - \rho\,b\,c\left(\frac{b}{2}\sqrt{5}\right)\frac{3\,g}{10\,b}\,\frac{1}{\sqrt{5}} = \frac{37}{20}\,\rho\,b\,c\,g$$

۶-۵۸.

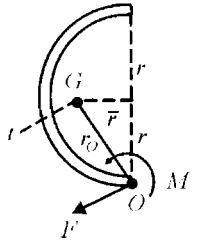
برای حلقه کامل $I_O = 2(2\,m)\,r^2$

برای حلقه نیم دایره‌ای $I_O = 2\,m\,r^2$

$$\Sigma M_O = I_O\alpha \;:\; M = 2\,m\,r^2\alpha \;\Rightarrow\; \alpha = \frac{M}{2\,m\,r^2}$$

$$\Sigma F_t = m\,\bar{a}_t \;:\; F = m\,r_O\alpha = m\sqrt{r^2 + \bar{r}^2}\;\frac{M}{2\,m\,r^2}$$

$$= \frac{M}{r}\sqrt{\frac{1}{4} + \frac{1}{\pi^2}} = 0.593\;M/r$$



۶-۵۹.

(الف) $\Sigma M_A = m\,\bar{a}\,d$:

$$M = m\,\bar{r}\omega^2\,\frac{l}{2}\cos\theta$$

$$= m\,\frac{r}{\cos\theta}\,\omega^2\,\frac{l}{2}\cos\theta = \frac{m\,r\,l\,\omega^2}{2}$$

$$\Sigma F_n = m\,\bar{a}_n \;:$$

$$V = m\,\bar{r}\omega^2\cos\theta = m\,\frac{r}{\cos\theta}\,\omega^2\cos\theta = m\,r\omega^2$$

$$\Sigma F_t = m\,\bar{a}_t \;:$$

$$T = m\,\bar{r}\omega^2\sin\theta = \frac{m\,l\,\omega^2}{2}$$

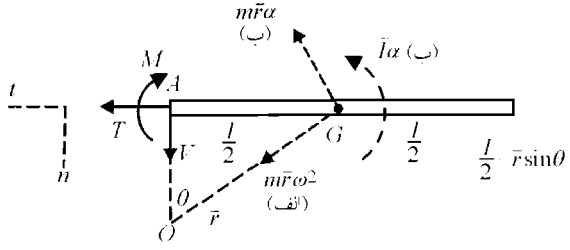
(ب) $\Sigma M_A = m\,\bar{a}\,d$:

$$M = -\,m\,\bar{r}\,\alpha\,\frac{l}{2}\sin\theta - \bar{I}\alpha$$

$$\Rightarrow \quad M = -\,m\,\alpha\,\frac{l^2}{4} - \frac{1}{12}\,m\,l^2\,\alpha = -\,m\,l^2\,\frac{\alpha}{3}$$

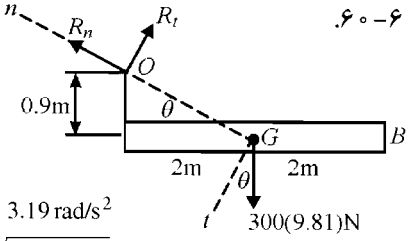
$$\Sigma F_n = m\,\bar{a}_n \;:\; V = -\,m\,\bar{r}\alpha\sin\theta = -\,\frac{m\,l\,\alpha}{2}$$

$$\Sigma F_t = m\,\bar{a}_t \;:\; T = m\,\bar{r}\alpha\cos\theta = m\,r\alpha$$



۶-۶۰.

$$\theta = \tan^{-1}\frac{0.9}{2} = 24.2°$$



$$I_O = I_G + mr^2 = \frac{1}{12}(300)(4^2) + 300(2^2 + 0.9^2)$$

$$= 1843\ \text{kg.m}^2$$

$$\curvearrowleft \Sigma M_O = I_O\alpha \ : \ 300(9.81)(2) = 1843\alpha \ \Rightarrow \ \alpha = 3.19\ \text{rad/s}^2$$

$$\Sigma F_t = m\bar{r}\alpha \ : \ 300(9.81)\cos 24.2° - R_t = 300\,(\sqrt{2^2 + 0.9^2}\,)3.19 \ \Rightarrow \ R_t = 582\ \text{N}$$

$$\Sigma F_n = m\bar{r}\omega^2 \ : \ R_n - 300(9.81)\sin 24.2° = 300(0) \ \Rightarrow \ R_n = 1208\ \text{N}$$

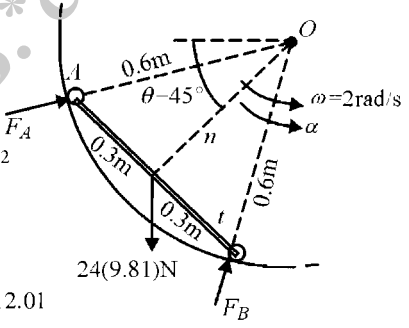
$$R = \sqrt{528^2 + 1208^2} = 1341\ \text{N}$$

۶-۶۱.

$$I_O = \bar{I} + md^2 = \frac{1}{12}(24)(0.6)^2 + 24(0.6\cos 30°)^2 = 7.2\ \text{kg.m}^2$$

$$\curvearrowright \Sigma M_O = I_O\alpha \ : \ 24(9.81)\left(0.6\,\frac{\sqrt{3}}{2}\cos 45°\right) = 7.2\alpha$$

$$\alpha = 12.01\ \text{rad/s}^2$$



$$\Sigma F_n = m\bar{a}_n = m\bar{r}\omega^2 \ :$$

$$(F_A + F_B)\cos 30° - 24(9.81)\cos 45° = 24(0.6\cos 30°)\,2^2$$

$$F_A + F_B = 250\ \text{N}$$

$$\Sigma F_t = m\bar{a}_t = m\bar{r}\alpha \ :$$

$$(F_A - F_B)\cos 60° + 24(9.81)\cos 45° = 24(0.6\cos 30°)\,12.01$$

$$F_A - F_B = -33.3\ \text{N}$$

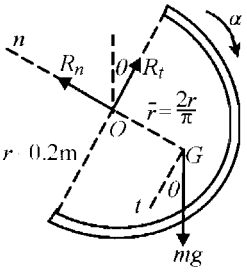
$$\Rightarrow \ F_A = 108.3\ \text{N} \quad , \quad F_B = 141.6\ \text{N}$$

۶-۶۲.

$$\Sigma M_O = I_O\alpha \ : \ m g\bar{r}\cos\theta = m r^2\alpha$$

$$\Rightarrow \ 24.5(0.1273)\cos 30° = 0.1\,\alpha$$

$$\alpha = \frac{2.70}{0.1} = 27.0\ \text{rad/s}^2$$



$$\Sigma F_t = m\bar{r}\alpha \ : \ mg\cos\theta - R_t = m\bar{r}\alpha$$

$$\Rightarrow \ R_t = 24.5\cos 30° - 2.5(0.1273)(27.0)$$

$$R_t = 12.63\ \text{N}$$

$$\Sigma F_n = m\bar{r}\omega^2 \ : \ R_n - mg\sin\theta = 0$$

$$\Rightarrow \ R_n = 24.5\sin 30° = 12.26\ \text{N}$$

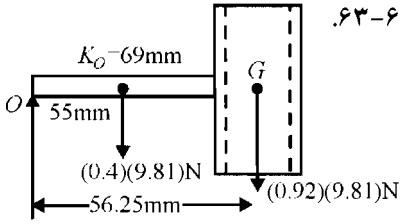
$$R = \sqrt{R_n^2 + R_t^2} = \sqrt{12.26^2 + 12.3^2} = 17.60\ \text{N}$$

$$mg = 2.5(9.81) = 24.5\ \text{N}$$

$$\bar{r} = \frac{2(0.2)}{\pi} = 0.1273\ \text{m}$$

$$I_O = m r^2 = 2.5(0.2)^2 = 0.1\ \text{kg.m}^2$$

۶-۶۳.



طبق پیوست د برای لوله :

$$I_G = \frac{1}{2}m\left(r^2 + \frac{l^2}{6}\right)$$

$$= \frac{1}{2}(0.92)\left[\left(\frac{62.5}{2}\right)^2 + \frac{100^2}{6}\right]$$

$$= 1215.9\,\text{kg.mm}^2$$

$$I_O = I_G + m\bar{r}^2 = 1215.9 + 0.92(156.25)^2 = 23676.8\,\text{kg.mm}^2$$

میله : $I_O = 0.4(69)^2 = 1904.4\,\text{kg.mm}^2$

$\Sigma M_O = I_O\alpha$:

$$0.92(9.81)(10^3)(156.25) + 0.4(9.81)(10^3)(55) = (23676.8 + 1904.4)\alpha \Rightarrow \alpha = 63.6\,\text{rad/s}^2$$

$\Sigma F_t = m\bar{a}_t$: $0.92(9.81) + 0.4(9.81) - O = [0.92(156.25\times10^{-3}) + 0.4(55\times10^{-3})]\,63.6$

$$\Rightarrow O = 2.41\,\text{N}$$

۶-۶۴.

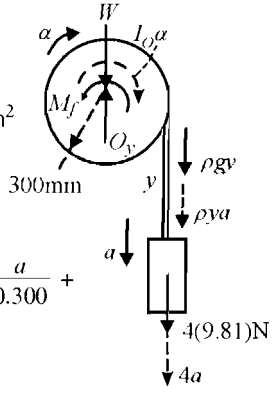
$\rho = 0.16\,\text{kg/m}$

$M_f = 4(9.81)(0.3) = 11.772\,\text{N.m}$

$(I_O)_{\text{قرقره}} = m k_O^2 = 160(0.200)^2 = 0.64\,\text{kg.m}^2$

$(I_O)_{\text{کابل}} = m\bar{r}^2 = (60 - y)(0.16)(0.300)^2 = 0.0144(60 - y)\,\text{kg.m}^2$

$\rho g y = 0.16(9.81)\,y = 1.5696\,y$



$\Sigma M_O = I_O\alpha + \Sigma m a r$ برای کل سیستم

$$[1.5696\,y + 4(9.81)]0.300 - 11.772 = [0.64 + 0.0144(60 - y)]\frac{a}{0.300} +$$

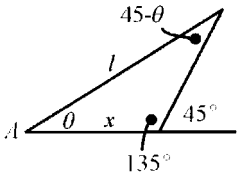
$$+ [0.16\,y + 4]\,a\,(0.300) \Rightarrow a = 0.0758\,y$$

۶-۶۵.

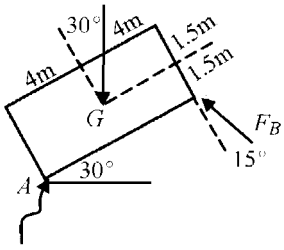
$$\frac{\sin(45^\circ - \theta)}{x} = \frac{\sin 135^\circ}{l} \Rightarrow \underbrace{-\dot{\theta}\cos(45^\circ - \theta)}_{0} = \dot{x}\frac{\sin 135^\circ}{l}$$

$$\Rightarrow -\ddot{\theta}\cos(45^\circ - \theta) + \dot{\theta}^2\sin(45^\circ - \theta) = \ddot{x}\frac{\sin 135^\circ}{l}$$

$$\Rightarrow \ddot{\theta} = -\frac{\ddot{x}}{l}\frac{\sin 135^\circ}{\cos(45^\circ - \theta)}$$



$l = 8\,\text{m}$, $\theta = 35°$, $\ddot{x} = 3\,\text{m/s}^2 \Rightarrow \ddot{\theta} = -0.275\,\text{rad/s}^2$ (CW)



۴-۴ طبق جدول د : $I_A = \frac{1}{3} m(b^2 + l^2)$

$= \frac{1}{3}(120)(b^3)[3^2 + 8^2] = 2920(10^3)\,\text{kg.m}^3$

$+\circlearrowright \Sigma M_A = I\alpha$: $1177(10^3)\cos 30°\,(4) - 1177(10^3)\sin 30°(1.5)$

$- 8F_B \cos 15° = 2920(10^3)(0.275)$

$F_B = 310.000\,\text{N}$ یا $310\,\text{kN}$

۶۶-۶.

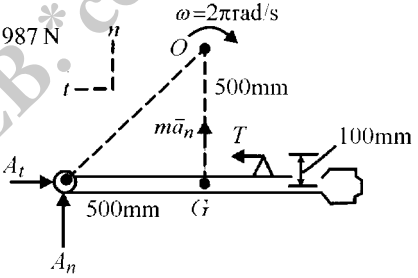
$\Sigma F_n = m\bar{r}\omega^2$: $A_n = 10(0.500)(2\pi)^2 = 197.4\,\text{N}$

$\Sigma M_A = m\bar{a}d$: $0.100\,T = 197.4(0.500) \Rightarrow T = 987\,\text{N}$

$\Sigma F_t = m\bar{r}\alpha$: $A_t = 987\,\text{N}$

$A = \sqrt{(197.4)^2 + (987)^2} = 1007\,\text{N}$

یا $A = 1.007\,\text{kN}$



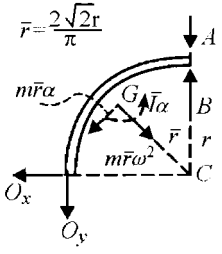
۶۷-۶.

(الف) $\alpha = 0$, $\Sigma M_C = 0$, $O_y = 0$

$\Sigma M_O = m\bar{a}d$: $Ar = m\frac{2\sqrt{2}\,r}{\pi}\omega^2\frac{r}{\sqrt{2}} \Rightarrow A = \frac{2}{\pi} m r\omega^2$

طبق تقارن $\Sigma M_A = m\bar{a}d$ یا $\Sigma F_n = m\bar{a}_n$

$O = -O_x = A = \frac{2}{\pi} m r\omega^2$



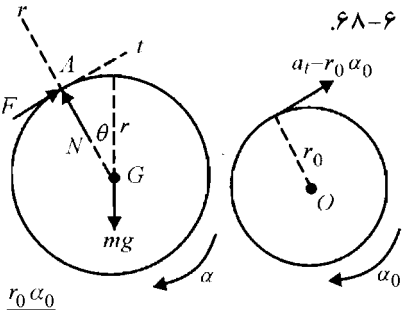
(ب) $\omega = 0$, $\Sigma M_C = I_C\alpha$, $O_y r = m r^2\alpha \Rightarrow O_y = m r\alpha$

$\Sigma F_x = m\bar{a}_x$: $O_x = m\frac{2\sqrt{2}\,r}{\pi}\alpha\frac{1}{\sqrt{2}} = \frac{2}{\pi} m r\alpha$

$\Sigma F_y = m\bar{a}_y$: $O_y - B = m\frac{2\sqrt{2}\,r}{m}\alpha\frac{1}{\sqrt{2}} \Rightarrow B = m r\alpha - \frac{2 m r\alpha}{\pi} = m r\alpha\left(1 - \frac{2}{\pi}\right)$

$\Rightarrow O = \sqrt{O_x^2 + O_y^2} = m r\alpha\sqrt{1 + \frac{4}{\pi^2}}$

۶-۶۸.



برای عدم لغزش : $a_t = r_0\alpha_0 = r\alpha \Rightarrow \alpha = \dfrac{r_0}{r}\alpha_0$

$+\circlearrowright \Sigma M_G = \bar{I}\alpha \;:\; Fr = mr^2\dfrac{r_0}{r}\alpha_0 \quad (1)$

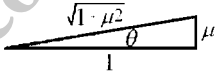
$\Sigma F_t = m\bar{a}_t = 0 \;:\; F - mg\sin\theta = 0 \quad (2)$

$\Sigma F_r = m\bar{a}_r = 0 \;:\; N - mg\cos\theta = 0 \quad (3)$

معادلات (1) و (2) : $mg\sin\theta = m r_0\alpha_0 \Rightarrow \theta = \sin^{-1}\dfrac{r_0\alpha_0}{g}$

$\left.\begin{array}{l}\sin\theta = \dfrac{r_0\alpha_0}{g} \\ \text{معادلات (2) و (3)} : \mu = \dfrac{F}{N} = \tan\theta\ ,\ \theta = \theta_{\max}\end{array}\right\} \Rightarrow \sin\theta = \dfrac{\mu}{\sqrt{1+\mu^2}} = \dfrac{r_0\alpha_0}{g}$

$\Rightarrow \mu = \mu_{\min} = \dfrac{1}{\sqrt{\left(\dfrac{g}{r_0\alpha_0}\right)^2 - 1}}\ ,\ r_0\alpha_0 < g$



۶-۶۹.

$b = \dfrac{2r}{\pi}$

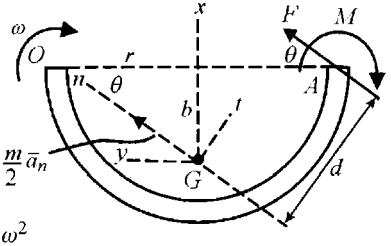
$\dfrac{m}{2}\bar{a}_n = \dfrac{m}{2}\overline{OG}\,\omega^2$

$\overline{OG} = \dfrac{r}{\cos\theta}\ ,\ d = 2r\sin\theta$

$\Sigma M_A = \dfrac{m}{2}\bar{a}_n d \;:\; M = \dfrac{m}{2}\dfrac{r}{\cos\theta}\omega^2 \times 2r\sin\theta$

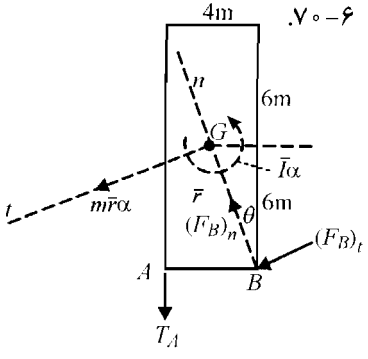
$\Rightarrow M = m r^2\omega^2\tan\theta$

از طرفی $\tan\theta = \dfrac{b}{r} = \dfrac{2r}{\pi r} = \dfrac{2}{\pi}$ $\Rightarrow M = \dfrac{2}{\pi} m r^2\omega^2$



$\Sigma F_y = \dfrac{m}{2}\bar{a}_y \;:\; V = F\cos\theta = \dfrac{m}{2}\overline{OG}\,\omega^2\cos\theta = \dfrac{m}{2}\dfrac{r}{\cos\theta}\omega^2\cos\theta \Rightarrow V = \dfrac{m}{2}r\omega^2$

۶-۷۰.



$\bar{r} = \sqrt{6^2 + 2^2} = 6.32\,\text{m}$

$\bar{I} = \dfrac{1}{12}(3000)(4^2 + 12^2) = 40000\,\text{kg.m}^2$

$\theta = \tan^{-1}\dfrac{2}{6} = 18.43°$

$+\circlearrowleft \Sigma M_B = \bar{I}\alpha + m\bar{a}\,\bar{r}$

$4T_A = 40000\,\alpha + 3000(6.32\,\alpha)(6.32)$

$\Rightarrow \alpha = 0.05\,\text{rad/s}$

$\Sigma F_n = m\,\bar{a}_n = 0 \;:\; (F_B)_n - 2000\cos 18.43° = 0 \;\Rightarrow\; (F_B)_n = 1897\,\text{N}$

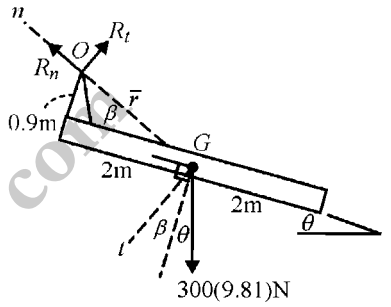
$\Sigma F_t = m\,\bar{a}_t \;:\; 2000\sin 18.43° + (F_B)_t = 3000(6.32)(0.05) \;\Rightarrow\; (F_B)_t = 316\,\text{N}$

$F_B = \sqrt{1897^2 + 316^2} = 1924\,\text{N}$

◀ ۶-۷۱. طبق پاسخ مسئله ۶-۶۰: $\beta = 24.2°\;,\; I_{(}) = 1843\,\text{kg.m}^2\;,\; \bar{r} = 2.19\,\text{m}$

$\curvearrowleft \Sigma M_{(}) = I_{(}\alpha \;:\; 300(9.81)(2\cos\theta - 0.9\sin\theta) = 1843\,\alpha$

$\alpha = 1.597(2\cos\theta - 0.9\sin\theta)$



$\int_0^{\omega}\omega\,d\omega = \int_0^{\theta}\alpha\,d\theta \;:\; \frac{\omega^2}{2} = 1.597\int_0^{\theta}(2\cos\theta - 0.9\sin\theta)\,d\theta$

$\omega^2 = 3.19(2\sin\theta - 0.9[1 - \cos\theta])$

$\frac{d(\omega^2)}{d\theta} = 3.19[2\cos\theta - 0.9\sin\theta] = 0$ (برای $\omega_{\max}$)

$\Rightarrow \theta = \tan^{-1}\frac{2}{0.9} = 65.8°$

$\Rightarrow \omega_{\max}{}^2 = 3.19(2\sin 65.8° - 0.9[1 - \cos 65.8°]) = 4.13\,(\text{rad/s})^2 \;\Rightarrow\; \omega_{\max} = 2.03\,\text{rad/s}$

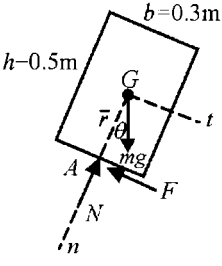
برای $\omega = \omega_{\max}$: $\alpha = 0$

$\Sigma F_n = m\,\bar{r}\omega^2 \;:\; R_n - 300(9.81)\sin\underbrace{(24.2° + 65.8°)}_{90°} = 300(2.19)4.13 \;\Rightarrow\; R_n = 5660\,\text{N}$

$\Sigma F_t = m\,\bar{r}\alpha \;:\; 300(9.81)\cos\underbrace{(24.2° + 65.8°)}_{90°} - R_t = 300(0) \;\Rightarrow\; R_t = 0$

$R = R_n = 5.66\,\text{kN}$

◀ ۶-۷۲.



طبق جدول د-۴ : $I_A = \frac{1}{12}m\,b^2 + \frac{1}{3}m\,h^2$

$I_A = \frac{m}{3}\left(\frac{0.3^2}{4} + 0.5^2\right) = 0.0908\,\text{m}$

$\Sigma M_A = I_A\alpha \;:\; mg\,(0.25)\sin\theta = 0.0908\,m\,\alpha$

$\alpha = 27\sin\theta\;\text{rad/s}^2$

$\Sigma F_t = m\,\bar{r}\alpha \;:\; mg\sin\theta - F = m\,(0.250)27\sin\theta$

$\Rightarrow F = m(g\sin\theta - 6.75\sin\theta) = 3.06\,m\sin\theta$

$\int_0^{\omega}\omega\,d\omega = \int_0^{\theta}\alpha\,d\theta \;:\; \frac{\omega^2}{2} = 27\int_0^{\theta}\sin\theta\,d\theta \;\Rightarrow\; \omega^2 = 54(1 - \cos\theta)$

$\Sigma F_n = m\bar{r}\omega^2 \;:\; mg\cos\theta - N = m(0.250)\,54(1-\cos\theta) \Rightarrow N = m(23.31\cos\theta - 13.5)$

$\theta = 30^\circ \;:\; F = 3.06m\sin 30^\circ = 1.53\,\text{m} \quad , \quad N = m(23.31\cos 30^\circ - 13.5) = 6.68\,\text{m}$

$\mu_s = \dfrac{F}{N} = \dfrac{1.53m}{6.68m} = 0.229$

(ب) تماس قطعه و گوشه وقتی قطع می‌شود که :

$N = 0 \Rightarrow 23.31\cos\theta - 13.5 = 0 \Rightarrow \theta = \cos^{-1}\dfrac{13.5}{23.31} = 54.6^\circ$

۶-۷۳.

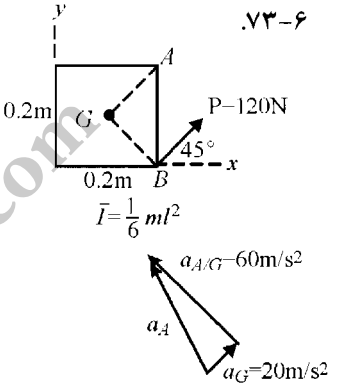
$\Sigma F = m\bar{a} : 120 = 6\bar{a} \Rightarrow \bar{a} = a_G = 20\,\text{m/s}^2$

$\Sigma M_G = \bar{I}\alpha \;:\; 120\dfrac{0.2}{\sqrt{2}} = \dfrac{1}{6}(6)(0.2^2)\alpha \Rightarrow \alpha = 424\,\text{rad/s}^2$

$\mathbf{a}_A = \mathbf{a}_G + \mathbf{a}_{A/G}$

$a_{A/G} = (a_{A/G})_t = \overline{AG}\,\alpha = \dfrac{0.2}{\sqrt{2}} \times 424 = 60\,\text{m/s}^2$

$a_A = \sqrt{60^2 + 20^2} = 63.2\,\text{m/s}^2$



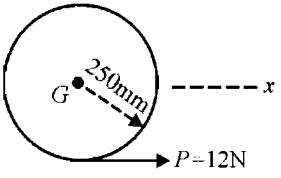
۶-۷۴.

$\Sigma F_x = m\bar{a}_x \;:\; 12 = 30a \Rightarrow a = 0.4\,\text{m/s}^2$

$v^2 = 2ax \;:\; v^2 = 2(0.4)(1.2) = 0.96 \Rightarrow v = 0.98\,\text{m/s}$

$\Sigma M_G = \bar{I}\alpha : 12(0.250) = \dfrac{1}{2}(30)(0.250)^2\alpha \Rightarrow \alpha = 3.2\,\text{rad/s}^2$

$\omega = \alpha t = 3.2(2) = 6.4\,\text{rad/s}$



۶-۷۵.

با فرض لغزش تیرک : $F = 0.3\,\text{N}$

$\Sigma F_y = 0 \;:\; N = 50(9.81)\text{N}$

$\Rightarrow F = 0.3(50)9.81 = 147.2\,\text{N}$

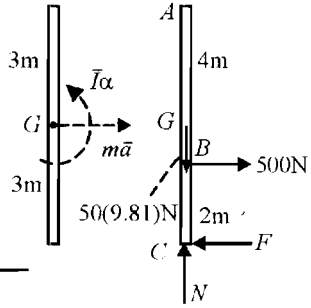
$\Sigma\bar{M} = \bar{I}\alpha \;:\; 500(1) - 147.2(3) = \dfrac{1}{12}50(6^2)\alpha$

$\alpha = 0.390\,\text{rad/s}^2 \quad \text{CCW}$

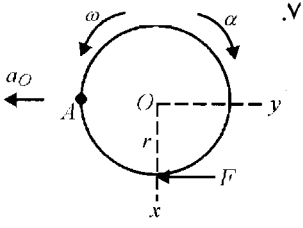
$\Sigma F_x = m\bar{a}_x \;:\; 500 - 147.2 = 50\bar{a} \Rightarrow \bar{a} = 7.06\,\text{m/s}^2$

$\mathbf{a}_A = \mathbf{a}_G + \mathbf{a}_{A/G} \;:\; a_{A/G} = \overline{AG}\,\alpha = 3(0.390) = 1.171\,\text{m/s}^2 \leftarrow$

$a_A = 7.06 - 1.171 = 5.89\,\text{m/s}^2 \rightarrow$



۶-۷۶.



$$\Sigma M_O = I_O\alpha \;:\; Fr = mk^2\alpha \;\Rightarrow\; \alpha = \frac{Fr}{mk^2}$$

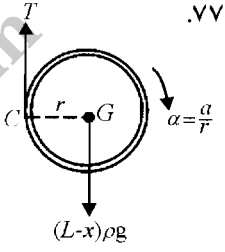
$$\Sigma F_y = ma_y \;:\; -F = m(-a_O) \;\Rightarrow\; a_O = \frac{F}{m}$$

$$\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{A/O}$$

$$(a_{A/O})_n = r\omega^2\mathbf{j} \;,\quad (a_{A/O})_t = -r\alpha\mathbf{i} = -\frac{Fr^2}{mk^2}\mathbf{i}$$

$$\mathbf{a}_A = -\frac{Fr^2}{mk^2}\mathbf{i} - \left(\frac{F}{m} - r\omega^2\right)\mathbf{j}$$

۶-۷۷.

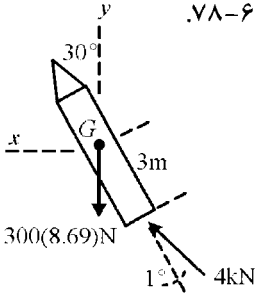


$$\Sigma M_C = I_C\alpha$$

$$(L-x)\rho g r = 2(L-x)\rho r^2\frac{a}{r}$$

$$a = \frac{g}{2} \quad \text{(ثابت)}$$

۶-۷۸.



$$\bar{I} = m\bar{k}^2 = 300(1.5)^2 = 675\,\text{kg.m}^2$$

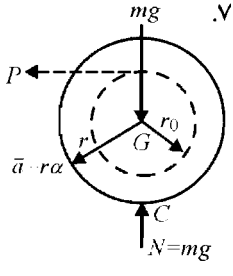
$$\Sigma M_G = \bar{I}\alpha \;:\; 4000\sin 1°(3) = 675\alpha \;\Rightarrow\; \alpha = 0.310\,\text{rad/s}^2$$

$$\Sigma F_x = m\bar{a}_x \;:\; 4000\sin 31° = 3000\,\bar{a}_x \;\Rightarrow\; \bar{a}_x = 6.87\,\text{m/s}^2$$

$$\Sigma F_y = m\bar{a}_y \;:\; 4000\cos 31° - 300(8.69) = 300\,\bar{a}_y$$

$$\Rightarrow\; \bar{a}_y = 2.74\,\text{m/s}^2$$

۶-۷۹.



$$I_C = \bar{I} + mr^2 = m(\bar{k}^2 + r^2)$$

$$\Sigma M_C = I_C\alpha \;:\; P(r + r_0) = m(\bar{k}^2 + r^2)\alpha$$

$$\Sigma F = m\bar{a} \;:\; P = m\bar{a} = mr\alpha$$

$$\Rightarrow\; mr\alpha(r + r_0) = m(\bar{k}^2 + r^2)\alpha \;\Rightarrow\; r^2 + rr_0 + \bar{k}^2 + r^2$$

$$\Rightarrow\; r_0 = \frac{\bar{k}^2}{r}$$

۸۰-۶

$$\Sigma F_y = 0 \;:\; N - 98.1\cos 60° = 0 \;\Rightarrow\; N = 49.0\,\text{N}$$

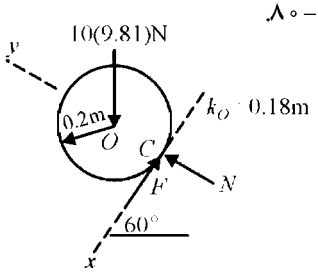
$$F = \mu_k N = 0.3(49.0) = 14.72\,\text{N}$$

$$\Sigma F_x = m\bar{a}_x \;:\; 10(9.81)\sin 60° - 14.72 = 10\,a_O$$

$$a_O = 7.02\,\text{m/s}^2$$

$$\Sigma M_O = I_O\alpha \;:\; 14.72(0.2) = 10(0.18^2)\,\alpha$$

$$\alpha = 9.08\,\text{rad/s}^2$$



۸۱-۶

$$\curvearrowleft \Sigma M_G = \bar{I}\alpha + m\bar{a}\,d \;:\; m\,g\,r\sin\theta = m\,k_O^2\,\alpha + m\,(r\alpha)\,r$$

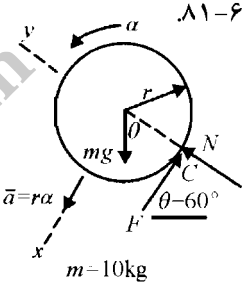
$$\alpha = \frac{gr\sin\theta}{k_O^2 + r^2} = \frac{9.81(0.2)\sin 60°}{0.18^2 + 0.2^2} = 23.4\,\text{rad/s}^2$$

$$\Sigma F_x = m\,a_x \;:\; mg\sin\theta - F = m\,r\alpha$$

$$\Rightarrow\; F = 98.1\sin 60° - 10(0.2)(23.4) = 39.0\,\text{N}$$

$$\Sigma F_y = 0 \;:\; N - mg\cos\theta = 0 \;\Rightarrow\; N = 98.1\cos 60° = 49.0\,\text{N}$$

$$\mu_s = \frac{F}{N} = 0.775 = (\mu_s)_{\min}$$



m=10kg
mg=10(9.81) = 98.1N
r=0.2m ,k_O=0.180m

۸۲-۶

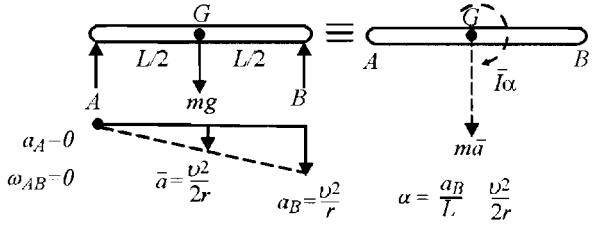
$$\Sigma M_A = m\bar{a}\frac{L}{2} + \bar{I}\alpha$$

$$mg\,\frac{L}{2} - BL = m\,\frac{v^2}{2r}\,\frac{L}{2} + \frac{1}{12}\,mL^2\,\frac{v^2}{L^2}$$

$$B = m\left(\frac{g}{2} - \frac{v^2}{3r}\right)$$

$$B = 0 \;\Rightarrow\; \frac{g}{2} - \frac{v^2}{3r} = 0$$

$$\Rightarrow\; v = \sqrt{\frac{3gr}{2}}$$



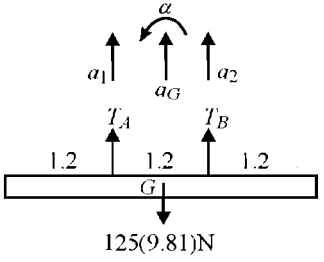
۸۳-۶

$$a_1 = r\alpha_1 = \frac{0.375}{2}4 = 0.75\,\text{m/s}^2$$

$$a_2 = r\alpha_2 = \frac{0.375}{2}6 = 1.125\,\text{m/s}^2$$

$$a_G = \frac{(1.125 + 0.75)}{2} = 0.9375\,\text{m/s}^2$$

$$\alpha = \frac{a_{G/A}}{\overline{AB}} = \frac{1.125 - 0.75}{1.2} = 0.3125\,\text{rad/s}^2$$



$$I_G = \frac{1}{12} mL^2 = \frac{1}{12}(125)(3.6)^2 = 135\ \text{kg.m}^2$$

$$\Sigma F = m a_G \ : \ T_A + T_B - 125(9.81) = 125(0.9375) \Rightarrow T_A + T_B = 1343.4$$

$$\Sigma M_G = I_G \alpha \ : \ (T_B - T_A)\frac{1.2}{2} = 135\,(0.3125) \Rightarrow T_B - T_A = 70.3$$

$$\Rightarrow T_A = 637\,\text{N} \ , \ T_B = 707\,\text{N}$$

۸۴-۶

$$\bar{I} = 2\left(\frac{1}{12}[2m][2r]^2\right) = \frac{4}{3} mr^2$$

$$\Sigma M_G = \bar{I}\alpha \ : \ Fr = \frac{4}{3} m r^2 \alpha \Rightarrow F = \frac{4}{3} m r \alpha = \frac{4}{3} ma$$

$$\Sigma F_x = m \bar{a}_x \ : \ -F + 4 m g \sin\theta = 4 m a \Rightarrow a = \frac{3}{4} g \sin\theta$$

$$F = m g \sin\theta$$

$$\mu_s = \frac{F}{N} = \frac{mg \sin\theta}{4 mg \cos\theta} = \frac{1}{4}\tan\theta$$

۸۵-۶

$$\bar{\mathbf{a}} = \mathbf{a}_G = \mathbf{a}_O + \mathbf{a}_{G/O}$$

$$v_O = r\omega = 0.15(10) = 1.5\,\text{m/s}$$

$$a_O = \frac{v_O^2}{R - r} = \frac{(1.5)^2}{2 - 0.15} = 1.216\,\text{m/s}^2 \uparrow$$

$$a_{G/O} = (a_{G/O})_n = \bar{r}\omega^2 = 0.01(10)^2 = 1\,\text{m/s}^2 \downarrow$$

$$\bar{a} = 1.216 - 1 = 0.216\,\text{m/s}^2 \uparrow$$

$$\Sigma F = m\bar{a} \ : \ P - 10(9.81) = 10(0.216) \Rightarrow P = 100.3\,\text{N}$$

۸۶-۶

$$\left.\begin{array}{l} \Sigma M_G = \bar{I}\alpha \\ \bar{I} = 0 \end{array}\right\} \Rightarrow \Sigma \bar{M} = 0 \Rightarrow \text{بدون نیروی اصطکاک} \Rightarrow \mu_s = 0$$

$$\Sigma F_x = m a_x \ : \ mg \sin\theta = m r \alpha_A \Rightarrow \alpha_A = \frac{g}{r}\sin\theta$$

$$\Sigma M_C = I_C \alpha \ : \ m g r \sin\theta = 2 m r^2 \alpha_B \Rightarrow \alpha_B = \frac{g}{2r}\sin\theta$$

$$\Sigma \bar{M} = \bar{I}\alpha \ : \ Fr = m r^2 \frac{g}{2r}\sin\theta \Rightarrow F = \frac{1}{2} m g \sin\theta$$

$$\mu_s = \frac{F}{N} = \frac{\frac{1}{2} mg \sin\theta}{mg\cos\theta} \Rightarrow \mu_s = \frac{1}{2}\tan\theta$$

۸۷-۶

$\mathbf{a}_A = \bar{\mathbf{a}} + \mathbf{a}_{A/G} \Rightarrow a_A = \bar{a} + (0.8 - b)\alpha$

$\bar{a} = 20 - (0.8 - b)\,18$

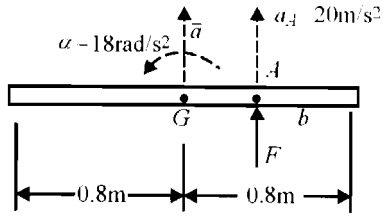
$\Sigma F = ma : F = m[20 - (0.8 - b)\,18]$

$= m[5.6 + 18\,b]$

$\Sigma M_G = \bar{I}\alpha : F(0.8 - b) = \frac{1}{12} m\,(1.6)^2(18)$

ترکیب دو رابطه : $18\,b^2 - 8.8\,b - 0.64 = 0$

$\Rightarrow b = 0.553$ m



۸۸-۶

استوانه داخلی :

$\Sigma M_{C_1} = I_{C_1}\alpha_1$:

$8(9.81)0.15\sin 10° - 0.15\,O_x =$

$\frac{3}{2}(8)(0.15)^2\,\alpha_1$

$\Rightarrow 2.044 - 0.15O_x = 0.27\alpha_1$ (1)

هر دو دیسک :

$\Sigma M_{C_2} = I_{C_2}\alpha_2$:

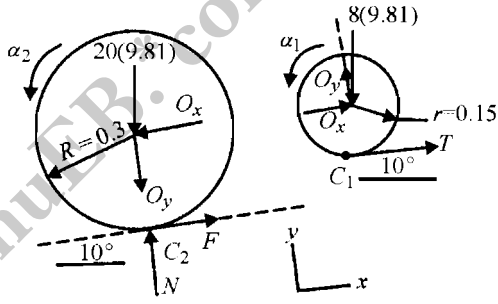
$20(9.81)(0.3)\sin 10° + 0.3\,O_x = \frac{3}{2}(20)(0.3)^2\alpha_2$

$\Rightarrow 10.22 + 0.3\,O_x = 2.7\alpha_2$ (2)

از طرفی $a_O = R\alpha_2 = r\alpha_1 \Rightarrow 0.3\alpha_2 = 0.15\alpha_1 \Rightarrow \alpha_1 = 2\alpha_2$ (3)

حل (1) تا (3) : $\alpha_1 = 7.57\,\text{rad/s}^2$, $\alpha_2 = 3.785\,\text{rad/s}^2$, $O_x = 0$

$a_O = 0.3(3.785) = 1.136\,\text{m/s}^2$



۸۹-۶

$\Sigma F_y = 0 \Rightarrow N = 25(9.81) = 245$ N

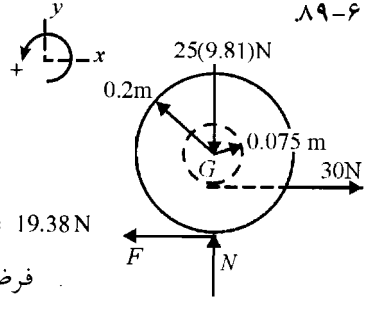
$\Sigma F_x = m\,\bar{a}_x : 30 - F = 25\,a$ (1)

$\Sigma M_G = \bar{I}\alpha : 30(0.075) - F(0.2) = 25(0.175)^2\alpha$ (2)

با فرض غلتش بدون لغزش : $a = -r\alpha$ (3)

حل (1) تا (3) : $a = 0.425\,\text{m/s}^2$, $\alpha = -2.12\,\text{rad/s}^2$, $F = 19.38$ N

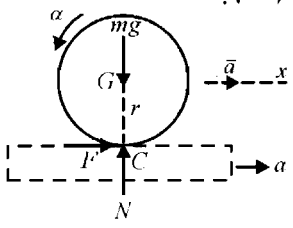
$F_{\max} = \mu_s N = 0.1(245) = 24.5 > F \Rightarrow$ فرض صحیح است



۶-۹۰.

$$\alpha = \frac{a_{G/O}}{r} = \frac{a - \bar{a}}{r}$$

$$\left.\begin{aligned} \Sigma F_x &= m a_x \Rightarrow F = m\bar{a} \\ \Sigma M_G &= \bar{I}\alpha \Rightarrow F.r = \frac{1}{2} m r^2 \frac{a-\bar{a}}{r} \end{aligned}\right\} \Rightarrow \bar{a} = \frac{1}{3}a$$



$$a_{G/O} = a - \bar{a} = \frac{2}{3}a \qquad \text{به طرف چپ}$$

نسبت به کامیون : $s = \frac{1}{2} a_{rel} t^2 \Rightarrow d = \frac{1}{2}\left(\frac{2}{3}a\right)t^2 \Rightarrow t^2 = \frac{3d}{a}$

کامیون : $s = \frac{1}{2} a t^2 \Rightarrow s = \frac{1}{2} a \left(\frac{3d}{a}\right) = \frac{3d}{2}$

۶-۹۱.

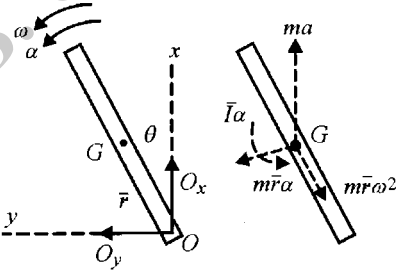
$$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d \quad , \quad \bar{I} = m(k_O^2 - \bar{r}^2)$$

$$0 = m(k_O^2 - \bar{r}^2)\alpha + m\bar{r}^2\alpha - m a \bar{r}\sin\theta$$

$$\alpha = \frac{a\bar{r}\sin\theta}{k_O^2}$$

$$\int_0^{\omega} \dot{\theta}\, d\dot{\theta} = \int_0^{\frac{\pi}{2}} \alpha\, d\theta \Rightarrow$$

$$\frac{\omega^2}{2} = \frac{a\bar{r}}{k_O^2}\left(-\cos\theta\right)_0^{\frac{\pi}{2}} = \frac{a\bar{r}}{k_O^2} \Rightarrow \omega = \frac{1}{k_O}\sqrt{2a\bar{r}}$$



۶-۹۲.

$$\bar{I}\alpha = \frac{1}{12} m l^2 \alpha = \frac{1}{12} m l^2 \frac{a}{l} = \frac{1}{12} m l a$$

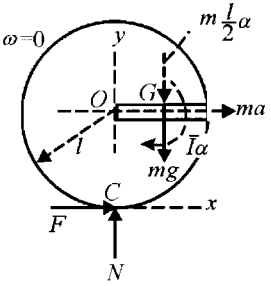
$$\Sigma M_C = \bar{I}\alpha + \Sigma m\bar{a}d$$

$$mg\frac{l}{2} = \frac{1}{12}mla + mal + m\frac{l}{2}\frac{a}{l}\frac{l}{2}$$

$$\Rightarrow a = \frac{3}{8}g \ , \ \alpha = \frac{a}{l} = \frac{3g}{8l}$$

$$\Sigma F_x = m\bar{a}_x : F = m\frac{3}{8}g = \frac{3}{8}mg$$

$$\Sigma F_y = m\bar{a}_y : N - mg = -m\frac{l}{2}\frac{3g}{8l} \Rightarrow N = \frac{13}{16}mg$$



۶-۹۳.

$$\bar{I} = I_O - m\bar{r}^2 = mr^2 - m\left(\frac{2r}{\pi}\right)^2 = mr^2\left(1 - \frac{4}{\pi^2}\right)$$

$$\Sigma F_x = m\bar{a}_x : F = mr\alpha$$

$\Sigma F_y = m\bar{a}_y$:

$mg - N = \frac{2}{\pi} m r\alpha \Rightarrow N = m\left(g - \frac{2r\alpha}{\pi}\right)$

$\Sigma M_G = \bar{I}\alpha$:

$N\left(\frac{2r}{\pi}\right) - Fr = mr^2\left(1 - \frac{4}{\pi^2}\right)\alpha$

حل همزمان : $\alpha = \frac{g}{\pi r}$

$\Rightarrow F = mr\alpha = mr\frac{g}{\pi r} = m\frac{g}{\pi}$, $N = mg\left(1 - \frac{2}{\pi^2}\right)$

$\mu_s = \frac{F}{N} = \frac{\frac{mg}{\pi}}{mg\left(1 - \frac{2}{\pi^2}\right)} = \frac{\pi}{\pi^2 - 2} = 0.399$

۶-۹۴.

$\bar{r} = \frac{4r}{3\pi}\sqrt{2}$

$\bar{I} = I_O - m\bar{r}^2 = \frac{1}{2}mr^2 - m\left[\frac{4r}{3\pi}\sqrt{2}\right]^2 = 0.1397\, mr^2$

$\curvearrowright \Sigma M_G = \bar{I}\alpha$

$N\left(\frac{4r}{3\pi}\right) - F\left(r + \frac{4r}{3\pi}\right) = 0.1397\, mr^2\alpha$ (1)

$\Sigma F_x = m\bar{a}_x$

$F = mr\alpha + m\left(\frac{4r}{3\pi}\sqrt{2}\right)\frac{1}{\sqrt{2}}\alpha$ (2)

$\Sigma F_y = m\bar{a}_y$

$N - mg = -m\left(\frac{4r}{3\pi}\sqrt{2}\right)\frac{1}{\sqrt{2}}\alpha$ (3)

حل (1) تا (3) : $F = 0.257\, mg$, $N = 0.923\, mg$, $\alpha = 0.1807\frac{g}{r}$

۶-۹۵.

$\Sigma M_A = \bar{I}\alpha + \Sigma m\bar{a}d$

$0 = 0.267\alpha + 2.0\alpha(0.4) - 20(0.4)$

$\alpha = 7.50\,\text{rad/s}^2$

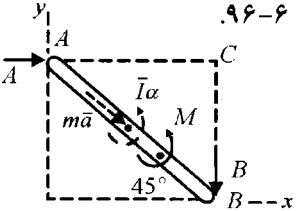
$\Sigma F_x = m\bar{a}_x$:

$A_x = 20 - 2.0(7.50) = 5\,\text{N}$

$\Sigma F_y = m\bar{a}_y$:

$A_y - 5(9.81) = 8 \Rightarrow A_y = 57.1\,\text{N}$

۹۶-۶



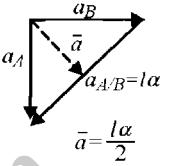
$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B} \quad , \quad a_{A/B} = (a_{A/B})_t = l\alpha$$

$$\Sigma M_C = \bar{I}\alpha + m\bar{a}d$$

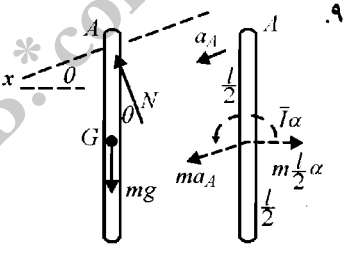
$$M = \frac{1}{12} m l^2 \alpha + m \frac{l\alpha}{2} \frac{l}{2} = \frac{1}{3} m l^2 \alpha \Rightarrow \alpha = \frac{3M}{m l^2}$$

$$\Sigma F_x = m\bar{a}_x : \quad A = m \frac{l\alpha}{2} \frac{1}{\sqrt{2}} = \frac{m l}{2\sqrt{2}} \frac{3M}{m l^2} \Rightarrow A = \frac{3M}{2\sqrt{2}\, l}\,\mathbf{i}$$

$$\Sigma F_y = m\bar{a}_y : \quad -B = m\left(-\frac{l\alpha}{2}\frac{1}{\sqrt{2}}\right) \Rightarrow B = -\frac{3M}{2\sqrt{2}\, l}\,\mathbf{j}$$



۹۷-۶



$$\Sigma M_A = \bar{I}\alpha + \Sigma m\bar{a}d$$

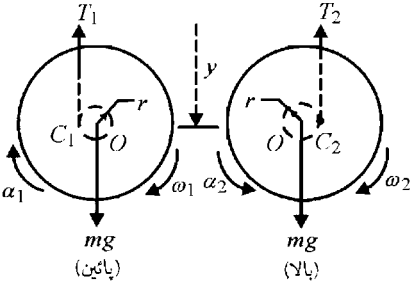
$$0 = \frac{1}{12} m l^2 \alpha + m \frac{l}{2}\alpha \frac{l}{2} - m a_A \frac{l}{2}\cos\theta \quad (1)$$

$$\Sigma F_x = m\bar{a}_x$$

$$mg\sin\theta = m\left(a_A - \frac{l}{2}\alpha\cos\theta\right) \quad (2)$$

$$(2)\ ,\ (1) : \quad a_A = \frac{g\sin\theta}{1 - \frac{3}{4}\cos^2\theta}$$

۹۸-۶



به طرف پائین : $\Sigma M_{C_1} = I_{C_1}\alpha_1$

$$mgr = m(k^2 + r^2)\alpha_1$$

$$(a_O)_1 = r\alpha_1 = \frac{g}{\frac{k^2}{r^2} + 1} \qquad \text{(ثابت)}$$

$$\Sigma F_y = m\bar{a}_y :$$

$$mg - T_1 = \frac{mg}{\frac{k^2}{r^2} + 1} \Rightarrow T_1 = \frac{mg}{1 + \frac{r^2}{k^2}}$$

$$v^2 = {v_O}^2 + 2as : \quad v = \sqrt{{v_O}^2 + \frac{2gL}{\frac{k^2}{r^2} + 1}}$$

به طرف بالا : $\Sigma M_{C_2} = I_{C_2}\alpha_2$

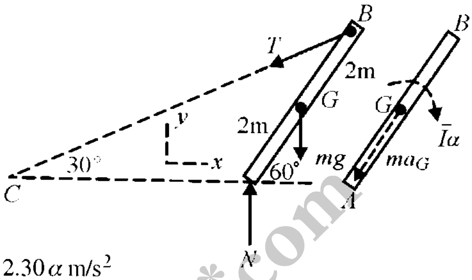
$$mgr = m(k^2 + r^2)\alpha_2$$

$$(a_O)_2 = r\alpha_2 = \frac{g}{\frac{k^2}{r^2} + 1} \qquad \text{(ثابت)}$$

$$\Sigma F_y = m\,\overline{a}_y \;:\; mg - T_2 = \frac{mg}{\frac{k^2}{r^2}+1} \;\Rightarrow\; T_2 = \frac{mg}{1+\frac{r^2}{k^2}}$$

$$\Rightarrow\; T = \frac{mg}{1+\frac{r^2}{k^2}} \;,\; a = \frac{g}{\frac{k^2}{r^2}+1}$$ برای هر دو حرکت

۶-۹۹.



$mg = 12(9.81)10^3 = 117.7(10^3)\,\text{N}$

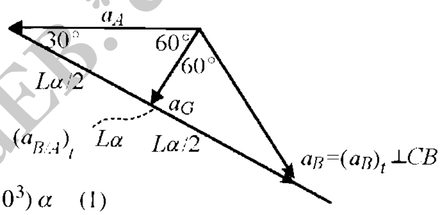
$\mathbf{a}_B = \mathbf{a}_A + (\mathbf{a}_{B/A})_t$

$\mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_t$

$(a_{B/A})_t = L\alpha \;,\; (a_{G/A})_t = \frac{L}{2}\alpha$

طبق دیاگرام شتاب :

$a_B = \frac{L}{2}\alpha \sec 30°\alpha = \frac{4}{2}\sec 30°\alpha = 2.30\alpha\ \text{m/s}^2$



چون $\mathbf{a}_G$ از نقطه A می‌گذرد :

$\Sigma M_A = \overline{I}\alpha$

$117.7(10^3)2\cos 60° - T\times 4\sin 30°$

$= \frac{1}{12}12(10^3)4^2\alpha \;\Rightarrow\; 117.7(10^3) - 2T = 16(10^3)\alpha \quad (1)$

$\Sigma F_x = m\,\overline{a}_x \;:\; T\cos 30° = 12(10^3)(a_G)_x$

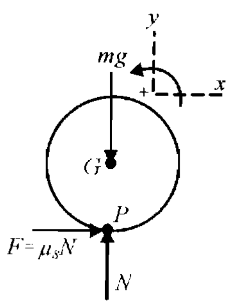
از طرفی $a_G = \frac{L}{2}\alpha\tan 30° \;\Rightarrow\; (a_G)_x = \frac{L}{2}\tan 30°\alpha\cos 60° = 0.577\alpha\ \text{m/s}^2$

$\Rightarrow\; T = 8(10^3)\alpha \quad (2)$

حل (1) و (2) : $\alpha = 3.68\ \text{rad/s}^2 \;,\; T = 29.4\ \text{kN}$

$a_A = \frac{L}{2}\frac{\alpha}{\cos 30} = \frac{4}{2}\frac{3.68}{\cos 30°} \;\Rightarrow\; a_A = 8.50\ \text{m/s}^2$

۶-۱۰۰. محدودیت عدم لغزش یعنی مساوی قرار دادن شتاب افقی نقطه P با شتاب گاری a_C



$(a_P)_{hor} = a_G + r\alpha = a_C \quad (1)$

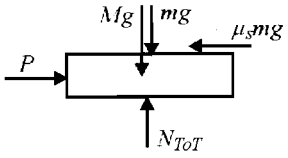
$\Sigma F_y = 0 \;\Rightarrow\; N = mg \quad (2)$

$\Sigma F_x = m\,a_G \;:\; \mu_s mg = m\,a_G \quad (3)$

$\Sigma M_G = \overline{I}\alpha \;:\; \mu_s m g r = m\overline{k}^2\alpha \quad (4)$

حل (1) و (3) و (4) : $a_G = \mu_s g \;,\; a_O = \mu_s g\left[1 + \frac{r^2}{k^2}\right] \;,\; \alpha = \mu_s \frac{gr}{k^2}$

گاری : $\Sigma F_x = m a_O$



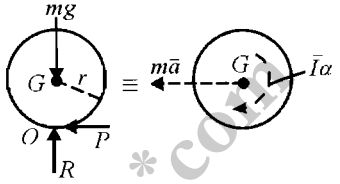
$$P - \mu_s mg = M\mu_s g\left[1 + \frac{r^2}{k^2}\right]$$

$$P = \mu_s g\left[m + M\left(1 + \frac{r^2}{k^2}\right)\right]$$

۶-۱۰۱.

$$\bar{I} = \frac{2}{5} m r^2$$

$$\Sigma M_O = \bar{I}\alpha - m\bar{a}r \;:$$

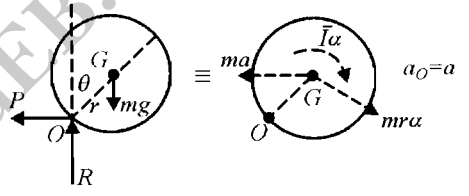


$$0 = \frac{2}{5} m r^2 \alpha - m\bar{a}r \;\Rightarrow\; \bar{a} = \frac{2}{5} r\alpha$$

$$\mathbf{a}_G = \mathbf{a}_O + \mathbf{a}_{G/O} \;:$$

$$\bar{a} = a - r\alpha = \frac{2}{5} r\alpha \;\Rightarrow\; \alpha = \frac{5}{7}\frac{a}{r}$$

$$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d$$



$$mgr\sin\theta = \frac{2}{5} m r^2 \alpha + m r^2 \alpha - mar\cos\theta$$

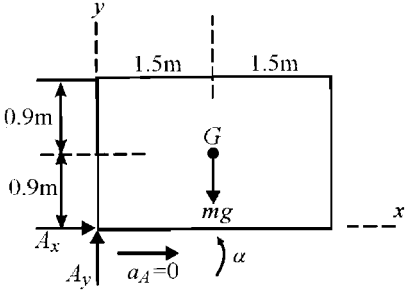
$$\alpha = \frac{5}{7r}(g\sin\theta + a\cos\theta)$$

$$\omega\, d\omega = \alpha\, d\theta \;:\; \int_0^{\omega} \omega\, d\omega = \frac{5}{7r}\int_0^{\theta}(g\sin\theta + a\cos\theta)d\theta \;\Rightarrow\; \omega = \sqrt{\frac{10}{7r}}\sqrt{g(1 - \cos\theta) + a\sin\theta}$$

۶-۱۰۲.

$$I_A = \frac{1}{12} m([1.8]^2 + [3.0]^2) + m([0.9]^2 + [1.5]^2)$$

$$= \frac{4}{3} m([0.9]^2 + [1.5]^2) = 4.08\,\text{m}\;\;\text{kg.m}^2$$



معادله ۶-۳ : $\Sigma \mathbf{M}_A = I_A \boldsymbol{\alpha} + \bar{\boldsymbol{\rho}} \times m\mathbf{a}$

$$-m(9.81)(1.5)\,\mathbf{k}$$

$$= 4.08m\alpha\,\mathbf{k} + (1.5\,\mathbf{i} + 0.9\,\mathbf{j}) \times ma\,\mathbf{i}$$

$$-14.72 = 4.08\alpha - 0.9a$$

وقتی $\alpha = 0$ $\Rightarrow$ $a = a_{\min} = \frac{14.72}{0.9} = 16.35\,\text{m/s}^2$

(حداکثر شتاب ممکن برای $\mu_s \approx 1$ تقریباً برابر است با $g = 9.81\,\text{m/s}^2$ بنابراین $a_{\min}$ امکان‌پذیر نیست)

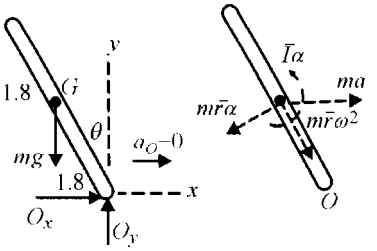
اگر شتاب $= 1.2a = 1.2(16.35)\,\text{m/s}^2$

$$\alpha = \frac{1}{4.08}(0.9[1.2]16.35 - 14.72) = 0.721\,\text{rad/s}^2$$

۶-۱۰۳.

$$m\bar{\mathbf{a}} = m\mathbf{a}_O + m\mathbf{a}_{G/O} = m\mathbf{a} + m\bar{r}\omega^2 + m\bar{r}\alpha \quad (\bar{r} = 1.8\,\text{m})$$

$$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d$$



$$mg(1.8\sin\theta)$$

$$= \frac{1}{12}m(3.6)^2\alpha + m(1.8\alpha)(1.8) - m(0.9)(1.8\cos\theta)$$

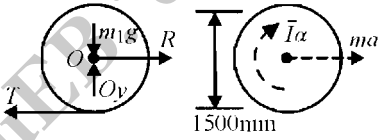
$$\alpha = 0.417(g\sin\theta + 0.9\cos\theta)$$

$$\int\omega\,d\omega = \int\alpha\,d\theta \;:\; \int_0^{\omega}\omega\,d\omega = 0.417\int_0^{\frac{\pi}{2}}(g\sin\theta + 0.9\cos\theta)\,d\theta$$

$$\omega^2 = 0.833\left[-g\cos\theta + 0.9\sin\theta\right]_0^{\frac{\pi}{2}} = 0.833[9.81 + 0.9] = 8.925 \Rightarrow \omega = 2.99\,\text{rad/s}$$

۶-۱۰۴.

$$a = r\alpha \Rightarrow \alpha = \frac{0.2(9.81)}{\frac{1.500}{2}} = 2.62\,\text{rad/s}^2$$



قرقره کابل :

$$\bar{I} = 140(0.530)^2 + 150\pi(1.5)(0.75)\left(\frac{1.5}{2}\right)^2 = 338\,\text{kg.m}^3$$

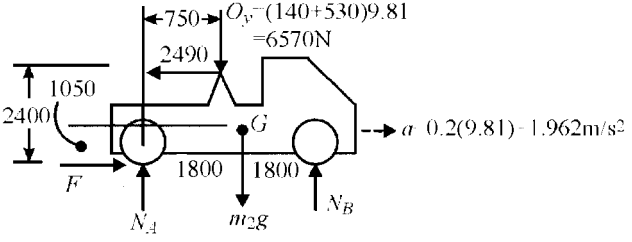
$$\Sigma M_O = I_O\alpha$$

$$T\left(\frac{1.500}{2}\right) = 338(2.62) \Rightarrow T = 1177\,\text{N}$$

$$\Sigma F = m\bar{a} \;:\; R - 1177 = \left(140 + 150\pi(1.5)(0.75)\right)(0.2)(9.81) \Rightarrow R = 2490\,\text{N}$$

$$m_2g = 2030(9.81) = 19910\,\text{N}$$

$$\Sigma M_A = \Sigma m\bar{a}d$$



$$6570(0.750) + 19910(1.8) - 2490(2.4) - 3.6N_B = 2030(1.962)(1.05)$$

$$\Rightarrow N_B = 8500\,\text{N}$$

$$\Sigma F_y = 0 \;:\; N_A + N_B - 6570 - 19910 = 0$$

$$\Rightarrow N_A = 17980\,\text{N}$$

۶-۱۰۵. $\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d$

$$mg\bar{r}\sin\theta = m(k_O^2 - \bar{r}^2)\alpha + m\bar{r}\alpha(\bar{r}) - ma_O\bar{r}\cos\theta$$

$$\alpha = \frac{1}{k_O^2}(g\bar{r}\sin\theta + a_O\bar{r}\cos\theta)$$

$$\dot{\theta}\,d\dot{\theta} = \ddot{\theta}\,d\theta$$

$$\int_0^{\omega}\dot{\theta}\,d\dot{\theta} = \frac{1}{k_O^2}\int(g\bar{r}\sin\theta + a_O\bar{r}\cos\theta)\,d\theta$$

$$\frac{\omega^2}{2} = \frac{1}{k_O^2}[g\bar{r}(1 - \cos\theta) + a_O r\sin\theta]$$

که در آن $(a_O = a)$

$$v = r\omega = \frac{r\sqrt{2}}{k_O}\sqrt{g\bar{r}(1 - \cos\theta) + a\bar{r}\sin\theta}$$

برای: $a = 10\,g$ ، $\theta = 45°$ ، $k_O = 0.55\,m$ ، $r = 0.8\,m$ خواهیم داشت: $\bar{r} = 0.45\,m$

$$v = \frac{0.80\sqrt{2}}{0.55}\sqrt{9.81(0.45)\left(1 - \frac{1}{\sqrt{2}}\right) + 10(9.81)(0.45)\frac{1}{\sqrt{2}}} = 11.73\,m/s$$

۶-۱۰۶.

$$I = mk^2 = 20(0.202)^2 = 0.816\,kg.m^2$$

$$\mathbf{a}_G = \mathbf{a}_O + (a_{G/O})_n + (a_{G/O})_t$$

$$a_O = r\alpha = 0.250\alpha$$

$$(a_{G/O})_n = 0.075(3)^2 = 0.675\,m/s^2$$

$$(a_{G/O})_t = 0.075\alpha$$

$$\Sigma M_G = \bar{I}\alpha:\ 0.25F + 0.075N = 0.816\alpha$$

$$\Sigma F_x = m\bar{a}_x:\ 20(9.81)\sin 10° - F = 20(0.25\alpha) - 20(0.675)$$

$$\Sigma F_y = m\bar{a}_y:\ N - 20(9.81)\cos 20° = -20(0.075\alpha)$$

$$\Rightarrow\ F = 2.62\,N,\quad N = 161\,N,\quad \alpha = 15.6\,rad/s^2\ CCW$$

۶-۱۰۷.

$$\mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_t$$

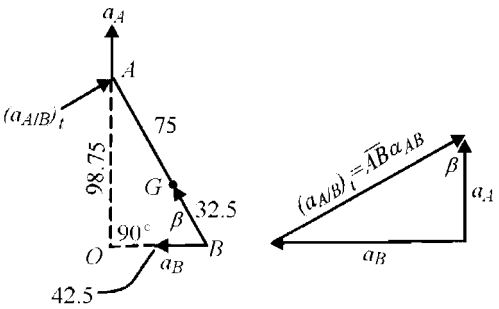
$$(a_{A/B})_n = \overline{AB}\ \omega_{AB}^2 = 0 \quad (\text{زیرا } v_A = v_B)$$

$$a_B = r\omega^2 = 0.0425(100\pi)^2 = 4194.6\ \text{m/s}^2$$

$$(a_{A/B})_t = \frac{4194.8}{\sin 66.7^\circ} = 4567\ \text{m/s}^2$$

$$\alpha_{AB} = \frac{4567}{0.1075} = 42.5(10^3)\ \text{rad/s}^2\ \text{CW}$$

$$a_A = \frac{4194.6}{\tan\beta} = 1806.5\ \text{m/s}^2$$



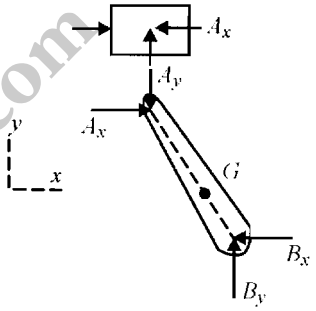
پیستون : $\Sigma F_y = m\,a_y$

$$A_y = 0.82(1806.5) = 1481\ \text{N}$$

شاتون : $\Sigma \mathbf{M}_B = I_B\boldsymbol{\alpha} + \bar{\boldsymbol{\rho}} \times m\,\mathbf{a}_B$

$$1481(0.0425) - A_x(0.09875) = 0.6[(0.028)^2 + (0.0325)^2] \times$$
$$\times (-42.5)(10^3) + 0.0325(0.6)(4194.6)\sin\beta$$

$$\Rightarrow A_x = 351\ \text{N}$$

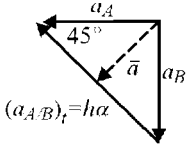


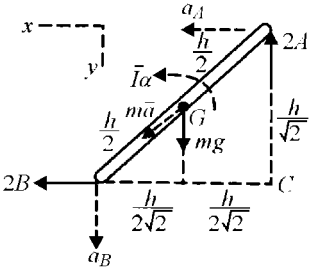
$$A = \sqrt{(1481)^2 + (351)^2} = 1522\ \text{N}$$

۱۰۸-۶.

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

بدون سرعت $a_{A/B} = (a_{A/B})_t = h\alpha$





$$a_A = \frac{h\alpha}{\sqrt{2}}\ ,\quad a_B = \frac{h\alpha}{\sqrt{2}}\ ,\quad \bar{a} = \frac{h\alpha}{2}$$

$$\Sigma M_C = \bar{I}\alpha + m\,\bar{a}\,d$$

$$mg\frac{h}{2\sqrt{2}} = \frac{1}{12}m\,h^2\alpha + m\frac{h\alpha}{2}\frac{h}{2} \Rightarrow \alpha = \frac{3g}{2\sqrt{2}\,h}$$

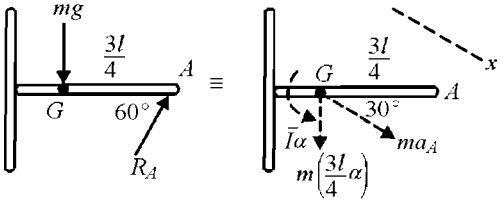
$$\Sigma F_x = m\,\bar{a}_x : 2B = m\frac{h\alpha}{2}\frac{1}{\sqrt{2}} \Rightarrow B = \frac{m\,h}{4\sqrt{2}}\frac{3g}{2\sqrt{2}\,h} \Rightarrow B = \frac{3}{16}mg$$

$$\Sigma F_y = m\,\bar{a}_y : mg - 2A = m\frac{h\alpha}{2}\frac{1}{\sqrt{2}} \Rightarrow 2A = mg - \frac{mh}{2\sqrt{2}}\frac{3g}{2\sqrt{2}\,h} \Rightarrow A = \frac{5}{16}mg$$

۶-۱۰۹.

$$\bar{r} = \frac{\frac{m}{2}\frac{l}{2} + \frac{m}{2}l}{m} = \frac{3l}{4}$$

$$\bar{I} = \left[\frac{1}{12}\frac{m}{2}l^2 + \frac{m}{2}\left(\frac{l}{4}\right)^2\right]2 = \frac{7}{48}ml^2$$



$\curvearrowleft \Sigma M_A = \bar{I}\alpha + \Sigma m\bar{a}d$:

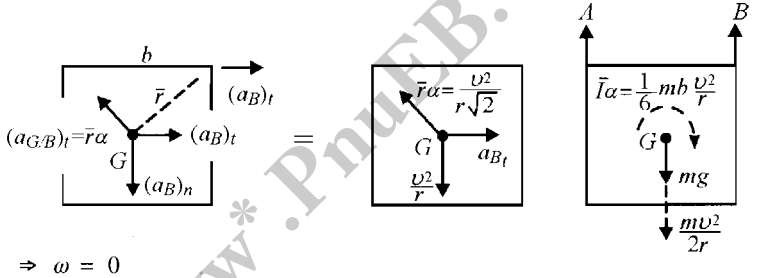
$$mg\left(\frac{3l}{4}\right) = \frac{7}{48}ml^2\alpha + ma_A\left(\frac{3l}{4}\cdot\frac{1}{2}\right) + m\frac{3l}{4}\alpha\left(\frac{3l}{4}\right)$$

$$\Sigma F_x = m\bar{a}_x \;:\; mg\sin 30^\circ = m\left(a_A + \frac{3l}{4}\alpha\frac{1}{2}\right)$$

$\Rightarrow$ حل همزمان

$$\begin{cases} \alpha = \dfrac{108}{109}\dfrac{g}{l} \\ a_A = \dfrac{14}{109}g \end{cases}$$

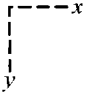
۶-۱۱۰.



$$v_{B/A} = 0 \;\Rightarrow\; \omega = 0$$

$$(a_{B/A})_t = b\alpha = \frac{v^2}{r}$$

$$\Rightarrow\; \alpha = \frac{v^2}{br}$$



$$\mathbf{a}_G = (\mathbf{a}_B)_n + (\mathbf{a}_B)_t + (\mathbf{a}_{G/B})_t$$

$$\Sigma F_x = m\bar{a}_x \;:\; 0 = m\left(a_{B_t} - \frac{v^2}{r\sqrt{2}}\frac{1}{\sqrt{2}}\right) \;\Rightarrow\; a_B = \frac{v^2}{2r}$$

$$a_G = \bar{a}_y = \frac{v^2}{r} - \frac{v^2}{r\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{v^2}{2r}$$

$$\bar{I}\alpha = \frac{1}{6}mb^2\frac{v^2}{br} = \frac{1}{6}mb\frac{v^2}{r}$$

$$\left.\begin{aligned} \Sigma M_G = \bar{I}\alpha \;:\; (A - B)\frac{b}{2} &= \frac{1}{6}mb\frac{v^2}{r} \\ \Sigma F_y = m\bar{a}_y \;:\; mg - (A + B) &= \frac{mv^2}{2r} \end{aligned}\right\} \Rightarrow \begin{cases} A = \dfrac{m}{2}\left(g - \dfrac{v^2}{6r}\right) \\ B = \dfrac{m}{2}\left(g - \dfrac{5v^2}{6r}\right) \end{cases}$$

۶-۱۱۱. ◀

$$(A)\quad v_O = r\omega = (R - r)(-\dot\theta) = -0.400\,\dot\theta \text{ m/s}$$

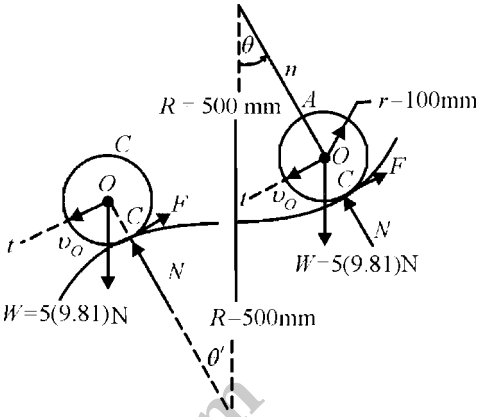
$$(a_O)_t = \dot v_O = -400\,\ddot\theta$$

$$\alpha = \frac{(a_O)_t}{r} = \frac{-400\,\ddot\theta}{0.100} = -4\,\ddot\theta$$

$$(C)\quad v_O = r\omega = (R + r)(\dot\theta') = 0.600\,\dot\theta'$$

$$(a_O)_t = \dot v_O = 0.600\,\ddot\theta'$$

$$\alpha = \frac{(a_O)_t}{r} = 6\,\ddot\theta'$$



$$I_O = m\,k^2 = 5(0.080)^2 = 0.032\,\text{kg.m}^2$$

$$I_C = 0.032 + 5(0.100)^2 = 0.082\,\text{kg.m}^2$$

$$(A)\ :\ \Sigma M_C = I_C\alpha\ :\ 5(9.81)(0.1\sin\theta) = 0.082\,\alpha = 0.082(-4\,\ddot\theta) \Rightarrow \ddot\theta = -14.95\sin\theta$$

$$\int_0^{\dot\theta}\dot\theta\,d\dot\theta = \int_{\frac{\pi}{4}}^{0}\ddot\theta\,d\theta \Rightarrow \frac{\dot\theta^2}{2} = -14.95\int_{\frac{\pi}{4}}^{0}\sin\theta\,d\theta \Rightarrow \dot\theta^2 = 8.76 \Rightarrow \dot\theta = -2.96 \text{ rad/s}$$

$$\left(= -\frac{3}{2}\dot\theta' \quad \text{در} \quad \theta = \theta' = 0\right)$$

$$(\Sigma F_n = m\,\bar a_n)_{\theta\to 0}\ :\ N - 5(9.81) = 5(0.400)8.76 \Rightarrow N = 66.6\,\text{N} \qquad (\text{الف})$$

$$(C)\ :\ \Sigma M_C = I_C\alpha\ :\ 5(9.81)(0.1\sin\theta') = 0.082\,\alpha = 0.082\,(6\,\ddot\theta') \Rightarrow \ddot\theta' = 9.97\sin\theta'$$

$$\int_{2.96\times\frac{2}{3}}^{\dot\theta'}\dot\theta'\,d\dot\theta' = \int_0^{\theta'}\ddot\theta'\,d\theta' \Rightarrow \frac{1}{2}(\dot\theta'^2 - 8.76\times\frac{4}{9}) = 9.97\int_0^{\theta'}\sin\theta'\,d\theta'$$

$$\Rightarrow \dot\theta'^2 = 23.8 - 19.94\cos\theta'$$

$$\Sigma F_n = m\,\bar a_n\ :\ 5(9.81)\cos\theta' - N = 5(0.600)(23.8 - 19.94\cos\theta')$$

$\theta' \to 0$ برای : $N = 37.4\,\text{N}$ (ب)

$N = 0$ برای : $8(9.81)\cos\theta' = 5(0.600)(23.8 - 19.94\cos\theta') \Rightarrow \cos\theta' = 0.657$

$$\Rightarrow \theta' = 48.9° \qquad (\text{ج})$$

۶-۱۱۲. ◀

A اتاقک : $+\circlearrowright \Sigma M_A = \bar I\alpha_A + \Sigma m\,a_A\,d$:

$$0 = \bar I\alpha_A + m\,h^2\alpha_A - m\,R\,\alpha\,h\sin\theta$$

از طرفی : $\bar I + m\,h^2 = I_A = mk^2$

$$\Rightarrow I_A\alpha_A = m\,R\,h\,\alpha\sin\theta \quad \text{یا} \quad m\,\alpha_A = m\,R\,h\,\alpha\sin\frac{\theta}{k^2}$$

$\Sigma F_\theta = m\,\bar{a}_\theta$:

$A_\theta + m\,g\cos\theta = m\,R\,\alpha - m\,h\,\alpha_A \sin\theta$

$\Rightarrow A_\theta = m\,(R\,\alpha - g\cos\theta) - m\,R\,\alpha\left(\frac{h\sin\theta}{k}\right)^2$ (1)

B اتاقک : $\curvearrowleft + \Sigma M_B = \bar{I}\alpha_B + \Sigma m\,a_B\,d$:

$0 = \bar{I}\alpha_B + m\,h^2\alpha_B - m\,R\,\alpha\,h\sin\theta$

$I_B\alpha_B = m\,R\,h\,\alpha\sin\theta$

یا $m\,\alpha_B = m\,R\,h\,\alpha\sin\frac{\theta}{k^2}$

($I_B = I_A = mk^2$ که در آن)

$\Sigma F_\theta = m\,\bar{a}_\theta$:

$B_\theta - m\,g\cos\theta = m\,R\,\alpha - m\,h\,\alpha_B\sin\theta$

$\Rightarrow B_\theta = m(R\,\alpha + g\cos\theta)$

$- m\,R\,\alpha\left(\frac{h\sin\theta}{k}\right)^2$ (2)

چرخ و فلک :

$\curvearrowright + \Sigma M_O = I_O\alpha$:

$[F - \Sigma(A_\theta + B_\theta)]R = I_O\alpha$ (3)

جاگذاری (1) و (2) در (3) :

$$FR - \sum_{1}^{\frac{n}{2}}\left[2m\,R\,\alpha - 2m\,R\,\alpha\left(\frac{h\sin\theta_n}{k}\right)^2\right]R = I_O\alpha$$

ساده‌سازی :

$$F = \left\{m\,R\left[n - 2\frac{h^2}{k^2}\left(\sin^2\theta_1 + \sin^2\theta_2 + \ldots + \sin^2\theta_{\frac{n}{2}}\right)\right] + \frac{I_O}{R}\right\}\alpha$$

($n = 0$ مربوط است به $\theta = 0$ و $\frac{n}{2}$ مربوط به $\theta < \pi$)

نکته: عبارت فوق را می‌توان به صورت زیر ساده نمود:

$$F = \left\{m\,R\,n\left(1 - \frac{h^2}{2k^2}\right) + \frac{I_O}{R}\right\}\alpha$$

۶-۱۱۳.

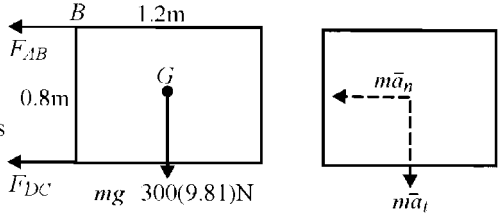
$U'_{1-2} = \Delta T + \Delta V_g$

$0 = \frac{1}{2}m(4^2 - 0^2) - mg(5)(1 - \cos\theta) \quad \Rightarrow \quad \theta = 33.2°$

۶-۱۱۴. ورق جابجایی منحنی‌الخط دارد لذا $T = \frac{1}{2} m v^2$

$$U = \Delta T \; : \; 300(9.81)(0.8\cos 60°) = \frac{1}{2} 300 v^2 \; \Rightarrow \; v = 2.80 \text{ m/s}$$

$\omega = \frac{v}{r}$:



سرعت زاویه‌ای میله‌ها $\omega = \frac{2.80}{0.8} = 3.50 \text{ rad/s}$

$$\Sigma F_t = m\,\bar{a}_t \; : \; \bar{a}_t = 9.81 \text{ m/s}^2$$

$$\bar{a}_n = \frac{v^2}{r} \; : \; \bar{a}_n = \frac{2.80^2}{0.8} = 9.81 \text{ m/s}^2$$

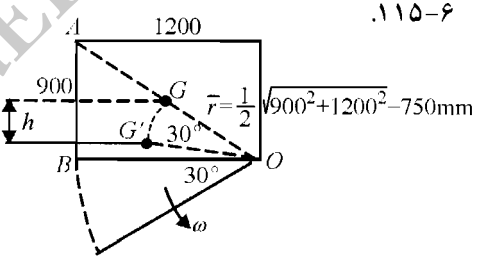
$$\Sigma M_B = m\,\bar{a}\,d \; : \; 300(9.81)(0.6) + F_x(0.8) = 300(9.81)(0.6) + 300(9.81)(0.4)$$

$$\Rightarrow \; F_{DC} = 1472 \text{ N}$$

۶-۱۱۵.

$$\angle AOB = \tan^{-1}\frac{900}{1200} = 36.9°$$

$$\angle G'OB = 36.9° - 30° = 6.87°$$



$$h = 750\sin 36.9° - 750\sin 6.87° = 360.3 \text{ mm}$$

$$U = \Delta T \; : \; mgh = \frac{1}{2} I_O \omega^2$$

$$120(9.81)(0.3603) = \frac{1}{2}\frac{1}{3} 120\,(0.9^2 + 1.2^2)\,\omega^2$$

$$\Rightarrow \; \omega = 3.07 \text{ rad/s}$$

جرم از دو طرف رابطه حذف می‌شود. بنابراین اثری روی نتیجه ندارد.

۶-۱۱۶. برای دوران $T_{rot} = \frac{1}{2} I_C \omega^2 = \frac{1}{2}(4)\left(\frac{1}{12} m b^2 + m\left(\frac{b}{2}\right)^2\right)\omega^2 = \frac{2}{3} m b^2 \omega^2$

برای انتقال $T_{tran} = \frac{1}{2}(4\,m)\,v^2 = 2\,m\,v^2$

$$T_{rot} = T_{tran} \; : \; 2\,m\,v^2 = \frac{2}{3} m b^2 \omega^2 \; \Rightarrow \; v = \frac{b\omega}{\sqrt{3}}$$

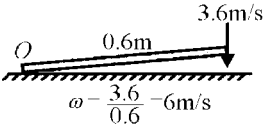
۶-۱۱۷.

$$U' = \Delta T + \Delta V_g$$

$$\Delta T = 2\left(\frac{1}{2} I_O \omega^2 - 0\right) = 2\left[\frac{1}{2}\left(\frac{1}{3} 30 \times 0.6^2\right) 6^2\right] = 129.6\,\text{J}$$

$$\Delta V_g = -mgh = -2\{30 \times 9.81 \times 0.3 \sin 60°\} = -152.9\,\text{J}$$

$$U' = 129.6 - 152.9 = -23.3\,\text{J} \Rightarrow Q = 23.3\,\text{J}$$



۶-۱۱۸.

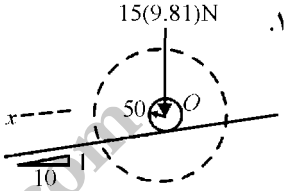
$$\theta = \tan^{-1}\frac{1}{10} \Rightarrow \sin\theta = 0.0995$$

$$U = \Delta T$$

$$U = 15(9.81)(0.0995)(3) = 43.92\,\text{J}$$

$$\Delta T = \frac{1}{2} m v^2 + \frac{1}{2} I_O \omega^2 - 0 = \frac{1}{2} 15 v^2 + \frac{1}{2} 15 (0.125)^2 \left(\frac{v}{0.05}\right)^2 - 0 = 54.375 v^2$$

$$\Rightarrow 43.92 = 54.375 v^2 \Rightarrow v = 0.899\,\text{m/s}$$



۶-۱۱۹.

$$U = \Delta T$$

$$U = M\theta = 22(0.150)6(2\pi) = 124.4\,\text{J}$$

$$\Delta T = \Sigma \frac{1}{2} I \omega^2 = \Sigma \frac{1}{2} m k^2 \omega^2 = \frac{1}{2}(1.8)(0.072)^2\left(\frac{\omega}{4}\right)^2 + \frac{1}{2}(0.55)(0.054)^2\omega^2$$

$$= 1.0935(10^{-3})\omega^2$$

$$\Rightarrow \omega^2 = \frac{124.4}{1.0935(10^{-3})} = 1.137(10^5) \Rightarrow \omega = 337\,\text{rad/s}$$

$$N = 337\left(\frac{60}{2\pi}\right) = 3220\,\text{rpm}$$

۶-۱۲۰. نکته: دیسک هیچ حرکتی در موقعیت‌های اولیه یا نهایی ندارد لذا $\Delta T_{wheel} = 0$

$$U' = \Delta V_g + \Delta T$$

$$U' = Fb\sin\theta$$

$$\Delta V_g = -2 m_0 g \frac{b}{2}\sin\theta$$

$$\Delta T = 2\left(\frac{1}{2} I_C \omega^2\right) = \frac{1}{3} m_0 b^2 \omega^2$$

$$\Rightarrow Fb\sin\theta = -m_0 g b \sin\theta + \frac{1}{3} m_0 b^2 \omega^2 \Rightarrow \omega = \sqrt{\frac{3(F + m_0 g)\sin\theta}{m_0 b}}$$

۶-۱۲۱.

$$\Delta V_g = -200(9.81)(1.25) = -2453\,\text{J}$$

هر فنر به اندازهٔ 1.25 m کشیده می‌شود بنابراین:

$$\Delta V_e = 2\left[\frac{1}{2}k(1.25)^2 - 0\right] = 1.563k\ \text{J}$$

$$\Delta T = \frac{1}{2}I_{()}\omega^2 = \frac{1}{2}200\left(\frac{1}{12}2.5^2 + 1.25^2\right)1.5^2 = 469\,\text{J}$$

$$\Delta T + \Delta V_g + \Delta V_e = 0 \;:\; 469 - 2453 + 1.563k = 0 \Rightarrow k = 1270\,\text{N/m} \quad \text{یا} \quad k = 1.270\,\text{kN/m}$$

۶-۱۲۲.

$$U'_{1\text{-}2} = \Delta T + \Delta V_g + \Delta V_e$$

$$U'_{1\text{-}2} = M\theta = \frac{\pi}{2}M = 1.571M\ \text{J}$$

$$\Delta T = \frac{1}{2}I_{()}\omega^2 - 0 = \frac{1}{2}(5.5)(0.250)^2(4)^2 = 2.75\,\text{J}$$

$$\Delta V_g = mgh = 5.5(9.81)(-0.200) = -10.791\,\text{J}$$

$$\Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}(525)\left([0.75 - 0.375\sqrt{2}\,]^2 - 0\right) = 12.67\,\text{J}$$

$$\Rightarrow 1.571M = 2.75 - 10.791 + 12.67 \Rightarrow M = 2.95\,\text{N/m}$$

۶-۱۲۳.

$$T_1 + U_{1\text{-}2} = T_2$$

$$T_1 = 0$$

$$U_{1\text{-}2} = \int_1^2 M d\theta = \int_0^{5(2\pi)} 2(1 - e^{-0.1\theta})\,d\theta = (2\theta + 20e^{-0.1\theta})\Big|_0^{5(2\pi)}$$

$$= 2(5)(2\pi) + 20e^{-0.1(5)(2\pi)} - 20 = 43.7\,\text{J}$$

$$T_2 = \frac{1}{2}I\omega^2 = \frac{1}{2}(50)(0.4)^2\omega^2 = 4\omega^2$$

$$\Rightarrow 0 + 43.7 = 4\omega^2 \Rightarrow \omega = 3.31\,\text{rad/s}$$

۶-۱۲۴.

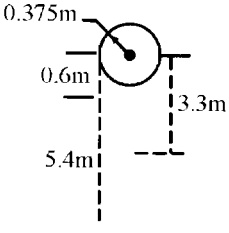
$$\Delta V_g + \Delta T = 0$$

$$\Delta V_g = -5.4(3.08)(9.81)(3.3) = -538\,\text{J}$$

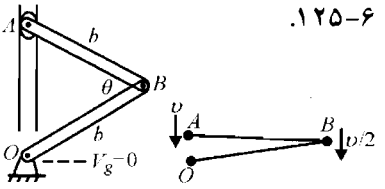
$$\Delta T = \frac{1}{2}6.0(3.08)(0.375\,\omega)^2 + \frac{1}{2}[41(0.30)^2 + (3.08)(18 - 6)(0.375)^2]\omega^2$$

$$= 1.299\omega^2 + 4.44\omega^2 = 5.77\omega^2$$

$$\Rightarrow -538 + 5.74\omega^2 = 0 \Rightarrow \omega = 9.68\,\text{rad/s}$$



۶-۱۲۵.



$$OB:\ \Delta V_g = -mg\frac{b}{2}\sin\frac{\theta}{2}$$

$$\Delta T = \frac{1}{2}I_O\omega^2 - 0 = \frac{1}{2}\frac{1}{3}mb^2\left(\frac{\frac{v}{2}}{b}\right)^2 = \frac{1}{24}mv^2$$

$$AB:\ \Delta V_g = -mg\frac{3b}{2}\sin\frac{\theta}{2}$$

$$\Delta T = \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}m\bar{v}^2 = \frac{1}{2}\frac{1}{12}mb^2\left(\frac{v}{2b}\right)^2 + \frac{1}{2}m\left(\frac{3v}{4}\right)^2 = \frac{7}{24}mv^2$$

$$U' = 0 = \Delta T + \Delta V_g$$

$$0 = \left(\frac{7}{24}+\frac{1}{24}\right)mv^2 - \left(\frac{3b}{2}+\frac{b}{2}\right)mg\sin\frac{\theta}{2} = 0$$

$$v^2 = 6gb\sin\frac{\theta}{2} \quad\Rightarrow\quad v = \sqrt{6gb\sin\frac{\theta}{2}}$$

۶-۱۲۶.

$$T_1 + U_{1\text{-}2} = T_2$$

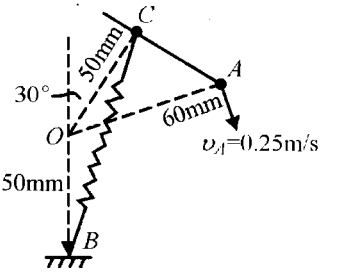
$$mg\left(\frac{l}{2}-x\right) = \frac{1}{2}\left[\frac{1}{12}ml^2 + m\left(\frac{l}{2}-x\right)^2\right]\omega^2 \quad\Rightarrow\quad \omega^2 = \frac{g\left(\frac{l}{2}-x\right)}{\frac{l^2}{6}-\frac{lx}{2}+\frac{x^2}{2}}$$

$$\frac{d\omega^2}{dx} = 0 \quad\Rightarrow\quad x = 0.789l \quad \text{یا} \quad x = 0.211l$$

$$\omega_{\max} = \omega_{x=0.22l} = \sqrt{\frac{g\left(\frac{l}{2}-0.211l\right)}{\frac{l^2}{6}-\frac{0.211l^2}{2}+\frac{(0.211l)^2}{2}}} = 1.861\sqrt{\frac{g}{l}}$$

(جواب $x = 0.789l$ منتهی به یک $\omega_{\max}$ خواهد شد تنها بعد از آنکه حرکت بر خلاف عقربه‌های ساعت شود)

۶-۱۲۷. فرض کنید l_0 طول آزاد فنر باشد.



$$\Delta V_e = \frac{1}{2}(1400)\left[(\overline{BC}-l_0)^2 - (0.10-l_0)^2\right]$$

$$\overline{BC} = 2(0.05)\cos 15° = 0.0966\,\text{m}$$

$$\Delta V_g = 0 \quad \text{(صفحه افق)}$$

$$\Delta T = \frac{1}{2}I_O\omega^2 = \frac{1}{2}(1.5)(0.55)^2\left(\frac{0.25}{0.06}\right)^2 = 0.0394\,\text{J}$$

$$U' = 0$$

$$U' = \Delta T + \Delta V_e = 0$$

$$0.0394 + 700[(0.0966-l_0)^2 - (0.10-l_0)^2] = 0 \quad\Rightarrow\quad l_0 = 0.0900\,\text{m} \quad \text{یا} \quad l_0 = 90.0\,\text{mm}$$

۶-۱۲۸. برای موقعیت بالایی $\omega_B = \frac{v}{0.080}$, $\omega_{OA} = \frac{v}{0.280}$

برای کل سیستم : $U'_{1-2} = \Delta T + \Delta V_g$

$U'_{1-2} = M\theta = 4\left(\frac{\pi}{2}\right) = 6.28\,\text{J}$

$\Delta T_{OA} = \frac{1}{2} I_O \omega_{OA}^2 = \frac{1}{2} 0.8\,(0.140^2)\left(\frac{v}{0.0280}\right)^2 = 0.1\,v^2\,\text{J}$

$\Delta T_B = \frac{1}{2} m v^2 + \frac{1}{2} \bar{I}\omega^2 = \frac{1}{2} 0.9 v^2 + \frac{1}{2}\left[\frac{1}{2} 0.9 \times 0.080^2\right]\left(\frac{v}{0.080}\right)^2 = 0.675\,v^2\,\text{J}$

$(\Delta V_g)_{OA} = m g h = 0.8(9.81)(0.100) = 0.785\,\text{J}$

$(\Delta V_g)_B = m g h = 0.9(9.81)(0.280) = 2.47\,\text{J}$

$\Rightarrow \quad 6.28 = 0.1\,v^2 + 0.675\,v^2 + 0.785 + 2.47 \quad \Rightarrow \quad v = 1.976\,\text{m/s}$

۶-۱۲۹. $0 = \Delta V_g + \Delta T$

$\Delta V_g = -100(9.81)\left[0.300\sin 30° + 0.450(1 - \cos 30°)\right] = -206.3\,\text{J}$

$\Delta T = \frac{1}{2} I_C(\omega_2^2 - \omega_1^2)$

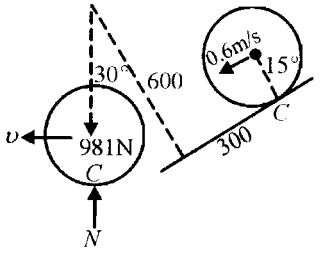
$I_C = 100(0.1^2 + 0.15^2) = 3.25\ \text{kg.m}^2$

$\omega_1 = \frac{0.6}{0.15} = 4\,\text{rad/s}$, $\omega_2 = \frac{v}{0.15} = 6.67\,v$

$\Rightarrow \quad \Delta T = \frac{1}{2}(3.25)[(6.67\,v)^2 - 4^2] = 72.2\,v^2 - 26\,\text{J}$

$\Rightarrow \quad 0 = -206.3 + 72.2\,v^2 - 26 \quad \Rightarrow \quad v^2 = 3.216 \quad \Rightarrow \quad v = 1.79\,\text{m/s}$

$\Sigma F_n = m\,\bar{a}_n$: $N - 100(9.81) = 100\left(\frac{3.216}{0.450}\right) \quad \Rightarrow \quad N = 1696\,\text{N}$



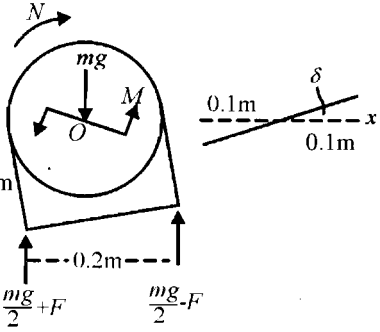
۶-۱۳۰. توان $P = M\omega = M\,\frac{2\pi N}{60} \Rightarrow$

$M = \frac{4000(60)}{2\pi(1725)} = 22.14\,\text{N/m}$

$\Sigma M_O = 0$: $2F(0.2) - 22.14 = 0 \quad \Rightarrow \quad F = 55.4\,\text{N}$

$F = kx$: $x = \frac{55.4}{15(10^3)} = 0.00369\,\text{m}$ یا $x = 3.69\,\text{mm}$

$\delta = \tan^{-1}\frac{x}{0.1} = \tan^{-1}\frac{3.69}{100} = 2.11°$



محور موتور در جهت عقربه‌های ساعت می‌چرخد.

۶-۱۳۱. برای کل سیستم $U' = \Delta T + \Delta V_g + \Delta V_e$

$U' = 0$

$\Delta T = \frac{1}{2} I_{(}\omega^2 = \frac{1}{2}\left[\frac{1}{3}4.5(0.500)^2\right](1.5)^2 = 0.42\,\text{J}$

$\Delta V_g = -mgh = -4.5(9.81)(0.250) = -11.04\,\text{J}$

$\Delta V_e = 2\left(\frac{1}{2}kx^2\right) = k(0.05)^2 = 2.5(10^{-3})k\,\text{J}$

$\Rightarrow 0 = 0.42 - 11.04 + 2.5(10^{-3})k \Rightarrow k = 4250\,\text{N/m}$ یا $4.25\,\text{kN/m}$

۶-۱۳۲. $\Delta V_e = \frac{1}{2}(1500)[(0.1 + 2\times 0.05)^2 - 0.1^2] = 22.5\,\text{J}$

$\Delta V_g = -(150)(9.81)(0.05) = -73.58\,\text{J}$

$\Delta T = \Sigma\frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}(150)v^2 + \frac{1}{2}(50)(0.3)^2\left(\frac{v}{0.4}\right)^2 = 75v^2 + 14.06v^2 = 89.06v^2$

$\Delta T + \Delta V_g + \Delta V_e = 0 \quad : \quad 89.06v^2 - 73.58 + 22.5 = 0 \Rightarrow v = 0.757\,\text{m/s}$

۶-۱۳۳. $I = mk^2 = 10(0.090)^2 = 0.081\,\text{kg.m}^2$

$M = I\dot{\omega} \Rightarrow \dot{\omega} = \frac{M}{I} = \frac{-2.10}{0.081} = -25.9\,\text{rad/s}^2$

$\omega_0 = 80000\left(\frac{2\pi}{60}\right) = 8380\,\text{rad/s}$

$P = \frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = I\omega\dot{\omega}$

(الف) $t = 0$

$P = I\omega\dot{\omega} = (0.081)(8380)(25.9) = 17590\,\text{W}$ یا $17.59\,\text{kW}$

(ب) $t = 120\,\text{s}$

$\omega = \omega_0 + \dot{\omega}t = 8380 - 25.9(120) = 5270\,\text{rad/s}$

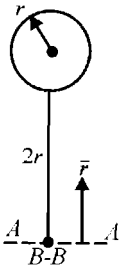
$P = I\omega\dot{\omega} = (0.081)(5270)(25.9)$

$11060\,\text{W}$ یا $11.06\,\text{kW}$

۶-۱۳۴.

جرم کل $m = 2r\rho + 2\pi r\rho = 2r\rho(1+\pi)$ (ρ = جرم واحد طول)

$$\bar{r} = \frac{\Sigma \bar{r} m}{\Sigma m} = \frac{2r\rho(r) + 2\pi r\rho(3r)}{2r\rho + 2\pi r\rho} = r\frac{1+3\pi}{1+\pi}$$



$$I_{B-B} = \frac{1}{3}(2r\rho)(2r)^2 + [2\pi r\rho r^2 + 2\pi r\rho(3r)^2] = \frac{4+30\pi}{3(1+\pi)} m r^2$$

$$I_{A-A} = \frac{1}{3}(2r\rho)(2r)^2 + [\frac{1}{2}2\pi r\rho r^2 + 2\pi r\rho(3r)^2] = \frac{8+57\pi}{6(1+\pi)} m r^2$$

$T_1 + U_{1-2} = T_2$

(الف) $0 + mgr\frac{1+3\pi}{1+\pi} = \frac{1}{2}\frac{8+57\pi}{6(1+\pi)} m r^2\omega^2 \Rightarrow \omega = 2\sqrt{\frac{3+9\pi}{8+57\pi}\frac{g}{r}}$

(ب) $0 + mgr\frac{1+3\pi}{1+\pi} = \frac{1}{2}\frac{4+30\pi}{3(1+\pi)} m r^2\omega^2 \Rightarrow \omega = \sqrt{\frac{3+9\pi}{2+15\pi}\frac{g}{r}}$

۶-۱۳۵.

$\Delta T_{\text{جابجایی}} = \frac{1}{2}mv^2 - 0 = \frac{1}{2}(10000)\left(\frac{72}{3.6}\right)^2 - 0 = 2(10^6)$ J

$\Delta T_{\text{دورانی}} = \frac{1}{2}I(\omega_2^2 - \omega_1^2) = \frac{1}{2}(1500)(0.5)^2\omega_2^2 - \left[\frac{400\times 2\pi}{60}\right]^2$

$= 187.5\omega_2^2 - 32.90(10^6)$ J

$\Delta E = 0.1(187.5\omega_2^2 - 32.90(10^6)) = 18.75\omega_2^2 - 3.29(10^6)$ J

$\Delta V_g = mgh = 10000(9.81)(20) = 1.96(10^6)$ J

$\Delta E = \Delta T + \Delta V_g$

$18.75\omega_2^2 - 3.29(10^6) = 2(10^6) + 187.5\omega_2^2 - 32.90(10^6) + 1.96(10^6)$

$168.75\omega_2^2 = 25.65(10^6) \Rightarrow \omega_2^2 = 152000 \Rightarrow \omega_2 = 390$ rad/s

$N = 390\left(\frac{60}{2\pi}\right) = 3720$ rpm

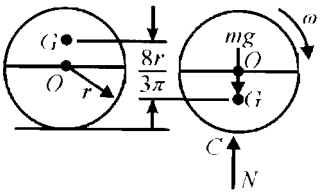
۶-۱۳۶.

$U = \Delta T : mg\left(\frac{8r}{3\pi}\right) = \frac{1}{2}I_C\omega^2$

$I_G = I_O - m\bar{r}^2$

$I_C = I_G + m(r-\bar{r})^2$

$\Rightarrow I_C = I_O - m\bar{r}^2 + m(r-\bar{r})^2$



$$I_C = m\left(\frac{1}{2}r^2 - \bar{r}^2 + r^2 - 2r\bar{r} + \bar{r}^2\right) = m\left(\frac{3}{2}r^2 - 2r\left(\frac{4r}{3\pi}\right)\right) = mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)$$

$$\Rightarrow mg\left(\frac{8r}{3\pi}\right) = \frac{1}{2}mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\omega^2 \Rightarrow \omega^2 = \frac{32}{9\pi - 16}\frac{g}{r} \Rightarrow \omega = \sqrt{\frac{g}{r}\frac{32}{9\pi - 16}}\ \text{rad/s}$$

$$\Sigma F_n = m\bar{a}_n : N - mg = m\bar{r}\omega^2 \Rightarrow N = mg + m\frac{4r}{3\pi}\omega^2 \Rightarrow N = mg\left(1 + \frac{128}{3\pi(9\pi - 16)}\right)$$

۶-۱۳۷. کشیدگی هر فنر 1.2 m است بنابراین:

$$\Delta V_e = 2\left(\frac{1}{2}kx^2\right) = 2\left(\frac{1}{2}700[1.2]^2\right) = 1008\,\text{J}$$

$$\Delta V_g = -100(9.81)(2.7 - 1.2) = -1471.5\,\text{J}$$

$$U' = \Delta T + \Delta V_g + \Delta V_e$$

$$0 = \frac{1}{2}(100)v^2 - 1471.5 + 1008 \Rightarrow v^2 = 9.27 \Rightarrow v = 3.04\ \text{m/s}$$

۶-۱۳۸.

$$U = \Delta T$$

$$T = 2\left(\frac{1}{2}\left[2\pi r\rho(r^2 + r^2)\frac{v^2}{r^2}\right] + \frac{1}{2}(\rho b)(2v)^2\right) = 2(2\pi\rho r v^2 + 2\rho b v^2)$$

$$U = M\theta = M\frac{s}{r} \Rightarrow M\frac{s}{r} = 2(2\pi\rho r v^2 + 2\rho b v^2) \Rightarrow M = 4\rho\frac{r}{s}v^2(\pi r + b)$$

۶-۱۳۹. فرض کنید x = مسافت طی شده توسط نقطه O برحسب متر باشد.

$$\theta = \tan^{-1}\frac{1}{5} = 11.31^\circ\ ,\ \sin\theta = 0.1961$$

$$\Delta V_g = mg\Delta h = mgx\sin\theta = 10(9.81)x(0.1961) = 19.24x$$

$$\Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}(600)\left[\left(0.225 - \frac{275}{200}x\right)^2 - (0.225)^2\right] = 567.2x^2 - 185.6x$$

$$\Delta T = \frac{1}{2}mv^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}(10)v^2 + \frac{1}{2}(10)(0.125)^2\left(\frac{v}{0.2}\right)^2 = 6.95v^2$$

برای سیستم : $U' = \Delta T + \Delta V_g + \Delta V_e$

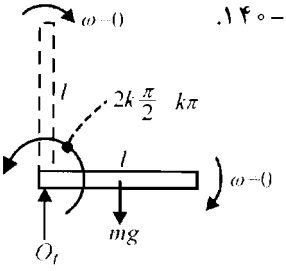
$$0 = 6.95v^2 + 19.24x + 567.2x^2 - 185.6x$$

$$v^2 = 23.93x - 81.57x^2$$

برای v_{max} : $\frac{dv^2}{dx} = 0 \Rightarrow x = 0.1467\,\text{m}$

$$v_{max}^2 = 23.93(0.1467) - 81.57(0.1467)^2 \Rightarrow v_{max} = 1.325\ \text{m/s}$$

۶-۱۴۰.



$$U' = \Delta T + \Delta V_g + \Delta V_e$$

$$U' = \Delta T = 0 \quad , \quad \Delta V_g = -mg\frac{l}{2}$$

$$\Delta V_e = 2\int_0^{\frac{\pi}{2}} k\theta\, d\theta = k\theta^2\Big|_0^{\frac{\pi}{2}} = \frac{\pi^2}{4}k$$

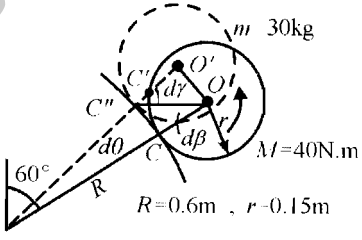
$$\Rightarrow 0 = -mg\frac{l}{2} + \frac{\pi^2}{4}k \Rightarrow k = \frac{2l}{\pi^2}mg$$

رها کردن از وضعیت : $\theta = \frac{\pi}{2}$

$$\Sigma M_O = I_O\alpha \; : \; \frac{2l}{\pi^2}mg\pi - mg\frac{l}{2} = \frac{1}{3}ml^2\alpha \Rightarrow \alpha = 0.410\frac{g}{l}$$

درب پائین نگه داشته نخواهد شد. $k = \frac{2l}{\pi^2}mg$ عملی نیست.

۶-۱۴۱. طی دوران خط شعاعی به اندازهٔ $d\theta$، دیسک بین خطوط OC' و $O'C''$ به اندازه $d\gamma$ دوران می‌کند.



$$CC' = CC'' \Rightarrow R\,d\theta = r\,d\beta$$

$$d\gamma = d\theta + d\beta = \left(1 + \frac{R}{r}\right)d\theta \Rightarrow \gamma = \left(1 + \frac{R}{r}\right)\theta$$

$$\theta = \frac{\pi}{3} \Rightarrow \gamma = \left(1 + \frac{0.6}{0.15}\right)\frac{\pi}{3} = 5\frac{\pi}{3}\ \text{rad}$$

$$U' = \Delta T + \Delta V_g$$

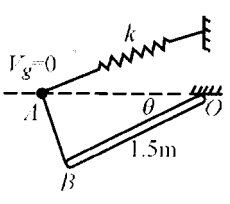
$$U' = M\gamma = 40\frac{5\pi}{3} = 209\,\text{J}$$

$$\Delta T = \frac{1}{2}I_C\omega^2 = \frac{1}{2}\left(\frac{3}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{3}{4}mv^2 = \frac{3}{4}(30)v^2 = 22.5v^2$$

$$\Delta V_g = mgh = mg(R + r)(1 - \cos 60°) = 30(9.81)(0.75)\left(\frac{1}{2}\right) = 110.4\,\text{J}$$

$$\Rightarrow 209 = 22.5v^2 + 110.4 \Rightarrow v^2 = 4.40 \Rightarrow v = 2.10\,\text{m/s}$$

۶-۱۴۲.



$$\Delta V_g = 100(9.81)\left(-\frac{1.5}{2}\sin\theta\right) = -735.75\sin\theta$$

$$\Delta V_e = \frac{1}{2}k\,(\overline{AB}^2 - 0) = \frac{1}{2}200\left(2\times 1.5\sin\frac{\theta}{2}\right)^2 = 900\sin^2\frac{\theta}{2}$$

$$\Delta T = \frac{1}{2}I_O\omega^2 = \frac{1}{2}\frac{1}{3}100\,(1.5)^2\omega^2 = 37.5\omega^2$$

$$\Delta T + \Delta V_g + \Delta V_e = 0$$

$$37.5\omega^2 - 735.75\sin\theta + 900\sin^2\frac{\theta}{2} = 0$$

$$\omega^2 = 19.62\sin\theta - 24\sin^2\frac{\theta}{2}$$

ω_{max} برای : $\frac{d\omega^2}{d\theta} = 19.62\cos\theta - 12\sin\theta = 0 \Rightarrow \theta = \tan^{-1}\frac{19.62}{12} = 58.5°$

$\sin\theta = 0.853$, $\sin\frac{\theta}{2} = 0.489$

$\omega^2 = 19.62(0.853) - 24(0.489)^2 = 11.0\,(\text{rad/s})^2 \Rightarrow \omega = 3.32\,\text{rad/s}$

۶-۱۴۳.

OA میله : $\Delta V_g = 2(9.81)(0.1)(1 - \cos 45°) = 0.575\,\text{J}$

$I_O = m(\bar{k}^2 + \bar{r}^2) = 2\left([0.060]^2 + [0.1]^2\right) = 0.0272\,\text{kg.m}^2$

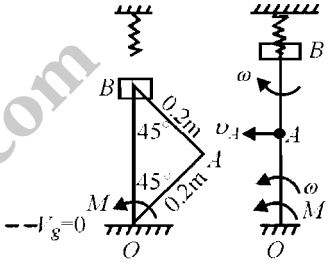
$\Delta T = \frac{1}{2} I_O \omega^2 = 0.0136\,\omega^2$

AB میله : $\Delta V_g = 2(9.81)[0.3 - 0.3\cos 45°] = 1.724\,\text{J}$

$\Delta T = 0.0136\,\omega^2$ (OA مشابه)

B لغزنده : $\Delta V_g = 3(9.81)(0.4)(1 - \cos 45°) = 3.448\,\text{J}$

$\Delta T = 0$



$\Delta V_e = \frac{1}{2} k x^2 = \frac{1}{2} 6000(0.050)^2 = 7.5\,\text{J}$

$U' = \int M\,d\theta = M\theta = 20\frac{\pi}{4} = 15.71\,\text{J}$

$U' = \Delta T + \Delta V_g + \Delta V_e$

$15.71 = 2(0.0136\,\omega^2) + (0.575 + 1.724 + 3.448) + 7.5$

$\Rightarrow \omega^2 = 90.5 \Rightarrow \omega = 9.51\,\text{rad/s}$

۶-۱۴۴.

$P = \frac{dU}{dt} = \frac{d}{dt}(T + V_g) + Rv$

$P = \frac{d}{dt}\left[\Sigma\frac{1}{2} m v^2 + \Sigma\frac{1}{2}\bar{I}\omega^2\right] + \frac{d}{dt}(mgh) + Rv$

$= \Sigma m v\frac{dv}{dt} + \Sigma\bar{I}\omega\frac{d\omega}{dt} + mgv\sin\theta + Rv$

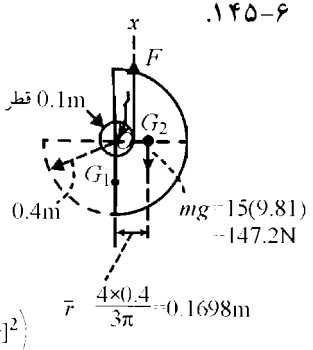
$= mv\alpha + 4\bar{I}\omega\alpha + (mg\sin\theta + R)v$

(الف) $a = 0$: $P = 0 + \left(500\times 9.81\times\frac{1}{\sqrt{101}} + 400\right)\frac{72}{3.6} = 17761\,\text{w}$ یا $P = 17.76\,\text{kW}$

(ب) $a = 3\,\text{m/s}^2$: $P = \left(500\frac{72}{3.6} + 4(40)(0.4)^2\frac{\frac{72}{3.6}}{0.6^2}\right)3 + 17761$

$= 30000 + 4267 + 17761 = 52028\,\text{W}$ یا $P = 52.0\,\text{kW}$

۶-۱۴۵.



برای تعادل : $\Sigma M_O = 0$

$0.05F_0 - 147.2(0.1698) = 0 \Rightarrow F_0 = 500\,\text{N}$

$F_0 = 2k\delta$ (کشیدگی اولیه فنر = δ)

در حالت تعادل $\delta = \dfrac{500}{2\times2.6\times10^3} = 0.0961\,\text{m}$

$U' = \Delta T + \Delta V_e + \Delta V_g = 0$

$\Delta T = \frac{1}{2}I_O\omega^2 - 0 = \frac{1}{2}\left(\frac{1}{2}\times15\times0.4^2\right)\omega^2 = 0.6\omega^2$

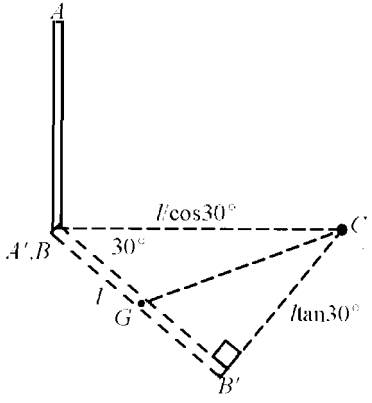
$\Delta V_e = 2\left(\frac{1}{2}k\Delta[x^2]\right) = 2.6\times10^3\left(0.0961^2 - [0.0961 + 0.05\,\frac{\pi}{2}]^2\right)$

$= 2.6\times10^3(0.0961^2 - 0.1746^2) = -55.3\,\text{J}$

$\Delta V_g = mg\Delta h = mg\bar{r} = 15\times9.81\times0.01698 = 25.0\,\text{J}$

$\Rightarrow 0 = 0.6\omega^2 - 55.3 + 25.0 \Rightarrow \omega^2 = 50.5 \Rightarrow \omega = 7.11\,\text{rad/s}$

۶-۱۴۶. نقطه C مرکز آنی دوران درست قبل از برخورد است.



$I_C = \frac{1}{12}ml^2 + m\left[\left(\frac{l}{2}\right)^2 + (l\tan30°)^2\right] = \frac{2}{3}ml^2$

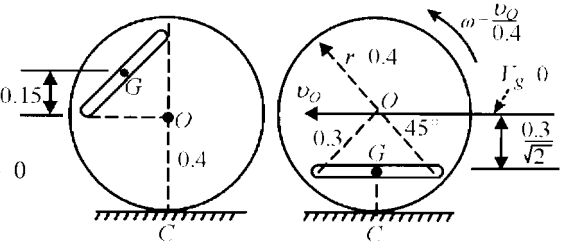
$T_1 + U_{1\text{-}2} = T_2$:

$0 + mg\left[\frac{l}{2} + \frac{l}{2}\sin30°\right] = \frac{1}{2}\left[\frac{2}{3}ml^2\right]\omega^2$

$\Rightarrow \omega = \frac{3}{2}\sqrt{\frac{g}{l}}$

$v_A = \frac{l}{\cos30°}\frac{3}{2}\sqrt{\frac{g}{l}} = \sqrt{3gl}$

۶-۱۴۷.



$v_G = \frac{\overline{CG}}{r}v_O$

$I_G = \frac{1}{12}9(0.3\sqrt{2})^2 = 0.318\,\text{kg.m}^2$

$U' = 0 = \Delta T + \Delta V_g \quad (1)$

$\Delta T_{\text{دیسک}} = \frac{1}{2}45v_O^2 + \frac{1}{2}\left[\frac{1}{2}45(0.4)^2\right]\left(\frac{v_o}{0.4}\right)^2 - 0$

$= 33.75v_O^2$

$\Delta T_{\text{میله}} = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$

$$= \frac{1}{2}9\left[\frac{0.4-\frac{0.3}{\sqrt{2}}}{0.4}v_0\right]^2 + \frac{1}{2}(0.318)\left(\frac{v_0}{0.4}\right)^2 - 0 = 1.986\,v_0^2$$

معادله (1) : $0 = (33.75 + 1.986)v_0^2 - 31.97 \Rightarrow v_0 = 0.95\ \text{m/s}$

۶-۱۴۸.

$$\bar{I} = 4\left[\frac{1}{12}\frac{m}{4}b^2 + \frac{m}{4}\left(\frac{b}{2}\right)^2\right] = \frac{1}{3}mb^2$$

$$I_B = \frac{1}{3}mb^2 + m\left(\frac{b\sqrt{2}}{2}\right)^2 = \frac{5}{6}mb^2$$

(الف) سقوط به اندازهٔ b $(v_{B'} = 0)$

$$T_1 + U_{1\text{-}2} = T_2 : \quad 0 + \frac{mgb}{2} = \frac{1}{2}\left[\frac{5}{6}mb^2\right]\omega^2$$

$$\omega = \sqrt{\frac{6g}{5b}} \Rightarrow v_A = b\omega\sqrt{2} = \sqrt{\frac{12}{5}gb}$$

(ب) سقوط به اندازهٔ $2b$ $(v_{C'} = 0)$

$$T_1 + U_{1\text{-}2} = T_2 : \quad 0 + mgb = \frac{1}{2}\left[\frac{5}{6}mb^2\right]\omega^2$$

$$\omega = \sqrt{\frac{12g}{5b}} \Rightarrow v_A = b\omega = \sqrt{\frac{12}{5}gb}$$

۶-۱۴۹. فرض کنید ρ = جرم واحد طول کاغذ

$$m = \rho(L-x)$$

$$\Delta T = \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}m\bar{v}^2 = \frac{1}{2}\frac{1}{2}m r^2\omega^2 + \frac{1}{2}m v^2$$

$$= \frac{3}{4}m v^2 = \frac{3}{4}\rho(L-x)v^2$$

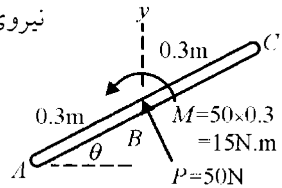
$$\Delta V_g = -\rho g(L-x)x\sin\theta - \rho g x\frac{x}{2}\sin\theta = -\rho g x\left(L - \frac{x}{2}\right)\sin\theta$$

$$U' = 0 = \Delta T + \Delta V_g : \quad 0 = \frac{3}{4}\rho(L-x)v^2 - \rho g x\left(L - \frac{x}{2}\right)\sin\theta$$

$$v^2 = \frac{4}{3}\frac{gx\left(L-\frac{x}{2}\right)\sin\theta}{L-x} \Rightarrow v = 2\sqrt{\frac{gx}{3}\frac{L-\frac{x}{2}}{L-x}\sin\theta}$$

وقتی $x \to L$ ، $v \to 0$ به طوری که افت انرژی پتانسیل $\left(-\rho g\frac{L}{2}\sin\theta\right)$ به انرژی جنبشی آخرین قسمت کاغذ تبدیل می‌شود. پایان ناگهانی حرکت باعث اتلاف ناگهانی انرژی در انتها می‌شود.

۶-۱۵۰.



$$U'_{1\text{-}2} = \Delta T + \Delta V_g$$

نیروی P را با یک نیروی P در B یک ممان $M = 15\ \text{N.m}$ جایگزین می‌کنیم

$$U'_{1\text{-}2} = \int_0^{0.3m} 50\cos\theta\, dy + 15\frac{\pi}{2}$$

$$\sin\theta = \frac{y}{0.3} \Rightarrow \cos\theta = \frac{10}{3}\sqrt{0.09 - y^2}$$

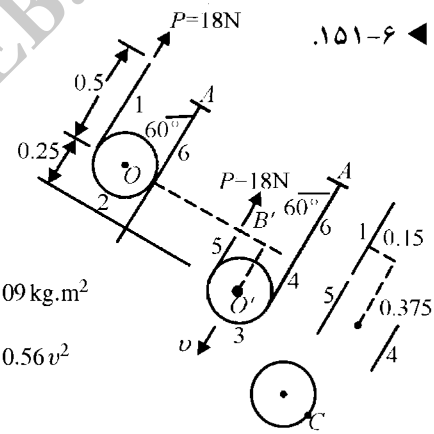
$$U'_{1\text{-}2} = \int_0^{0.3m} \frac{500}{3}\sqrt{0.09 - y^2}\, dy + 7.5\pi = 11.25\pi = 35.3\ \text{J}$$

$$\Delta T = \frac{1}{2} I_B \omega^2 = \frac{1}{2}\left(\frac{1}{12} 4 \times 0.6^2\right)\left(\frac{v}{0.3}\right)^2 = \frac{2}{3} v^2 = 0.667 v^2$$

$$\Delta V_g = mgh = 4 \times 9.81 \times 0.3 = 11.77\ \text{J}$$

$$\Rightarrow 35.3 = 0.667 v^2 + 11.77 \Rightarrow v^2 = 35.4 \Rightarrow v = 5.95\ \text{m/s}$$

◄ ۶-۱۵۱.



$$U'_{1\text{-}2} = \Delta T + \Delta V_g$$

$$U'_{1\text{-}2} = -18(0.5) = -9\ \text{J}$$

$$\Delta T_3 = \frac{1}{2} T_{\text{کل تسمه}} = \frac{1}{2}\left(\frac{1}{2} I_C \omega^2\right)$$

$$(I_C)_{\text{کل تسمه}} = 2 m r^2$$

$$= 2(1.2 \times 2\pi \times 0.150)(0.150)^2 = 0.0509\ \text{kg.m}^2$$

$$\Rightarrow \Delta T_3 = \frac{1}{2}\left(\frac{1}{2} 0.0509\right)\left(\frac{v}{0.15}\right)^2 = 0.56 v^2$$

$$\Delta T_5 = \frac{1}{2}\left(0.8 \times 0.250\right)(2v)^2 = 0.4 v^2$$

$$\Delta T_{\text{چرخ}} = \frac{1}{2} I_C \omega^2 = \frac{1}{2}[m(k_O{}^2 + r^2)]\omega^2 = \frac{1}{2} 7([0.1]^2 + [0.15]^2)\left(\frac{v}{0.15}\right)^2 = 5.06 v^2$$

(مقایسه 1 با 4 و 5) $\Delta V_g = -1.2(9.81)(0.5)(0.15\cos 60° + 0.375\sin 60°) = -2.35\ \text{J}$

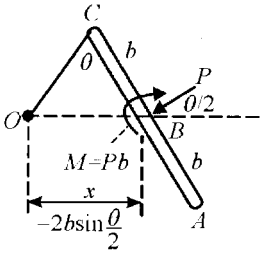
(مقایسه 2 با 3) $\Delta V_g = -1.2(9.81)(0.15\pi)\, 0.25\sin 60° = -1.2\ \text{J}$

چرخ $\Delta V_g = -7(9.81)(0.25)\sin 60° = -14.8\ \text{J}$

$$\Rightarrow -9 = (0.56 + 0.4 + 5.06) v^2 - (2.35 + 1.2 + 14.8)$$

$$\Rightarrow v^2 = 1.55\ (\text{m/s})^2 \Rightarrow v = 1.24\ \text{m/s}$$

◀ **۶-۱۵۲.** نیروی P را با نیروی P در B و ممان $M = Pb$ جایگزین کنید.



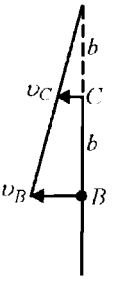
$$dU = M\left(-\frac{d\theta}{2}\right) + P\cos\frac{\theta}{2}(-dx)$$

$$= \frac{-M}{2}d\theta - P\cos\frac{\theta}{2}\left(b\cos\frac{\theta}{2}\right)d\theta$$

$$= \left(-\frac{M}{2} - Pb\cos^2\frac{\theta}{2}\right)d\theta$$

$$U = -\int_{\pi}^{0}\left(\frac{Pb}{2} + Pb\cos^2\frac{\theta}{2}\right)d\theta$$

$$U = \frac{Pb\pi}{2} - Pb\left[\frac{\theta}{2} + \frac{\sin\theta}{2}\right]_{\pi}^{0} = \frac{Pb\pi}{2} + \frac{Pb\pi}{2} = Pb\pi$$



$$\theta = 0 \;:\; v_B = 2v_C \;,\; \omega = \frac{v_B}{2b}$$

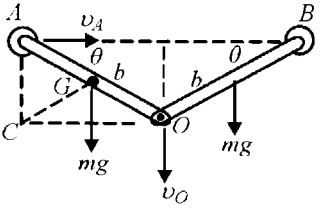
$$\Delta T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m(2b\omega)^2 + \frac{1}{2}\left(\frac{1}{12}m[2b]^2\right)\omega^2$$

$$= \frac{13}{6}mb^2\omega^2$$

$$U = \Delta T \;:\; Pb\pi = \frac{13}{6}mb^2\omega^2 \;\Rightarrow\; \omega = \sqrt{\frac{6P\pi}{13mb}}$$

۶-۱۵۳. C = مرکز آنی دوران برای AO



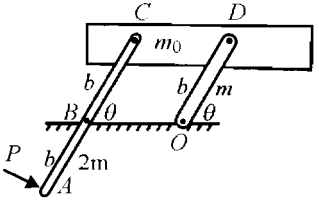
$$dU = dT$$

$$dU = 2mgd\left(\frac{b}{2}\sin\theta\right) = mgb\cos\theta\,d\theta$$

$$dT = 2d\left(\frac{1}{2}I_C\omega^2\right) = 2I_C\omega\,d\omega = \frac{2}{3}mb^2\alpha\,d\theta$$

$$\Rightarrow\; mgb\cos\theta\,d\theta = \frac{2}{3}mb^2\ddot{\theta}\,d\theta \;\Rightarrow\; \alpha = \ddot{\theta} = \frac{3g\cos\theta}{2b}$$

۶-۱۵۴.



$$dU' = dT + dV_g$$

$$dU' = Pd(b\theta) = Pb\,d\theta$$

$$dT_{\text{سکو}} = d\left(\frac{1}{2}m_0v^2\right) = m_0v\,dv = m_0a_t\,ds = m_0a_tb\,d\theta$$

$$dT_{OI} = d\left(\frac{1}{2}I_O\omega^2\right) = \frac{1}{3}mb^2\omega\,d\omega = \frac{1}{3}mb^2\alpha\,d\theta$$

$$dT_{AC} = d\left(\frac{1}{2}I_B\omega^2\right) = 2\,dT_{OI} = \frac{2}{3}mb^2\alpha\,d\theta$$

$$dV_g = d(\Sigma mgh) = m_0gd(b\sin\theta) + mgd\left(\frac{b}{2}\sin\theta\right) = \left(m_0 + \frac{m}{2}\right)gb\cos\theta\,d\theta$$

$$\Rightarrow P\,b\,d\theta = \frac{1}{3}m\,b^2\alpha\,d\theta + \frac{2}{3}m\,b^2\alpha\,d\theta + m_0\,b^2\alpha\,d\theta + \left(m_0 + \frac{m}{2}\right)g\,b\cos\theta\,d\theta$$

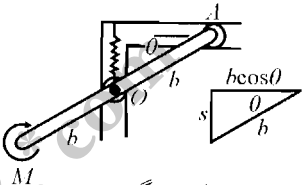
$$= (m + m_0)\,b^2\alpha\,d\theta + \left(m_0 + \frac{m}{2}\right)g\,b\cos\theta\,d\theta$$

$$\alpha = \ddot{\theta} = \frac{P - \left(\frac{m}{2} + m_0\right)g\cos\theta}{b(m + m_o)}$$

اگر $P > \left(\frac{m}{2} + m_0\right)g\cos\theta$:

در غیر این صورت $\alpha = 0$

۶-۱۵۵. برای موقعیت تعادل کار انجام شده توسط فنر برابر و مخالف کار انجام شده توسط وزن می‌باشد.



$$dU' = dT + dV_e + dV_g \quad \Rightarrow \quad dU' = dT$$

$$dU' = M(-d\theta)$$

$$dT = d\left(\frac{1}{2}m\,v_O^2 + \frac{1}{2}I_O\omega^2\right) = m\,a_o\,ds + I_o\ddot{\theta}\,d\theta$$

(α را در جهت ساعت بگیرید
در نتیجه $\alpha = -\ddot{\theta}$)

$$s = b\sin\theta \Rightarrow \dot{s} = b\dot{\theta}\cos\theta \Rightarrow a = \ddot{s} = b\ddot{\theta}\cos\theta - b\dot{\theta}^2\sin\theta = b\ddot{\theta}\cos\theta \qquad (\text{چون } \dot{\theta} = 0)$$

$$\Rightarrow M(-d\theta) = m\,b\ddot{\theta}\cos\theta\,(b\cos\theta\,d\theta) + \frac{1}{12}m\,(2\,b)^2\ddot{\theta}\,d\theta$$

$$-M = mb^2\left(\cos^2\theta + \frac{1}{3}\right)\ddot{\theta} \Rightarrow M = m\,b^2\left(\cos^2\theta + \frac{1}{3}\right)\alpha \Rightarrow \alpha = \frac{M}{m\,b^2\left(\cos^2\theta + \frac{1}{3}\right)}\ \text{CW}$$

۶-۱۵۶.

$$dU' = 0 = dT + dV_e + dV_g$$

$$dV_g = 2(2.7\times 9.81)\,d\,(0.2\cos\theta) + (4.5\times 9.81)(0.45\cos\theta)$$

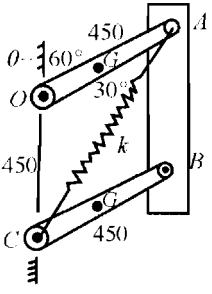
$$= 30.46\,d\,(\cos\theta) = -30.46\sin\theta\,d\theta = -26.38\,d\theta$$

$$dT_{\text{ستون}} = d\left(\frac{1}{2}m\,v^2\right) = m\,v\,dv = m\,a_t\,ds$$

$$= 4.5(0.45\,\alpha)\,0.45\,d\theta = 0.91\,\alpha\,d\theta$$

$$dT_{\text{میله‌ها}} = 2\,d\left(\frac{1}{2}I_O\omega^2\right) = 2\,I_O\,\omega\,d\omega = 2\,I_O\,\alpha\,d\theta$$

$$= 2(2.7(0.25)^2)\,\alpha\,d\theta = 0.3375\,\alpha\,d\theta$$



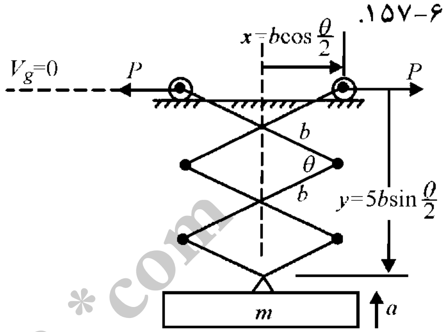
$$CA = 2(0.45)\cos 30° = 0.78\ \text{m} \qquad \text{کشیدگی } x = 2(0.45)\cos\frac{\theta}{2} - 0.45$$

$$dx = 0.9\left(-\sin\frac{\theta}{2}\,\frac{d\theta}{2}\right)$$

$$dV_e = d\left(\frac{1}{2}kx^2\right) = kx\,dx = -220(0.45)\left(2\cos\frac{\theta}{2} - 1\right)0.9\left(\sin\frac{\theta}{2}\,\frac{d\theta}{2}\right) = -16.306\,d\theta$$

$$\Rightarrow\ 0 = (0.91 + 0.3375)\,\alpha\,d\theta - 16.306\,d\theta - 26.38\,d\theta \ \Rightarrow\ \alpha = 34.2\ \text{rad/s}^2$$

۶-۱۵۷.



$$\dot{y} = \frac{5}{2}b\dot{\theta}\cos\frac{\theta}{2}$$

$$\ddot{y} = \frac{5}{2}b\ddot{\theta}\cos\frac{\theta}{2} - \frac{5}{4}b\dot{\theta}\sin\frac{\theta}{2}$$

$$\dot{\theta} = 0\ ,\ a = -\ddot{y} \Rightarrow a = \frac{-5}{2}b\ddot{\theta}\cos\frac{\theta}{2}$$

$$dU' = dT + dV_g$$

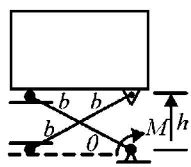
$$dU' = 2P\,d\left(b\cos\frac{\theta}{2}\right) = -Pb\sin\frac{\theta}{2}\,d\theta$$

$$dT = d\left(\frac{1}{2}mv^2\right) = mv\,dv = ma(-dy) = -ma\left(\frac{5}{2}b\cos\frac{\theta}{2}\right)d\theta$$

$$dV_g = d(-mgy) = -mg\frac{5}{2}b\cos\frac{\theta}{2}\,d\theta$$

$$\Rightarrow\ -Pb\sin\frac{\theta}{2}\,d\theta = -\frac{5}{2}mab\cos\frac{\theta}{2}\,d\theta - \frac{5}{2}mgb\cos\frac{\theta}{2}\,d\theta \ \Rightarrow\ a = \frac{2P}{5m}\tan\frac{\theta}{2} - g$$

۶-۱۵۸.



$$dU' = dT + dV_g \quad ,\quad dU' = M\,d\theta$$

$$dT = ma\,dh = ma\,d(2b\sin\theta) = 2mba\cos\theta\,d\theta$$

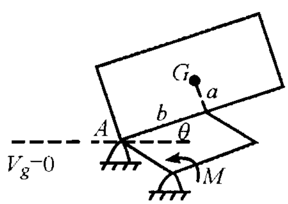
$$dV_g = mg\,dh = 2mgb\cos\theta\,d\theta$$

$$\Rightarrow\ M\,d\theta = 2mb\cos\theta(a+g)d\theta \ \Rightarrow\ a + g = \frac{M}{2mb\cos\theta}$$

از طرفی $2b\sin\theta = h \ \Rightarrow\ \cos\theta = \dfrac{\sqrt{4b^2 - h^2}}{2b} = \sqrt{1 - \left(\dfrac{h}{2b}\right)^2}$ $\Rightarrow$

$$\Rightarrow\ a = \frac{M}{2mb\sqrt{1 - \left(\frac{h}{2b}\right)^2}} - g$$

۶-۱۵۹.



$$dU' = dT + dV_g$$

$$dU' = M\,d\theta$$

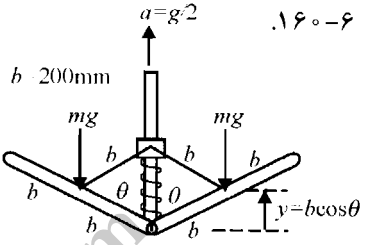
$$dT = d\left(\frac{1}{2}I_A\omega^2\right) = I_A\omega\,d\omega = I_A\alpha\,d\theta$$

$$dV_g = d(mgb\sin\theta + a\cos\theta) = mg(b\cos\theta - a\sin\theta)\,d\theta$$

$$\Rightarrow\ M\,d\theta = I_A\,\alpha\,d\theta + mg(b\cos\theta - a\sin\theta)\,d\theta$$

$$\Rightarrow\ \alpha = \frac{1}{I_A}[M - mg(b\cos\theta - a\sin\theta)]$$

۶-۱۶۰.



$$\delta U' = \delta T + \delta V_e$$

$$\delta U' = 2mg(-\delta y) = 2(2\times 9.81)(-\delta[b\cos\theta]) = -39.24(0.200)(-\sin\theta)\delta\theta = 7.85\sin\theta\,\delta\theta\ \text{J}$$

$$\delta T = \Sigma m\,\bar{\mathbf{a}}\cdot\delta\mathbf{s} = 2(2)\frac{9.81}{2}\delta y = 19.62\,\delta(b\cos\theta) = -19.62(0.200)\sin\theta\,\delta\theta = -3.92\sin\theta\,\delta\theta$$

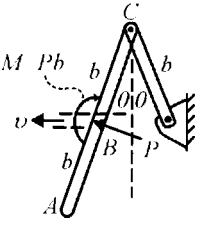
$$\delta V_e = kx\,\delta x = 130(2b - 2b\cos\theta)\,\delta(2b - 2b\cos\theta) = 520b^2(1-\cos\theta)\sin\theta\,\delta\theta$$

$$\Rightarrow\ 7.85\sin\theta\,\delta\theta = -3.92\sin\theta\,\delta\theta + 520b^2(1-\cos\theta)\sin\theta\,\delta\theta$$

$$\Rightarrow\ [(7.85 + 3.92) - 520(0.200)^2(1-\cos\theta)]\sin\theta\,\delta\theta = 0$$

$$\Rightarrow\ 1 - \cos\theta = \frac{11.77}{520(0.200)^2}\ \Rightarrow\ \cos\theta = 1 - 0.5660 = 0.4340\ \Rightarrow\ \theta = 64.3^\circ$$

۶-۱۶۱. P را با نیروی P در B و ممان $M = Pb$ جایگزین کنید.



$$dU = dT$$

$$dU = P\cos\theta\,d(2b\sin\theta) + Pb\,d\theta = Pb(2\cos^2\theta + 1)\,d\theta$$

$$dT_{AC} = d\left(\frac{1}{2}2mv^2 + \frac{1}{2}\bar{I}\omega^2\right) = 2mv\,dv + \bar{I}\omega\,d\omega = 2ma\,dx + \bar{I}\alpha\,d\theta$$

که در آن : $x = 2b\sin\theta$, $v = 2b\dot\theta\cos\theta$, $a = 2b(\ddot\theta\cos\theta - \dot\theta^2\sin\theta) = 2b\ddot\theta\cos\theta$

(چون $\dot\theta = 0$)

$$\Rightarrow\ dT_{AC} = 2m(2b\ddot\theta\cos\theta)\,d(2b\sin\theta) + \frac{1}{12}(2m)(2b)^2\ddot\theta\,d\theta = 2mb^2\left(4\cos^2\theta + \frac{1}{3}\right)\ddot\theta\,d\theta$$

$$dT_{OC} = d\left(\frac{1}{2}I_O\omega^2\right) = I_O\omega\,d\omega = I_O\alpha\,d\theta = \frac{1}{3}mb^2\ddot\theta\,d\theta$$

$$\Rightarrow\ dT = 2mb^2\left(4\cos^2\theta + \frac{1}{3}\right)\ddot\theta\,d\theta + \frac{1}{3}mb^2\ddot\theta\,d\theta = mb^2(8\cos^2\theta + 1)\ddot\theta\,d\theta$$

$$\Rightarrow \quad Pb(2\cos^2\theta + 1)\,d\theta = m b^2 (8\cos^2\theta + 1)\,\ddot{\theta}\,d\theta$$

$$\Rightarrow \quad \ddot{\theta} = \alpha = \frac{P(2\cos^2\theta + 1)}{m b (8\cos^2\theta + 1)}$$

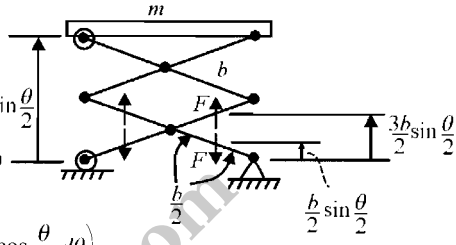
۶-۱۶۲.

$$dU' = dT + dV_g$$

$$dU' = 2F\,d\left(\frac{3b}{2}\sin\frac{\theta}{2}\right) - 2F\,d\left(\frac{b}{2}\sin\frac{\theta}{2}\right)$$

$$= 2F\,d\left(b\sin\frac{\theta}{2}\right) = Fb\cos\frac{\theta}{2}\,d\theta$$

$$dV_g = d\left(mg\,4b\sin\frac{\theta}{2}\right) = 2mgb\cos\frac{\theta}{2}\,d\theta$$



$$dT = d\left(\frac{1}{2}mv^2\right) = mv\,dv = ma\,dy = ma\left(2b\cos\frac{\theta}{2}\,d\theta\right)$$

$$\Rightarrow \quad Fb\cos\frac{\theta}{2}\,d\theta = 2mgb\cos\frac{\theta}{2}\,d\theta + 2mab\cos\frac{\theta}{2}\,d\theta \quad \Rightarrow \quad a = \frac{F}{2m} - g$$

همانطور که می‌بینید هر دوی b و θ حذف شدند بنابراین a مستقل از b و θ می‌باشد.

۶-۱۶۳.

$$x^2 + y^2 = l^2 \quad \Rightarrow \quad x\,dx + y\,dy = 0$$

$$\theta = 45^\circ : \quad x = y \quad , \quad dx = -dy$$

$$dU' = dT + dV_g$$

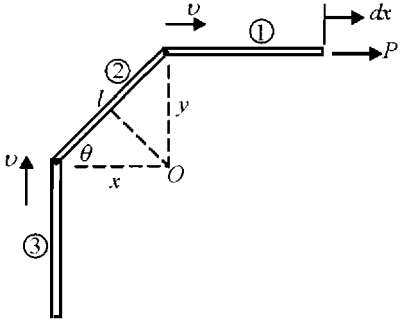
$$dU' = P\,dx$$

$$dT = d(T_1 + T_2 + T_3)$$

$$= d\left(\frac{1}{2}mv^2 + \frac{1}{2}I_O\omega^2 + \frac{1}{2}mv^2\right)$$

$$= 2mv\,dv + I_O\omega\,d\omega = 2ma\,dx + I_O\alpha\,|d\theta|$$

$$= 2ma\,dx + \frac{1}{3}ml^2\,\frac{a}{\sqrt{\frac{l}{2}}}\,\frac{dx}{\sqrt{\frac{l}{2}}} = \frac{8}{3}ma\,dx$$



$$dV_g = d(V_{g_1} + V_{g_2} + V_{g_3}) = 0 + mg\,\frac{dx}{2} + mg\,dx = \frac{3}{2}mg\,dx$$

$$\Rightarrow \quad P\,dx = \frac{8}{3}ma\,dx + \frac{3}{2}mg\,dx \quad \Rightarrow \quad a = \frac{3}{8}\left(\frac{P}{m} - \frac{3g}{2}\right)$$

۶-۱۶۴.

شعاع تا هر وزنه $r = 6 + 36\sin\theta$ mm $\Rightarrow \quad \delta r = 36\cos\theta\,\delta\theta$

$$\delta T = 2(mr\omega^2)(-\delta r) = 2(0.350)\left(\frac{6 + 36\sin\theta}{1000}\right)\omega^2\left(\frac{-36\cos\theta}{1000}\right)\delta\theta$$

از طرفی $2(36) - 2(36)\cos\theta = 15\sin\beta \quad , \quad \beta = 15^\circ \quad \Rightarrow \quad \cos\theta = \frac{72 - 15\sin 15^\circ}{72} = 0.461$

$$\Rightarrow \quad \theta = 18.90^\circ$$

$$\Rightarrow \quad \delta T = -4.21(10^{-4})\,\omega^2\,\delta\theta$$

$$\delta V_e = k\,x\,\delta x = 900[2(0.036)(1-\cos\theta)]\,\delta\{2(0.036)(1-\cos\theta)\} = 4.66(1-\cos\theta)\sin\theta\,\delta\theta$$

$$= 4.66(1-\cos 18.90°)\sin 18.90°\,\delta\theta = 0.0813\,\delta\theta$$

$$\delta U + \delta T + \delta V_e = 0 \quad : \quad -4.21(10^{-4})\,\omega^2\,d\theta + 0.0813\,\delta\theta = 0$$

$$\Rightarrow \quad \omega^2 = 193.3 \quad \Rightarrow \quad \omega = 13.9\ \text{rad/s}$$

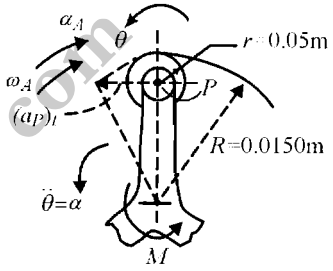
$$N = 13.4\left(\frac{60}{2\pi}\right) = 132.8\ \text{rpm}$$

۶-۱۶۵.

$$ds_P = (R-r)d\theta \quad \Rightarrow \quad d\theta_A = \frac{ds_P}{r} = \left(\frac{R}{r}-1\right)d\theta$$

$$v_P = (R-r)\,\dot\theta \quad \Rightarrow \quad \omega_A = \frac{v_P}{r} = \left(\frac{R}{r}-1\right)\dot\theta$$

$$(a_P)_t = (R-r)\,\alpha \quad \Rightarrow \quad \alpha_A = \frac{(a_P)_t}{r} = \left(\frac{R}{r}-1\right)\alpha$$



$dU = dT_{\text{سه شاخه}} + dT_{\text{چرخدنده‌ها}}$

$$dU = M\,d\theta$$

$$dT_{\text{سه شاخه}} = d\left(\frac{1}{2}I_O\,\omega^2\right) = I_O\,\omega\,d\omega = I_O\,\alpha\,d\theta$$

$$dT_{\text{چرخدنده‌ها}} = 3\left\{d\left(\frac{1}{2}I_A\,\omega_A{}^2\right) + d\left(\frac{1}{2}m_A\,v_P{}^2\right)\right\} = 3\left\{I_A\,\alpha_A\,d\theta_A + m_A\,(a_P)_t\,ds_P\right\}$$

$$= 3\left\{I_A\left(\frac{R}{r}-1\right)^2\alpha\,d\theta + m_A\,(R-r)^2\,\alpha\,d\theta\right\} = 3(R-r)^2\left(\frac{I_A}{r^2}+m_A\right)\alpha\,d\theta$$

$$\Rightarrow \quad M\,d\theta = \left[I_O + 3(R-r)^2\left(\frac{I_A}{r^2}+m_A\right)\right]\alpha\,d\theta$$

$$5 = \left[1.2\times 0.60^2 + 3(0.150-0.05)^2\left(\frac{0.8\times 0.30^2}{0.050^2}+0.8\right)\right]\alpha$$

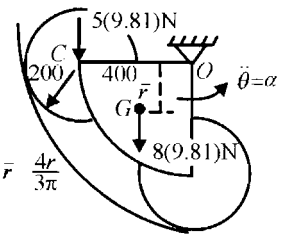
$$= [0.00432 + 0.03\times 1.088]\,\alpha \quad \Rightarrow \quad \alpha = 135.3\ \text{rad/s}^2$$

۶-۱۶۶.

هر چرخ : $dT = m_w\,\bar a_w\,d\,s_w + \bar I_w\,\alpha_w\,d\theta_w$

$$= 5(0.4\,\alpha)(0.4\,d\theta) + \frac{1}{2}5(0.2)^2(2\,\alpha)(2\,d\theta)$$

$$= 1.2\,\alpha\,d\theta \quad (\alpha_w = 2\alpha\ ,\ d\theta_w = 2\,d\theta \text{ که در آن})$$



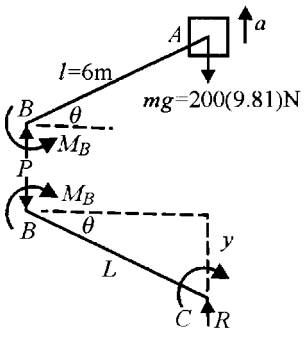
قطاع : $dT = I_O\,\alpha\,d\theta = \frac{1}{2}8(0.4)^2\,\alpha\,d\theta = 0.64\,\alpha\,d\theta$

ترکیب : $dT = 2(1.2\,\alpha\,d\theta) + 0.64\,\alpha\,d\theta = 3.04\,\alpha\,d\theta$

$$dU = 5(9.81)\,0.4\,d\theta + 8(9.81)\,\frac{4(0.4)}{3\pi}\,d\theta = 32.94\,d\theta$$

$$dU = dT \quad : \quad 32.94\,d\theta = 3.04\,\alpha\,d\theta \quad \Rightarrow \quad \alpha = 10.84\ \text{rad/s}^2$$

۶-۱۶۷.



بازوی بالایی : $dU = dT$

$$P\,dy - m\,g2\,dy + M_B\,d\theta = d\left(\frac{1}{2}m\,v^2\right)$$

$$y = l\sin\theta \quad \Rightarrow \quad dy = l\cos\theta\,d\theta$$

$$d\left(\frac{1}{2}m\,v^2\right) = m\,a\,d(2\,y) = 2\,m\,a\,l\cos\theta\,d\theta$$

$$\left.\begin{array}{l}\Rightarrow\ (P - 2\,m\,g)\,l\cos\theta + M_B = 2\,m\,a\,l\cos\theta \\ \text{از طرفی}\ \ P - mg = ma\end{array}\right\} \Rightarrow M_B = m\,g\,l\cos\theta\left(\frac{a}{g}+1\right) =$$

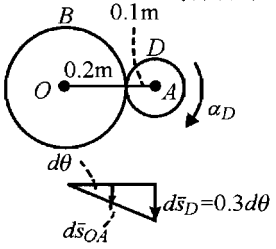
$$= 200(9.81)(6)(0.866)\left(\frac{1.2}{9.81}+1\right) = 11440\ \text{N.m} \quad \text{یا} \quad 11.44\ \text{kN.m}$$

بازوی پائینی : $\Sigma M = 0$: $M + M_B - P\,l\cos\theta = 0$

$$M = -m\,g\,l\cos\theta\left(\frac{a}{g}+1\right) + mg\left(\frac{a}{g}+1\right)l\cos\theta \quad \Rightarrow \quad M = 0$$

$M = 0$ با بررسی شکل نیز می‌تواند به‌دست آید، چون m مستقیماً بالای C قرار دارد. مسئله را مستقیماً می‌توان با معادلات $F - m - a$ نیز حل نمود.

۶-۱۶۸.



$$dU' = dT + dV_g$$

$$dU' = \Sigma m_i a_i . ds_i + \Sigma I_i \alpha_i . d\theta_i + \Sigma m_i g\,dh_i$$

فرض کنید α = شتاب زاویه‌ای OA ، $d\theta$ = جابجایی زاویه‌ای OA

OA بازوی : $\bar{a} = \frac{0.3}{2}\alpha$ ، $d\bar{s} = \frac{0.3}{2}d\theta$ ، $dh = -\frac{0.3}{2}d\theta$

$$\bar{I} = \frac{1}{12}(4)(0.3)^2 = 0.03\ \text{kg.m}^2$$

$$dU'_{\text{بازو}} = 4\left(\frac{0.3}{2}\alpha\right)\left(\frac{0.3}{2}d\theta\right) + 0.03\,\alpha\,d\theta - 4(9.81)\left(\frac{0.3}{2}d\theta\right) = 0.12\,\alpha\,d\theta - 5.89\,d\theta$$

چرخدنده D : $\bar{a} = a_A = 0.3\,\alpha$ ، $d\bar{s} = 0.3\,d\theta$ ، $d\bar{h} = -0.3\,d\theta$

$$\alpha_D = 3\,\alpha \ , \ d\theta_D = 3\,d\theta$$

$$\bar{I} = m\,\bar{k}^2 = 5(0.064)^2 = 0.0205\ \text{kg.m}^2$$

$$dU'_D = 5(0.3\,\alpha)(0.3\,d\theta) + 0.0205(3\,\alpha)(3\,d\theta) - 5(9.81)(0.3\,d\theta)$$

$$= 0.634\,\alpha\,d\theta - 14.72\,d\theta$$

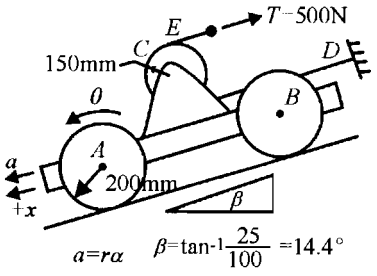
برای سیستم : $dU' = dU'_{\text{بازو}} + dU'_D = 0$

$$0.12\,\alpha\,d\theta - 5.89\,d\theta + 0.634\,\alpha\,d\theta - 14.72\,d\theta = 0 \quad \Rightarrow \quad \alpha = 27.3\ \text{rad/s}^2$$

۶-۱۶۹.

$$dU'_{1\text{-}2} = dT + dV_g$$

فرض کنید x = جابجایی ارابه به سمت پائین سمت شیبدار



$a = r\alpha$ $\beta = \tan^{-1}\frac{25}{100} = 14.4°$

$$\bar{I}_A = \bar{I}_B = m k^2 = 140(0.150)^2 = 3.15\,\text{kg.m}^2$$

$$\bar{I}_C = 40(0.100)^2 = 0.4\,\text{kg.m}^2$$

$$dU'_{1\text{-}2} = -500(2\,dx) = -1000\,dx$$

$$d(T_{\text{چرخها}}) \text{ فقط دوران} = 2\,d\left(\frac{1}{2}\bar{I}_A\omega^2\right) = 2\bar{I}_A\,\omega\,d\omega = 2\bar{I}_A\,\alpha\,d\theta = 2\bar{I}_A\,\frac{a}{r_A}\,\frac{dx}{r_A}$$

$$= 2\times 3.15\,\frac{x\,dx}{0.2^2} = 157.5\,a\,dx$$

$$d(T_{\text{قرقره}}) \text{ فقط دوران} = d\left(\frac{1}{2}\bar{I}_C\omega_C^2\right) = \bar{I}_C\,\omega_C\,d\omega_C = \bar{I}_C\alpha_C\,d\theta_C = \bar{I}_C\,\frac{a}{r_C}\,\frac{dx}{r_C}$$

$$= 0.4\,\frac{a\,dx}{0.15^2} = 17.78\,a\,dx$$

$$dT \text{ جابجایی ارابه} = d\left(\frac{1}{2}m v^2\right) = m\,v\,dv = m\,a\,dx = 520\,a\,dx$$

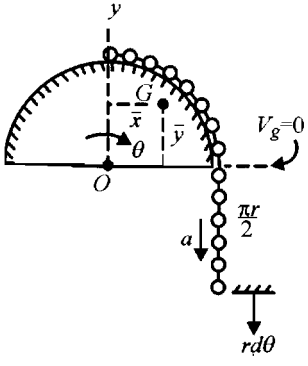
$$dV_g = -m\,g\,dh = -520(9.81)\,dx\sin 14.04° = -1237\,dx$$

$$\Rightarrow -1000\,dx = (157.5 + 17.78 + 520)\,a\,dx - 1237\,dx \Rightarrow a = 0.314\,\text{m/s}^2$$

۶-۱۷۰. ◀

$dU' = dT + dV_g$ برای کل زنجیر

$$dU' = 0$$



$$dT = d\left(\frac{1}{2}m v^2\right) = m\,v\,dv = m\,a\,.\,r\,d\theta = \rho\,\pi\,r^2\,a\,d\theta$$

$$dV_g = -\rho\,g\,\frac{\pi r}{2}(\bar{r}\,d\theta\cos 45°) - \rho\,\frac{\pi\,r}{2}\,g\,r\,d\theta$$

$$= -\rho\,g\,\frac{\pi\,r}{2}\,\frac{2\,r}{\pi}\sqrt{2}\;d\theta\,\frac{1}{\sqrt{2}} - \rho\,g\,\frac{\pi\,r^2}{2}\,d\theta$$

$$= -\rho\,g\,\pi\,r^2\left(\frac{1}{\pi} + \frac{1}{2}\right)d\theta$$

$$\Rightarrow 0 = \rho\,\pi\,r^2\,a\,d\theta - \rho\,g\,\pi\,r^2\left(\frac{1}{\pi} + \frac{1}{2}\right)d\theta \Rightarrow a = \left(\frac{1}{\pi} + \frac{1}{2}\right)g$$

۶-۱۷۱.

$$\int \Sigma F\,dt = \Delta G$$

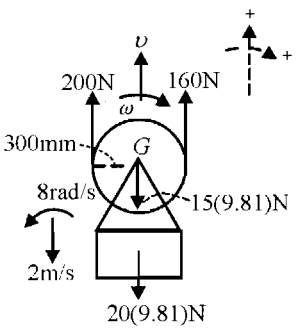
$$(360 - 35\times 9.81)5 = 35(v - [-2])$$

$\Rightarrow v = 0.379$ m/s به طرف بالا

$$\int \Sigma M_G\,dt = \Delta H_G$$

$$(200 - 160)\,0.3(5) = 15(0.25)^2(\omega - [-8])$$

$\Rightarrow \omega = 56.0$ rad/s CW



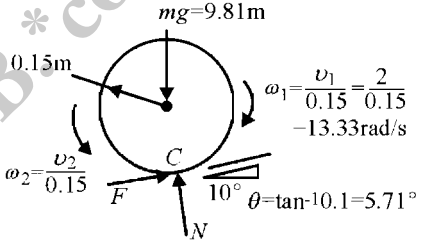
۶-۱۷۲.

$$\Sigma M_C = I_C\alpha$$

$$\Sigma M_C\,t = I_C(\omega_2 - \omega_1)$$

$$9.81\,m\sin 5.71^\circ(0.15)\,6 = m(0.090^2 + 0.150^2)\left(\frac{v_2}{0.15} - [-13.33]\right)$$

$v_2 = 2.31$ m/s



۶-۱۷۳.

طی برخورد $\Sigma M_A = \dot{H}_A = 0$

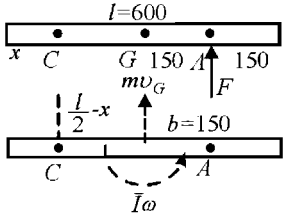
A یک نقطه ثابت روی خط اثر نیرو می‌باشد.

$H_A =$ ثابت $= 0$

$$H_A = \bar{I}\omega - m v_G b = 0$$

$$\Rightarrow \frac{1}{12}m l^2\omega = m\left(\frac{l}{2} - x\right)\omega b \Rightarrow \frac{1}{12}l^2 = \left(\frac{l}{2} - x\right)b \Rightarrow \frac{600^2}{12} = (300 - x)150$$

$$\Rightarrow x = 100\text{ mm}$$



۶-۱۷۴.

$H_{O_1} = H_{O_2}$ + ↻ برای سیستم

$$m v h = (I_O + m h^2)\omega$$

$$0.028(500)(1.075) = \left[25(0.925)^2 + 0.028(1.075)^2\right]\omega \Rightarrow \omega = 0.703\text{ rad/s}$$

۶-۱۷۵.

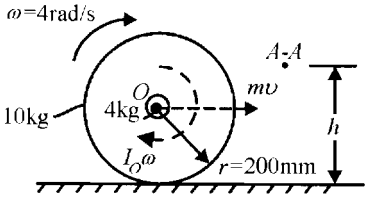
$v = r\omega = 0.2(4) = 0.8\,\text{m/s}$

$I_O = 10(0.180)^2 + \overbrace{I_{shaft}}^{\sim 0} = 0.324\,\text{kg.m}^2$

$\curvearrowright + H_{A-A} = I_O\omega - \mu v d$:

$0.324(4) - (10 + 4)(0.8)(h - 0.2) = 0$

$h = 0.316\,\text{m}$ یا $316\,\text{mm}$



۶-۱۷۶.

سیستم $\Sigma \xrightarrow[+]{} \int_0^{10} F\,dt = \Delta G$: $400(10) = (1200 + 800)[v - (-1.5)] \Rightarrow v = 0.5\,\text{m/s}$

(به طرف راست)

قرقره $\curvearrowright + \int \Sigma M_O\,dt = \Delta H_O$: $400(0.500)(10)=800(0.480)^2[\omega - (-3)] \Rightarrow \omega = 7.85\,\text{rad/sCW}$

دوران قرقره روی مومنتوم خطی سیستم تأثیری ندارد لذا $v = 0.5\,\text{m/s}$ مستقل از ω می‌باشد.

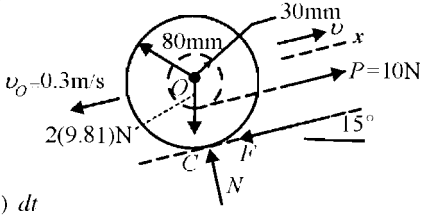
۶-۱۷۷.

$\Sigma M_C = I_C\alpha$: عدم لغزش مرکز جرم در O

$\Rightarrow \int \Sigma M_C\,dt = \Delta H_C = \Delta(I_C\omega)$

$I_C = m(k_O^2 + r^2) = 2(0.060^2 + 0.080^2)$

$= 0.2\,\text{kg.m}^2$



$\curvearrowright + \int_0^5 (10[0.080 - 0.030] - 2(9.81)(0.080)\sin 15°)\,dt$

$= 0.02\left(\frac{v}{0.080} - \left[\frac{-0.3}{0.080}\right]\right)$

$0.469 = 0.25(v + 0.3) \Rightarrow v = 1.575\,\text{m/s}$ به طرف بالای سطح شیبدار

راه‌حل دوم :

$\curvearrowleft + \int \Sigma M_G\,dt = \Delta H_G$: $\int_0^5 (F \times 0.080 - 10 \times 0.030)\,dt = 2 \times 0.060^2\left(\frac{v}{0.080} - \left[\frac{-0.3}{0.080}\right]\right)$

$0.080F(5) - 0.3(5) = 0.09(v + 0.3)$ (1)

$\int \Sigma F_x\,dt = \Delta G_x$: $\int_0^5 (10 - F - 2(9.81)\sin 15°)dt = 2(v - [-0.3])$

$50 - 25.4 - 5F = 2v + 0.6$ (2)

حل (1) و (2) : $v = 1.575\,\text{m/s}$

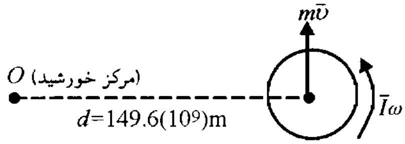
$\left(F = 4.17\,\text{N}\ ,\ N = 2(9.81)\cos 15° = 18.95\,\text{N}\ ,\ (\mu_s)_{\min} = \frac{4.17}{18.95} = 0.220\right)$

۶-۱۷۸.

$$\bar{H} = \bar{I}\omega = \frac{2}{5} m r^2 \left(\frac{2\pi}{\tau}\right)$$

$$= \frac{2}{5}(5.976 \times 10^{24})(6.371 \times 10^6)^2 \frac{2\pi}{23.9344(3600)}$$

$$= 7.08(10^{33})\ \text{kg.m}^2/\text{s}$$



$$\bar{v} = \sqrt{\frac{G m_s}{d}} = \sqrt{\frac{6.673(10^{-11})(333000)(5.976 \times 10^{24})}{149.6(10^9)}} = 29800\ \text{m/s}$$

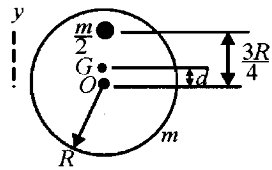
$$m\bar{v}d = 5.976(10^{24})(29800)(149.6 \times 10^9) = 2.66(10^{40})\ \text{kg.m}^2/\text{s}$$

$$\bar{H} = \bar{I}\omega + m\bar{v}d = 2.66(10^{40})\ \text{kg.m}^2/\text{s}$$

(جملهٔ $\bar{I}\omega$ در مقایسه با $m\bar{v}d$ قابل ملاحظه نیست)

۶-۱۷۹.

$$d = \frac{\Sigma \bar{y} m}{\Sigma m} = \frac{\left(\frac{m}{2}\right)\left(\frac{3R}{4}\right)}{m + \frac{m}{2}} = \frac{R}{4}$$



$$I_O = \frac{1}{2} m R^2 + \frac{m}{2}\left(\frac{3R}{4}\right)^2 = \frac{25}{32} m R^2$$

$$\bar{I} = I_O - m d^2 = \frac{25}{32} m R^2 - \frac{3}{2} m \left(\frac{R}{4}\right)^2 = \frac{11}{16} m R^2$$

$$H_G = \bar{I}\omega = \frac{11}{16} m R^2 \left(\frac{v_O}{R}\right) = \frac{11}{16} m R v_O \circlearrowright$$

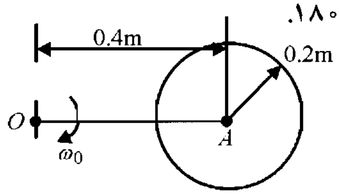
$$H_O = \bar{I}\omega + m\bar{v}d = \frac{11}{16} m R v_O + \frac{3}{2} m \left(\frac{5R}{4}\frac{v_O}{R}\right)\left(\frac{R}{4}\right) = \frac{37}{32} m R v_O \circlearrowright$$

۶-۱۸۰.

(الف) $\omega_0 = 4$ rad/s

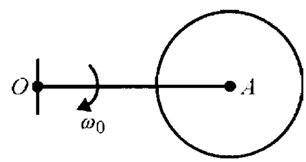
$\omega =$ سرعت زاویه‌ای دیسک

$$\bar{I} = I_A = \frac{1}{2} m r^2 = \frac{1}{2} 25 (0.2)^2 = \frac{1}{2}\ \text{kg.m}^2$$



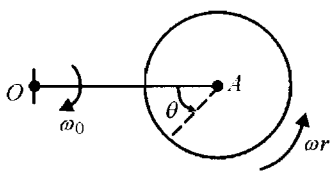
(ب) $\omega = \omega_0$

$$H_O = I_O \omega = \left[\frac{1}{2} + 25(0.4)^2\right]4 = 18\ \text{kg.m}^2/\text{s}$$



(ج) $\omega = \omega_0 - \omega_r = 4 - 8 = -4$ rad/s

$$H_O = \bar{I}\omega + m\bar{v}d = \frac{1}{2}(-4) + 16 = 14\ \text{kg.m}^2/\text{s}$$



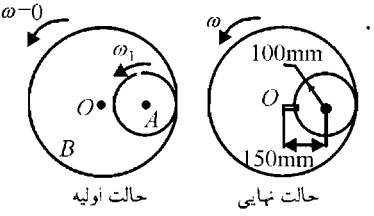
۶-۱۸۱.

$$\bar{I}_A = \frac{1}{2} m r^2 = \frac{1}{2} 8 (0.1)^2 = 0.04\,\text{kg.m}^2$$

$$\bar{I}_B = m k^2 = 60(0.2)^2 = 2.4\,\text{kg.m}^2$$

حالت اولیه : $\bar{H}_A = (H_A)_O = \bar{I}_A \omega_1 = 0.04(80)$

$= 3.2\,\text{kg.m}^2/\text{s}$



حالت نهایی : $H_O = (I_{AO} + \bar{I}_B)\omega = (0.04 + 8 \times \overline{0.15}^2 + 2.4)\omega = 2.62\omega$

$\Delta H_O = 0 \;:\; 3.2 = 2.62\omega \;\Rightarrow\; \omega = 1.221\,\text{rad/s}$

۶-۱۸۲. در حالت اول بدن شناگر را مانند یک میله باریک و در حالت دوم مانند یک کره در نظر بگیرید.

پایستاری مومنتوم زاویه‌ای : $H_1 = H_2$

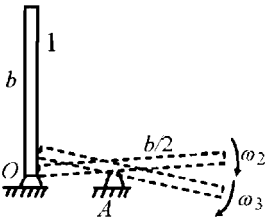
$$\frac{1}{2} m l^2 N_1 = \frac{2}{5} m r^2 N_2 \;\Rightarrow\; \frac{1}{2}(2)^2(0.3) = \frac{2}{5}\left(\frac{0.7}{2}\right)^2 N_2 \;\Rightarrow\; N_2 = 2.04\,\text{rps}$$

۶-۱۸۳.

از 1 تا 2 : $\Delta T + \Delta V_g = 0$

$$\frac{1}{2} I_O \omega_2^2 - 0 - m g \frac{b}{2} = 0 \;\Rightarrow\; \frac{1}{3} m b^2 \omega_2^2 = m g b$$

$$\Rightarrow \omega_2 = \sqrt{\frac{3g}{b}}$$



طی برخورد با A : $\Delta H_A = 0 \;\Rightarrow\; H_{A_2} = H_{A_3}$

$$H_{A_2} = \bar{I}\omega_2$$
$$H_{A_3} = \bar{I}\omega_3$$
$$\Rightarrow \omega_3 = \omega_2 = \sqrt{\frac{3g}{b}}$$

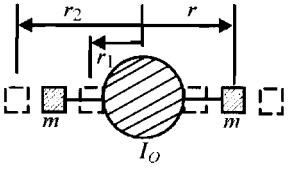
۶-۱۸۴.

$$H = I_O \omega_O + 2 m r^2 \omega_O$$

$$\dot{H} = 4 m r \dot{r} \omega_O$$

$$r = r_1 + \frac{\Delta r}{\Delta t} t = 1.2 + \frac{4.5 - 1.2}{120} t = 1.2 + 0.02750\,t$$

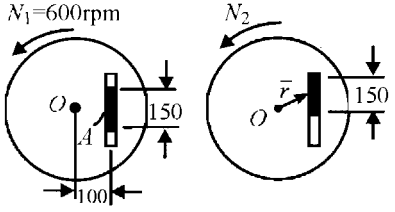
$$\dot{r} = 0.02750\,\text{m/s}$$



$$M = \dot{H} \;\Rightarrow\; 2T(1.1) = 4(10)(1.2 + 0.02750\,t)(0.02750) \times (1.25)$$

$$T = 0.750 + 0.01719\,t \;\text{N}$$

۶-۱۸۵. برای سیستم : $\Sigma M_O = 0 \Rightarrow \Delta H_O = 0 \Rightarrow H_1 = H_2$



$(H_1)_{\text{دیسک}} = 3.6(0.15)^2 \dfrac{600 \times 2\pi}{60}$

$(H_1)_{\text{میله}} = \left[\dfrac{1}{2}(0.9)(0.15)^2 + (0.9)(0.1)^2\right] \dfrac{600 \times 2\pi}{60}$

$(H_2)_{\text{دیسک}} = 3.6(0.15)^2 \cdot \dfrac{N_2 \times 2\pi}{60}$

$(H_2)_{\text{میله}} = \left[\dfrac{1}{2}(0.9)(0.15)^2 + (0.9)(0.125)^2\right] \dfrac{N_2 \times 2\pi}{60}$

$\bar{r}^2 = 75^2 + 100^2 = 125^2\,\text{mm}$

$$\Rightarrow \left[3.6 \times (0.15)^2 + \frac{0.9}{12}(0.15)^2 + (0.9)(0.1)^2\right]600 =$$

$$= \left[3.6 \times (0.15)^2 + \frac{0.9}{12}(0.15)^2 + (0.9)(0.125)^2\right]N_2$$

$$\Rightarrow N_2 = 569\,\text{rpm}$$

نیروهای اصطکاک در شیار داخلی هستند لذا اثری روی ΣM_O ندارند. از این رو مقدار نهایی N_2 و نیز اتلاف انرژی تحت تأثیر قرار نمی‌گیرند.

۶-۱۸۶. پایستاری مومنتوم زاویه‌ای حول محور عمودی چرخش سکو: $H_1 = H_2$

$$|10(0.3)^2|\left(250\frac{2\pi}{60}\right) = \left[I + \frac{1}{2}(10)(0.3)^2 + 10(0.6)^2\right]\left(30\frac{2\pi}{60}\right)$$

$$I = 3.45\,\text{kg.m}^2$$

۶-۱۸۷. $\Delta H = 0$

حالت اولیه : $H_{\text{میله‌ها}} : 2I\omega = 2(1.5)(0.060)^2 \dfrac{300 \times 2\pi}{60}$

$H_{\text{پایه}} : m k^2 \omega = 4(0.040)^2 \dfrac{300 \times 2\pi}{60}$

حالت نهایی : $H_{\text{میله‌ها}} = 2[\bar{I} + m d^2]\omega = 2m\left[\dfrac{l^2}{12} + d^2\right]\dfrac{2\pi N}{60}$

$$= 2(1.5)\left[\frac{0.3^2}{12} + (0.150 + 0.060)^2\right] \cdot \frac{2\pi N}{60} = 0.1548\left(\frac{2\pi N}{60}\right)$$

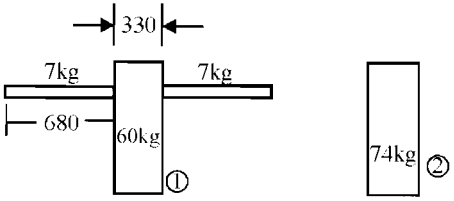
$$H_{\text{پایه}} = 4(0.040)^2 \frac{2\pi N}{60} = 0.0064\left(\frac{2\pi N}{60}\right)$$

$$\Rightarrow \left[3(0.06)^2 + 4(0.04)^2\right]300 = [0.1548 + 0.0064]N \Rightarrow N = 32.0\,\text{rpm}$$

۶-۱۸۸. پایستاری مومنتوم زاویه‌ای حول محور عمودی : $H_1 = H_2$

$$\left\{\frac{1}{2}60\left(\frac{0.330}{2}\right)^2 + 2\left[\frac{1}{12}7(0.680)^2 + 7\left(\frac{0.330 + 0.680}{2}\right)^2\right]\right\} \times 1$$

$$= \left\{\frac{1}{2}74\left(\frac{0.330}{2}\right)^2\right\} N \Rightarrow N = 4.89\,\text{rps}$$



۶-۱۸۹.

$$v_1 = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.3} = 2.43\,\text{m/s}$$

ضربه وزن طی برخورد ناچیز است. بنابراین :

$$\Delta H_A = 0$$

$$H_{A_1} = mvd = 78 \times 2.43 \times 0.125$$

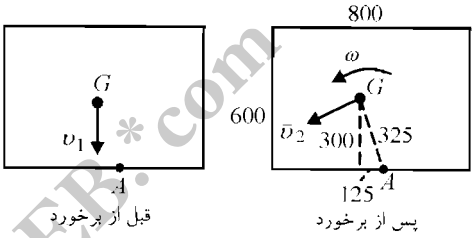
$$= 23.65\,\text{N.m.s}$$

$$H_{A_2} = I_A \omega$$

$$= \left[\frac{1}{12}78\,(0.8^2 + 0.6^2) + 78(0.325)^2\right]\omega$$

$$= 14.74\,\omega$$

$$\Rightarrow 23.65 = 14.74\,\omega \Rightarrow \omega = 1.6\,\text{rad/s}$$



$$T_1 = \frac{1}{2}mv_1^2 = mgh = 78(9.81)(0.3) = 229.6\,\text{J}$$

$$T_2 = \frac{1}{2}I_A\omega^2 = \frac{1}{2}(14.74)(1.6)^2 = 18.87\,\text{J}$$

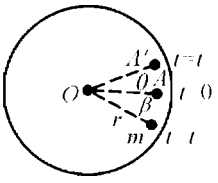
$$n = \frac{229.6 - 18.87}{229.6} \times 100\% = 91.7\%$$

۶-۱۹۰.

فرض کنید v = سرعت مطلق مرد

v_r = سرعت مرد نسبت به میزدوار

ω = سرعت زاویه‌ای میزدوار



$$v = v_r - r\omega \quad (1)$$

$$\Sigma M_O = 0 \Rightarrow \Delta H_O = 0 \Rightarrow mvr = I\omega$$

$$m(v_r - r\omega)r = I\omega \Rightarrow \omega = \frac{mrv_r}{I + mr^2} \quad (2) \qquad \text{یا} \quad \omega = \frac{v_r}{r}\,\frac{1}{1 + \frac{I}{mr^2}}$$

معادلات (1) و (2) $\Rightarrow \quad \dot{\theta} = \dfrac{mr(r\dot{\beta} + r\dot{\theta})}{I + mr^2} \quad \Rightarrow \quad I\dot{\theta} = mr^2\dot{\beta} \quad$ یا $\quad I\theta = mr^2\beta$

$\omega = \dot{\theta} \ , \ v = r\dot{\beta}$

$\theta + \beta = 2\pi \quad \Rightarrow \quad \theta = 2\pi \dfrac{1}{1 + \dfrac{I}{mr^2}}$

۶-۱۹۱. برای سیستم $\int \Sigma M_{(}dt = \Delta H_{(}$

دیسک : $\Delta H_{(} = 4(0.15)^2\omega_B = 0.09\omega_B \qquad$ ($\bar{I}\omega_{(}$ تغییر نمی‌کند)

بازو : $\Delta H_{(} = \dfrac{1}{12}2.5(0.3)^2\omega_B = 0.01875\omega_B$

میله : $\Delta H_{(} = 3(0.15)^2\omega_B = 0.0675\omega_B$

سیستم $\Delta H = (0.09 + 0.01875 + 0.0675)\omega_B = 0.17625\omega_B$

$\Rightarrow \quad 1.5(4) = 0.17625\omega_B \quad \Rightarrow \quad \omega_B = 34.0\,\text{rad/s}$

۶-۱۹۲. (برای سیستم) $H_{z_1} = H_{z_2}$

$0 = (I - I_w)\omega_s + I_w(\omega_s + \omega_{w/s})$

برای چرخ به تنهایی : $\int \Sigma M_G dt = \Delta H_G \quad \Rightarrow \quad Mt = I_w(\omega_s + \omega_{w/s})$

$\Rightarrow \quad 0 = (I - I_w)\omega_s + Mt \quad \Rightarrow \quad \omega_s = -\dfrac{Mt}{I - I_w}$

$\omega_{w/s} = \dfrac{Mt}{I_w} - \omega_s = \dfrac{I}{I_w}\dfrac{Mt}{(I - I_w)}$

۶-۱۹۳.

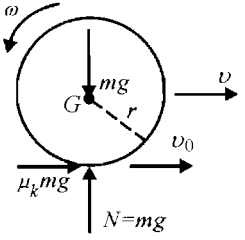
$\int \Sigma F dt = \Delta G \ : \ \mu_k mgt = m(v - 0) \quad \Rightarrow \quad v = \mu_k gt$

$\int M_G dt = \Delta H :$

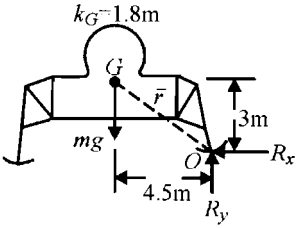
$\mu_k mgrt = \dfrac{1}{2}mr^2\dfrac{v_0 - v}{r} \quad \Rightarrow \quad v_0 = 2\mu_k gt + v = 3\mu_k gt$

$s = \dfrac{1}{2}\dot{v}t^2 = \dfrac{1}{2}\mu_k gt^2 \quad \Rightarrow \quad t = \sqrt{\dfrac{2s}{\mu_k g}}$

$\omega = \dfrac{v_0 - v}{r} = \dfrac{2\mu_k gt}{r} = \dfrac{2\mu_k g}{r}\sqrt{\dfrac{2s}{\mu_k g}} = \dfrac{2}{r}\sqrt{2\mu_k gs}$



۶-۱۹۴. ضربه mg حین بازهٔ برخورد کوچک و قابل صرفنظر است.



قبل از برخورد : $\bar{v} = \frac{8}{3.6} = 2.22\,\text{m/s}$

$\curvearrowleft_+ \Delta H_O = 0$

$m\bar{v}d = m(\bar{k}^2 + \bar{r}^2)\omega$

$2.22(4.5) = ((1.8)^2 + (4.5)^2 + (3)^2)\omega$

$\omega = 0.308\,\text{rad/s}$

۶-۱۹۵.

$\Delta H_o = 0 : I\omega_1 + 4mr_1^2\omega_1 - (I + 4mr_2^2)\omega_2 = 0$

$6(175)^2(120) + 4(0.15)(100)^2(120) = [6(175)^2 + 4(0.15)(200)^2]\omega$

$\omega = 109.6\,\text{rpm}$

$$|\Delta E| = T_1 - T_2 = \frac{1}{2}I\omega_1^2 + 4\left(\frac{1}{2}\right)m(r_1\omega_1)^2 - \frac{1}{2}I\omega_2^2 - 4\left(\frac{1}{2}\right)m(r_2\omega_2)^2$$

$$= \frac{1}{2}\left(120\frac{2\pi}{60}\right)^2[6(0.175)^2 + 4(0.15)(0.100)^2]$$

$$- \frac{1}{2}\left(109.6\frac{2\pi}{60}\right)^2[6(0.175)^2 + 4(0.15)(0.200)^2]$$

$$= 1.298\,\text{J}$$

صرفنظر کردن از قطر ساچمه‌ها منجربه صرفنظر کردن از $I\bar{\omega}$ ساچمه در مقایسه با ممان $mr^2\omega$ از مومنتوم خطی آن می‌شود.

۶-۱۹۶.

$\theta = 0 : (I_{\text{صفحه}})_z = \frac{1}{12}ml^2 + md^2 = \frac{1}{12}8(1.8)^2 + 8(1.5)^2 = 20.16\,\text{kg.m}^2$

$(I_{\text{بدنه}})_z = 160(0.45)^2 = 32.4\,\text{kg.m}^2$

$\theta = 90° : (I_{\text{صفحه}})_z = \frac{1}{12}m(a^2 + b^2) + md^2 = \frac{1}{12}8(1.2^2 + 1.8^2) + 8(1.5)^2 = 21.12\,\text{kg.m}^2$

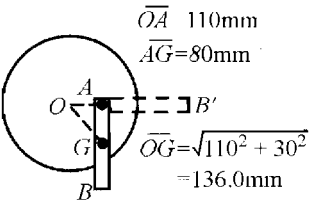
$H_{\theta=0} = (32.4 + 20.16 + 20.16)(1.0) = 72.72\,\text{kg.m}^2/\text{s}$

$H_{\theta=\frac{\pi}{2}} = (32.4 + 21.12 + 21.12)\omega = 74.64\omega\,\text{kg.m}^2/\text{s}$

$\Delta\bar{H}_z = 0 : 72.72 = 74.67\omega \Rightarrow \omega = 0.974\,\text{rad/s}$

۶-۱۹۷.

$\Sigma M_O = 0 = \Delta H_O \Rightarrow H_{O_1} = H_{O_2}$



شرایط اولیه :

$(I_O)_{\text{هر میله}} = 0.84\left(\frac{1}{12}\times 0.160^2 \times 0.136^2\right) = 0.01733\,\text{kg.m}^2$

$(I_O)_{\text{دیسک}} = 30(0.090)^2 = 0.243\,\text{kg.m}^2$

$\omega_1 = 600\times\frac{2\pi}{60} = 62.8\,\text{rad/s}$

$H_{O_1} = [4(0.01733) + 0.243]\,62.8 = 19.62\,\text{kg.m}^2/\text{s}$

شرایط نهایی :

$(I_O)_{\text{هر میله}} = 0.84\left(\frac{1}{12}\times 0.160^2 + [0.110 + 0.080]^2\right) = 0.0321\,\text{kg.m}^2$

$(I_O)_{\text{دیسک}} = 0.243\,\text{kg.m}^2$

$H_{O_2} = [4(0.321) + 0.243]\omega_2 = 0.371\,\omega_2$

$\Rightarrow 19.62 = 0.371\,\omega_2 \Rightarrow \omega_2 = 52.8\,\text{rad/s} \Rightarrow N = 504\,\text{rpm}$

اتلاف انرژی :

$T_1 = \Sigma\frac{1}{2} I_O \omega^2 = \frac{1}{2}(4\times 0.01733\times 0.243)(62.8)^2 = 617\,\text{J}$

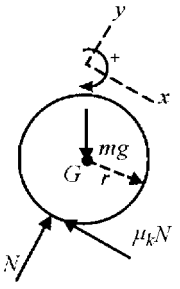
$T_2 = \Sigma\frac{1}{2} I'_O \omega'^2 = \frac{1}{4}(4\times 0.0321 + 0.243)(52.8)^2 = 518\,\text{J}$

$\Delta Q = T_1 - T_2 = 617 - 518 = 98.1\,\text{J}$

جهت دوران و یا ترتیب رها شدن بازوها اثری روی جوابها ندارد.

۶-۱۹۸.

$\int_0^t \Sigma F_y\,dt = m(v_y - v_{y_0}) = 0 \Rightarrow N = mg\cos\theta$



$\int_0^t \Sigma F_x\,dt = m(v_x - v_{x_0})$:

$(-\mu_k mg\cos\theta + mg\sin\theta)t = m(v - v_0) \quad (1)$

$\int_0^t \Sigma M_G\,dt = \overline{I}(\omega - \omega_0)$:

$(\mu_k mg\cos\theta\, r)\,t = \frac{2}{5} m r^2 \omega \quad (2)$

t در مدت زمان : $v = r\omega \quad (3)$

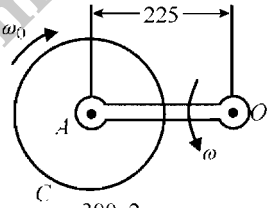
حل معادلات (1) تا (3) :

$$\begin{cases} t = \dfrac{2v_0}{g(7\mu_k\cos\theta - 2\sin\theta)} \\ v = \dfrac{5v_0\mu_k}{7\mu_k - 2\tan\theta} \\ \omega = \dfrac{\dfrac{5v_0\mu_k}{r}}{7\mu_k - 2\tan\theta} \end{cases}$$

برای لغزش تا توقف :

$$\mu_k > \frac{2}{7}\tan\theta \quad \text{یا} \quad 7\mu_k\cos\theta > 2\sin\theta$$

۶-۱۹۹.



$\omega_{rel} = \frac{300\times 2\pi}{60} = 31.4 \text{rad/s}$

سرعت زاویه‌ای واقعی دیسک و آرمیچر $\omega_0 = \omega_{rel} - \omega$

$\Sigma M_O = 0 \Rightarrow \Delta H_O = 0$

$H_{O\,\text{اولیه}} = 0 \Rightarrow H_{O\,\text{نهایی}} = 0$

$OA : H_O = I_O\omega = 4.5(0.175)^2\omega = 0.1378\omega$ CCW

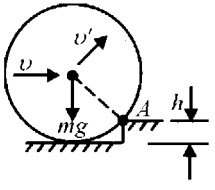
$C : H_O = I_{z'}\omega_0 - mr\omega = 7(0.100)^2[31.4 - \omega] + 7(0.225)^2\omega$

$= 2.198 - 0.424375\omega$ CW

$\Rightarrow 0.1378\omega = 2.198 - 0.424375\omega \Rightarrow \omega = 3.9 \text{ rad/s}$

$N = \frac{3.9\times 60}{2\pi} = 37.4 \text{ rpm}$

۶-۲۰۰. ضربه زاویه‌ای mg ناچیز است.



قبل از برخورد : $H_A = \bar{I}\omega + mv(r - b)$

$= mk^2\frac{v}{r} + mv(r - b)$

درست بعد از برخورد : $H'_A = I_A\frac{v'}{r} = m(k^2 + r^2)\frac{v'}{r}$

$\Delta H_A = 0 : mv\left(\frac{k^2}{r} + r - h\right) = m(k^2 + r^2)\frac{v'}{r}$

$$v' = v\left(1 - \frac{rh}{k^2 + r^2}\right)$$

طی غلتش از روی مانع : $\Delta T + \Delta V_g = 0$

$$\left[0 - \frac{1}{2}m(k^2 + r^2)\right]\frac{v'^2}{r^2} + [mgh - 0] = 0 \Rightarrow v = \frac{r}{k^2 + r^2 - rh}\sqrt{2gh(k^2 + r^2)}$$

۶-۲۰۱. B میلهٔ : $U'_{1\text{-}2} = 0 = \Delta T + \Delta V_g$

$\Delta V_g = -mgh = -8(9.81)(0.180) = -14.13\,\text{J}$

$\Delta T = \frac{1}{2} I \omega_B^2 = \frac{1}{2}(8)(0.220)^2 \omega_B^2 = 0.1936\,\omega_B^2$

$\Rightarrow 0 = 0.1936\,\omega_B^2 - 14.13 \Rightarrow \omega_B = 8.54\,\text{rad/s}$

قبل از برخورد : $H_O = I\omega_B = 8(0.220)^2(8.54) = 3.31\,\text{kg.m}^2/\text{s}$

برای سیستم بعد از برخورد : $H_O = I_{\text{tot}}\,\omega = [2.20(0.3)^2 + 8(0.220)^2]\omega = 3.99\,\omega$

$\Delta H_O = 0 : 3.99\,\omega - 3.31 = 0 \Rightarrow \omega = 0.830\,\text{rad/s}$

بعد از برخورد : $U'_{1\text{-}2} = 0 = \Delta T + \Delta V_g$

$\Delta V_g = mgh = 2.20(9.81)(0.25)(1 - \cos\theta) + 8(9.81)(0.18) - (1 - \cos\theta)$

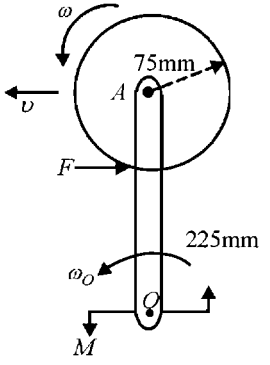
$= 112.2(1 - \cos\theta)$

$\Delta T = 0 - \frac{1}{2} I\omega^2 = -\frac{1}{2}[2.20(0.3)^2 + 8(0.220)^2](0.830)^2 = -1.372\,\text{J}$

$\Rightarrow 0 = 112.2(1 - \cos\theta) - 1.372 \Rightarrow \theta = 8.97°$

اتلاف انرژی : $\Delta Q = (V_g)_{\text{قبل}} - (V_g)_{\text{بعد}} = 14.13 - 11.2(1 - \cos 8.97°) = 12.75\,\text{J}$

۶-۲۰۲. برای هر چرخدنده کوچک به تنهایی $\Sigma M_G = \bar{I}\alpha$



$(0.075)F = \frac{1}{2}3(0.075)^2(20) \Rightarrow F = 2.25\,\text{N}$

که در آن : $\alpha_{\text{چرخدنده}} = \frac{\dot{v}}{0.075}$, $\dot{v} = 0.225\,\frac{20}{3} = 1.5\,\text{m/s}^2$

$\Rightarrow \alpha_{\text{چرخدنده}} = \frac{1.5}{0.075} = 20\,\text{rad/s}^2$

$v = 0.225\,\omega_o = 0.225(20) = 4.5\,\text{m/s}$

$\omega = \omega_{\text{چرخدنده}} = \frac{v}{r} = \frac{4.5}{0.075} = 60\,\text{rad/s}$

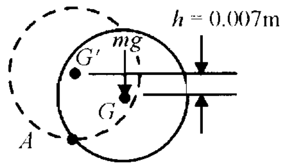
$H_{O\text{چرخدنده}} = \bar{I}\omega + m\bar{v}d = \frac{1}{2}3(0.075)^2 60 + 3(4.5)(0.225)$

$= 3.54375\,\text{kg.m}^2/\text{s}$

$H_{O\text{بازو}} = I_O\omega_O = 2(0.15)^2 20 = 0.9\,\text{kg.m}^2/\text{s}$

$\int \Sigma M_O\,dt = \Delta H_O : (M - 2.25\times 0.15)3 = 3.54375 + 0.9 \Rightarrow M = 1.819\,\text{N.m}$

۶-۲۰۳. غلتش حول نقطه ثابت A : فرآیند II

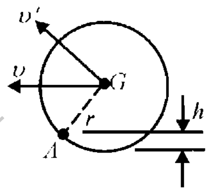


$$U_{1\text{-}2} = \Delta T$$

$$-mgh = \frac{1}{2}\left(\frac{1}{2}mr^2\right)(\omega'^2 - 0) \Rightarrow$$

$$\omega' = \sqrt{\frac{4gh}{3r^2}} = \sqrt{\frac{4(9.81)(0.007)}{3(0.035)^2}} = 8.65\,\text{rad/s}$$

برخورد در A : فرآیند I

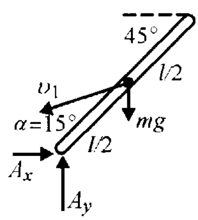


$$\Delta H_A = 0 \;:\; mv(r-h) = I_A\omega' = \left(\frac{3}{2}mr^2\right)\omega'$$

$$v = 0.5\Omega \Rightarrow \frac{1}{2}\Omega(r-b) = \frac{3}{2}r^2\omega'$$

$$\Omega = \frac{3r^2\omega'}{r-h} = \frac{3(0.035)^2(8.65)}{0.035 - 0.007} = 1.135\,\text{rad/s}$$

۶-۲۰۴. از ضربه وزن طی برخورد صرفنظر کنید بنابراین $\Delta H_A = 0$



$$mv_l\frac{l}{2}\sin 15° = \frac{1}{3}ml^2\omega_2$$

$$\omega_2 = \frac{3v_1}{2l}\sin 15°$$

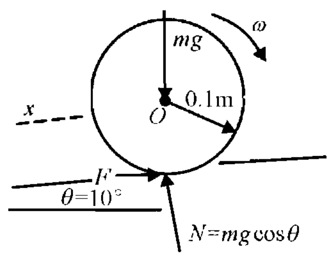
طی دوران حول A : $U = \Delta T$

$$-mg\frac{l}{2}(1 - \cos 45°) = 0 - \frac{1}{2}I_A\omega_2^2$$

$$\omega_2 = \sqrt{\frac{3g}{l}\left(1 - \frac{\sqrt{2}}{2}\right)}$$

$$\Rightarrow \sqrt{\frac{3g}{l}\left(1 - \frac{\sqrt{2}}{2}\right)} = \frac{3v_1}{2l}\sin 15° \Rightarrow v_1 = 2.41\sqrt{gl}$$

◀ **۶-۲۰۵.** طی لغزش : $(a_O)_x = 0 \Rightarrow \Sigma F_x = 0$



$$\left.\begin{aligned} &\Rightarrow F - mg\sin\theta = 0 \\ &\text{از طرفی}\quad F = \mu_k mg\cos\theta \end{aligned}\right\} \Rightarrow mg\sin\theta = \mu_k mg\cos\theta$$

$$\Rightarrow \mu_k = \tan\theta = \tan 10° = 0.1636$$

$$\Sigma M_O \times t = \Delta H_O \;:$$

$$0.1763(30)(9.81)\cos 10°(0.1)\,t = 0 - (-30 \times 0.075^2)\frac{300 \times 2\pi}{60} \Rightarrow t = 1.037\,\text{s}$$

(فرض عدم لغزش) طی غلتش

$$\int_0^4 \Sigma F_x\,dt = m\Delta v_x : (30 \times 9.81\sin 10° - F)4 = 30(v - 0) \Rightarrow 204 - 4F = 30v$$

$$\int_0^4 \Sigma M_O\,dt = I_O\Delta\omega : 0.1F \times 4 = 30 \times 0.075^2\left(\frac{v}{0.1}\right) \Rightarrow 4F = 16.88v$$

ترکیب دو رابطه : $F = 18.40\,\text{N}$, $v = 4.36\,\text{m/s}$

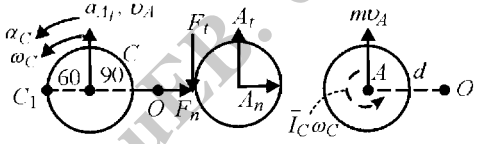
چک کردن : $F_{\max} = \mu_s N$

$$\mu_k N = 0.1763 \times 30 \times 9.81\cos 10° = 51.1\,\text{N} < \mu_s N$$

$\Rightarrow 18.40 < \mu_k N < \mu_s N \Rightarrow$ فرض عدم لغزش صحیح نیست

◀ ۲۰۶-۶.

برای کل سیستم $\Sigma M_O = \dot{H}_O$



از طرفی $\Sigma M_O =$

(گشتاور حول O ناشی از نیروهای مخالفی که از خارج به چرخدنده وارد می‌شوند) $M +$

ابتدا نیروی وارده توسط چرخدنده رینگی بر چرخدندهٔ C را تعیین می‌کنیم:

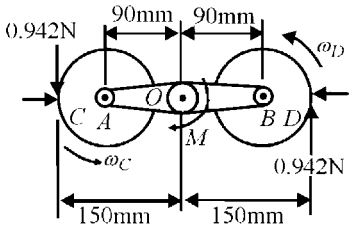
$$\alpha_{OA} = \frac{\omega}{t} = \frac{600 \times 2\pi}{60 \times 5} = 12.57\,\text{rad/s} \quad \text{(ثابت)}$$

$$a_{A_t} = r\alpha_{OA} = 0.090(12.57) = 1.131\,\text{m/s}^2$$

$$\alpha_C = \frac{a_{A_t}}{r} = \frac{1.131}{0.060} = 18.85\,\text{rad/s}^2$$

برای C : $\Sigma M_A = I_A\alpha$: $F_t \times 0.060 = (1.2 \times 0.050^2)18.85 \Rightarrow F_t = 0.942\,\text{N}$

دیاگرام آزاد سیستم :



$$\zeta_+ \Sigma M_O = \dot{H}_O \Rightarrow \Sigma M_O\,t = \Delta H_O = H_O$$

$$I_{AOB} = 2(2.1 \times 0.075^2) = 0.0236\,\text{kg.m}^2$$

$$\omega_{AOB} = 600 \times \frac{2\pi}{60} = 62.8\,\text{rad/s}$$

$$I_C = I_D = 1.2 \times 0.050^2 = 0.003\,\text{kg.m}^2$$

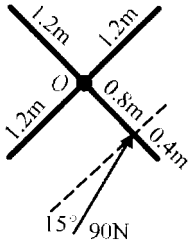
$$\omega_C = \omega_D = \frac{90}{60} \times 62.8 = 94.2\,\text{rad/s}$$

$$H_{O_{AOB}} = 0.0236 \times 62.8 = 1.484\,\text{kg.m}^2\text{/s CW}$$

$$H_{O_C} = m v_A d - I_C\omega_C = 1.2(0.090 \times 62.8)(0.090) - 0.033(94.2) = 0.328\,\text{kg.m}^2\text{/s} = H_{O_D}$$

$$\Rightarrow [M - 2(0.924)(0.150)]5 = 1.484 + 2(0.328) \Rightarrow M = 0.711\,\text{N.m}$$

۶-۲۰۷.



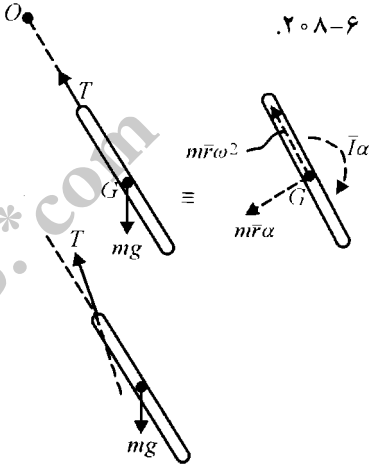
$$I_O = 4\left(\frac{1}{3}ml^2\right) = 4\left(\frac{1}{3}60(1.2)^2\right) = 115.2\,\text{kg.m}^2$$

$$\curvearrowright + \Sigma M_O = I_O\alpha$$

$$(90\cos 15°)(0.8) = 115.2\alpha$$

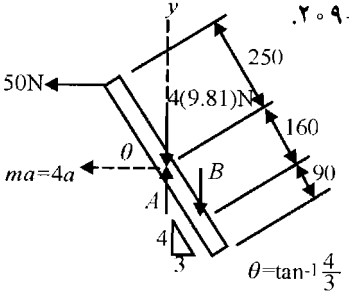
$$\alpha = 0.604\,\text{rad/s}^2$$

۶-۲۰۸.



فرض کنید میله همراستای طناب باقی بماند. برای انطباق سرعت زاویه‌ای میله با طناب، دیاگرام سینتیک نیاز به یک گشتاور در جهت عقربه‌های ساعت حول G دارد که در حالت مفروض فراهم نخواهد شد. لذا فرض غلط می‌باشد. دیاگرام آزاد صحیح به صورت شکل پائینی خواهد بود.

۶-۲۰۹.



مرکز جرم در A

$$\curvearrowright \Sigma M_A = 0 \;:\; B\times 160\times\frac{3}{5} - 50\times 250\times\frac{4}{5} = 0$$

$$B = 104.2\,\text{N}$$

$$\Sigma F_y = 0 \;:\; A - 4(9.81) - 104.2 = 0$$

$$A = 143.4\,\text{N}$$

۶-۲۱۰. حداکثر توان موقعی است که $\frac{dV_g}{dt}$ بیشترین مقدار باشد یعنی وقتی که $\overline{v}_y$ در شروع حداکثر باشد.

$$\overline{v}_y = 1.500\,\omega = 1.500\,\frac{4\pi}{18} = 0.1047\,\text{m/s}$$

$$P = mg\overline{v}_y = 1600(5)(9.81)(0.1047) = 8218\,\text{W} \quad \text{یا} \quad P = 8.22\,\text{kW}$$

۶-۲۱۱.

(II) $\Sigma M_N = m\bar{a}d$

$1600(9.81)(1.2) - 900(9.81)(1.2)$

$= 900a(1.2)$

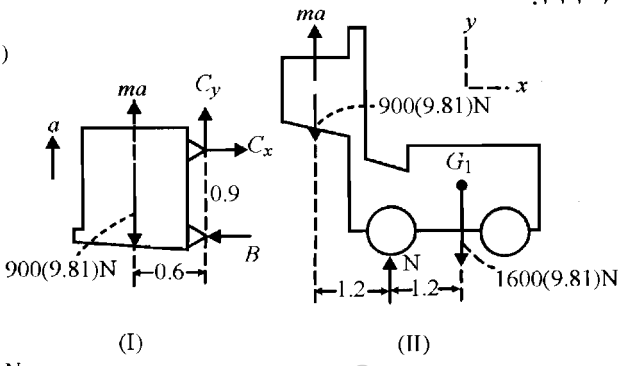
$a = 7.63\,\text{m/s}^2$

(I) $\Sigma M_G = m\bar{a}d$

$0.9B - 0.6(900)(9.81)$

$= 900(7.63)(0.6)$

$B = 10464\,\text{N}$ یا $B = 10.46\,\text{kN}$



۶-۲۱۲.

برای کل فضاپیما : $\Sigma M_x = I_x\alpha$: $10^{-6} = 150000\alpha \Rightarrow \alpha = 6.67\times10^{-12}\,\text{rad/s}^2$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\frac{1}{3600}\left(\frac{\pi}{180}\right) = 0 + 0 + \frac{1}{2}(6.67\times10^{-12})t^2 \Rightarrow t = 1206\,\text{s}$

۶-۲۱۳.

برای سیستم : $U = \Delta T + \Delta V_g + \Delta V_e$

$U = 0$

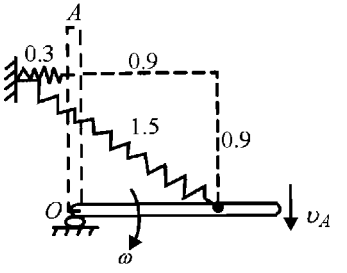
$\Delta T = \frac{1}{2}I_O\omega^2 = \frac{1}{2}\left(\frac{1}{3}ml^2\right)\left(\frac{v_A}{l}\right)^2$

$= \frac{1}{6}m{v_A}^2 = \frac{1}{6}30{v_A}^2 = 5{v_A}^2$

$\Delta V_e = \frac{1}{2}kx^2 - 0 = \frac{1}{2}150(1.5 - 0.3)^2 = 108\,\text{J}$

$\Delta V_g = -30(9.81)(0.6) = -176.58\,\text{J}$

$\Rightarrow 0 = 5{v_A}^2 - 176.58 + 108 \Rightarrow v_A = 3.70\,\text{m/s}$



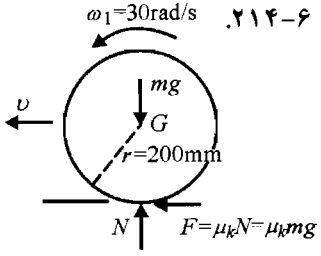
۶-۲۱۴.

لغزش رخ می‌دهد تا $v = r\omega$

$\int \Sigma F\,dt = \Delta G$: $\mu_k mgt = mv - 0 \Rightarrow \mu_k gt = v$

$\curvearrowright + \int \Sigma M_G\,dt = \Delta H_G$: $\mu_k mgrt = mr^2\left[-\frac{v}{r} - (-\omega_1)\right]$

$\mu_k gt = -v + r\omega_1$



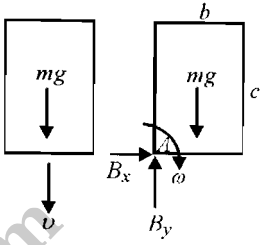
v حذف : $2\mu_k g t = r\omega_1 \Rightarrow \mu_k = \dfrac{r\omega_1}{2gt} = \dfrac{0.2(30)}{2(9.81)(1.5)} = 0.204$

$v = \mu_k g t = \dfrac{r\omega_1}{2} = \dfrac{0.2(30)}{2} = 3\ \text{m/s}$

۶-۲۱۵.

$v = \sqrt{2gh}$

$\Delta H_B = 0 : I_A \omega - m v \dfrac{b}{2} = 0$



$I_A = \dfrac{1}{12} m (b^2 + c^2) + m\left[\left(\dfrac{b}{2}\right)^2 + \left(\dfrac{c}{2}\right)^2\right] = \dfrac{1}{3} m (b^2 + c^2)$

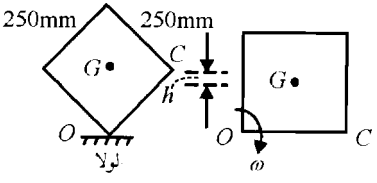
$\Rightarrow \dfrac{1}{3} m (b^2 + c^2)\omega - m \sqrt{2gh}\dfrac{b}{2} = 0 \Rightarrow \omega = \dfrac{3b\sqrt{2gh}}{2(b^2 + c^2)}$

درصد اتلاف انرژی $n = \dfrac{|\Delta E|}{E} = \dfrac{\frac{1}{2} m v^2 - \frac{1}{2} I_A \omega^2}{\frac{1}{2} m v^2} = 1 - \dfrac{I_A \omega^2}{m v^2}$

$b = c$ برای : $n = 1 - \dfrac{\frac{2c^2}{3}}{2gh} \dfrac{9c^2(2gh)}{4(2c^2)} = 1 - \dfrac{3}{8} = \dfrac{5}{8}$ یا $n = 62.5\%$ اتلاف

۶-۲۱۶.

$h = 0.25\left(\dfrac{1}{\sqrt{2}} - \dfrac{1}{2}\right) = 0.5178\ \text{m}$



(الف) $\bar{I} = \dfrac{1}{6} m (0.25)^2 = 0.01042\, m$

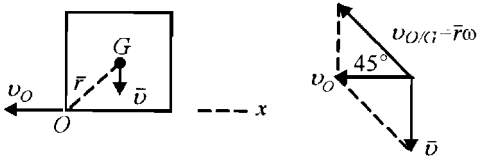
$I_O = \bar{I} + m\left(\dfrac{0.25}{\sqrt{2}}\right)^2 = 004167\, m$

$\Delta V_g + \Delta T = 0$

$-mgh + \dfrac{1}{2} I_O \omega^2 = 0 \Rightarrow \omega^2 = \dfrac{2mgh}{I_O} = \dfrac{2m(9.81)(0.05178)}{0.04167\, m} = 24.38 \Rightarrow \omega = 4.94\ \text{rad/s}$

(ب) $\Sigma F_x = 0 \Rightarrow$ در نتیجه $\bar{v}$ عمودی می‌مانند $\bar{a}$

$\bar{v} = \dfrac{\bar{r}\omega}{\sqrt{2}} = \dfrac{0.25}{2}\omega = 0.125\,\omega$

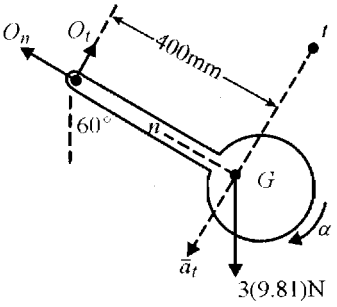


$\Delta V_g + \Delta T = 0$

$\Delta T = \dfrac{1}{2} m \bar{v}^2 + \dfrac{1}{2} \bar{I}\omega^2 = \dfrac{1}{2} m (0.125\,\omega)^2 + \dfrac{1}{2}(0.01042\, m)\omega^2 = 0.01302\, m\,\omega^2$

$\Rightarrow -mg(0.05178) + 0.01302\, m\,\omega^2 = 0 \Rightarrow \omega = 6.25\ \text{rad/s}$

۶-۲۱۷.



$\Sigma F_n = m\bar{a} = m\bar{r}\omega^2 = 0$:

$O_n - 3(9.81)\cos 60° = 0 \Rightarrow O_n = 14.715\,\text{N}$

$\Sigma F_t = m\bar{a}_t = m\bar{r}\alpha$:

$3(9.81)\sin 60° - O_t = 3(0.4)\alpha \quad (1)$

$\Sigma M_O = I_O\alpha$:

$[3(9.81)\sin 60°](0.4) = 3(0.425)^2\alpha$

$\alpha = 18.814\,\text{rad/s}^2 \Rightarrow$ (1) در $O_t = 2.91\,\text{N}$

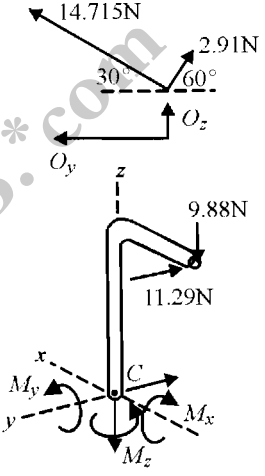
$O_y = 14.715\cos 30° - 2.91\cos 60° = 11.29\,\text{N}$

$O_z = 14.715\sin 30° + 2.91\sin 60° = 9.88\,\text{N}$

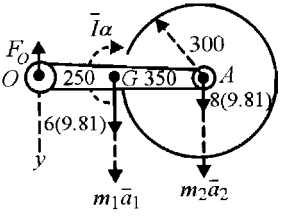
CA تعادل : $M_x = -11.29(0.5) = -5.64\,\text{N.m}$

$M_y = +9.88(0.2) = 1.976\,\text{N.m}$

$M_z = -11.29(0.2) = -2.26\,\text{N.m}$



۶-۲۱۸. دیسک هیچ گشتاوری حول مرکز خود ندارد لذا تحت جابجایی منحنی‌الخط بدون هیچ $\bar{I}\alpha$ قرار می‌گیرد.



(الف) OA برای : $m_1\bar{a}_1 = 6(0.250)\alpha = 1.5\alpha$

$\bar{I}\alpha = 6[(0.375)^2 - (0.250)^2]\alpha = 0.46875\alpha$

برای دیسک : $m_2\bar{a}_2 = 8(0.600)\alpha = 4.8\alpha$

$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d$

$6(9.81)(0.250) + 8(9.81)(0.600) = 0.46875\alpha + 1.5\alpha(0.250) + 4.8\alpha(0.600)$

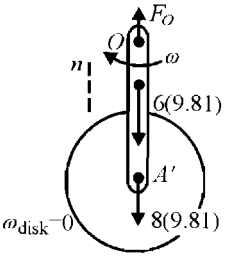
$\alpha = 16.6\,\text{rad/s}$

$\Sigma F_y = m\bar{a}_y$

$8(9.81) + 6(9.81) - F_O = (1.5 + 4.8)(16.6) \Rightarrow F_O = 32.8\,\text{N}$

(ب) $U = \Delta T$: $6(9.81)(0.250) + 8(9.81)(0.600) = \frac{1}{2}6(0.375)^2\omega^2 + \frac{1}{2}8(0.600\,\omega)^2$

$\omega^2 = 33.19\,(\text{rad/s})^2$



$\Sigma F_n = \Sigma m\,\bar{a}_n$:

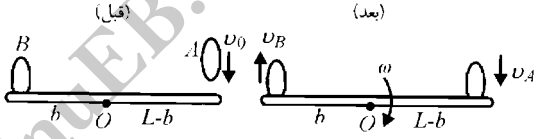
$F_O - 6(9.81) - 8(9.81) = 6(0.250)(33.19) + 8(0.600)(33.19)$

$\Rightarrow F_O = 347\,\text{N}$

۶-۲۱۹.

قبل : $H_O = m_A v_0 (L - b)$

بعد : $H_O = m_A v_A (L - b) + m_B v_B b$



$\left.\begin{aligned}\Delta H_O &= 0\\ \omega &= \frac{v_B}{b} = \frac{v_A}{L-b}\end{aligned}\right\} \Rightarrow m_A v_0 (L - b) = m_A \frac{L-b}{b} v_B (L - b) + m_B v_B b$

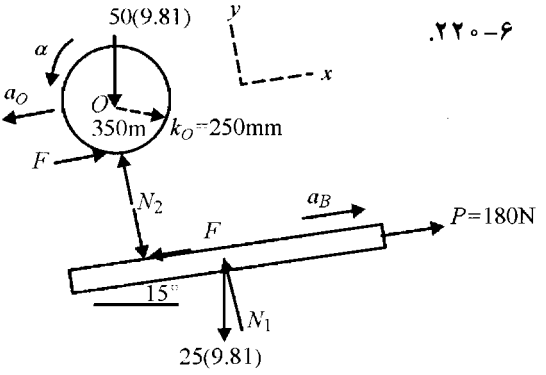
$$v_B = v_0 \frac{1}{\frac{L-b}{b} + n\frac{b}{L-b}} \qquad \left(n = \frac{m_B}{m_B} \text{ که در آن}\right)$$

$$\frac{dv_B}{db} = v_0 \frac{-\left(\frac{L}{b^2} + n\frac{L-b-b(-1)}{(L-b)^2}\right)}{\left(\frac{L-b}{b} + n\frac{b}{L-b}\right)^2} = v_0 \frac{L\left(\frac{1}{b^2} - \frac{n}{(L-b)^2}\right)}{\left(\frac{L-b}{b} + n\frac{b}{L-b}\right)^2} = 0 \qquad (\text{برای } v_{B_{\max}})$$

$\Rightarrow \frac{1}{b^2} = \frac{n}{(L-b)^2} \Rightarrow b = \frac{L}{1 \pm \sqrt{n}}$ (علامت + منتهی به v_B مثبت می‌شود)

$\Rightarrow b = \frac{L}{1+\sqrt{n}} \Rightarrow v_B = \frac{v_0}{2\sqrt{n}}$

۶-۲۲۰.



تخته :

$\Sigma F_x = m\bar{a}_x$:

$180 - 36(9.81)\sin 15° - F = 25\,a_B$ (1)

چرخ :

$\Sigma F_x = m\bar{a}_x$:

$F - 50(9.81)\sin 15° = 50(-a_O)$ (2)

$\Sigma M_O = I_O\alpha$:

$F(0.350) = 50(0.250)^2\alpha \Rightarrow F = 8.93\alpha$ (3)

شتاب نسبی : $\dfrac{(a_B + a_O)}{0.350} = \alpha \Rightarrow a_O + a_B = 0.350\alpha$ (4)

حل (1) تا (4) : $a_B = 1.758\ \text{m/s}^2$ (در جهت مثبت xها)

$a_O = 1.087\ \text{m/s}^2$ (در جهت منفی xها)

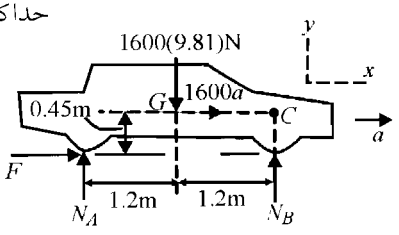
$F = 72.6\ \text{N}$

$$(\mu_s)_{\min} = \frac{F}{N_2} = \frac{72.6}{50(9.81)\cos 15°} = 0.1532$$

۶-۲۲۱.

(الف) $F = \mu N_A = 0.8\,N_A$

حداکثر شتاب وقتی رخ می دهد که: $\curvearrowleft_+ \Sigma M_G = mad = 0$

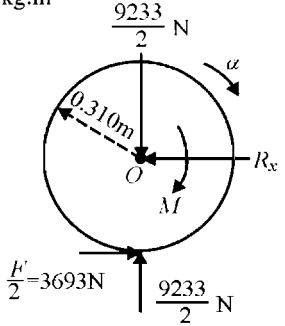


$1600(9.81)(1.2) - 2.4\,N_A + 0.8\,N_A(0.95) = 0$

$N_A = 9233\,N \Rightarrow F = 0.8(9233) = 7386\ \text{N}$

$\Sigma F_x = m\,a_x$: $7386 = 1600\,a \Rightarrow a = 4.62\ \text{m/s}^2$

(ب) هر یک از چرخ‌های عقب $I_O = m\,k^2 = 32(0.210)^2 = 1.411\ \text{kg.m}^2$



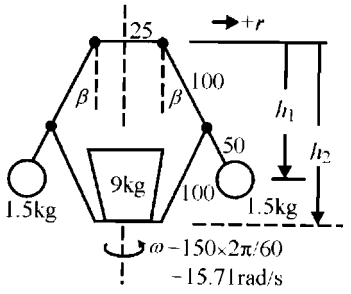
$\alpha = \dfrac{a}{r} = \dfrac{4.62}{0.310} = 14.89\ \text{rad/s}^2$

$\Sigma M_O = I_O\alpha$: $M - 3693(0.310) = 1.411(14.90)$

$M = 1166\ \text{N.m}$

۶-۲۲۲.

$\delta T_{\text{هر دو گوی}} = 2\,m\,a_r\,\delta r$



$r = (0.025 + 0.150\sin\beta)$

$a_r = -r\omega^2 = -r(15.71)^2 = -6.17(1 + 6\sin\beta)$

$\delta T = -2(1.5)6.17(1 + 6\sin\beta)\,0.15\cos\beta\,\delta\beta$

$= -2.7765(1 + 6\sin\beta)\cos\beta\,\delta\beta$

$\delta V_g = -9(9.81)\,\delta h_2 - 2(1.5\times 9.81)\,\delta h_1$

$\delta h_1 = \delta(0.150\cos\beta) = -0.15\sin\beta\,\delta\beta$

$\delta h_2 = \delta(2\times 0.100\cos\beta) = -0.2\sin\beta\,\delta\beta$

$\delta V_g = [17.658 + 4.4145]\sin\beta\,\delta\beta = 22.0725\sin\beta\,\delta\beta$

$\delta U = \delta T + \delta V_g = 0$

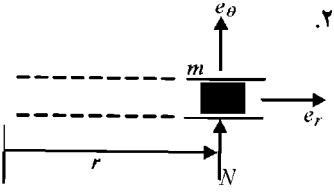
$-2.7765(1 + 6\sin\beta)\cos\beta\,\delta\beta + 22.0725\sin\beta\,\delta\beta = 0$

$1 + 6\sin\beta = 7.95\tan\beta$

حل با روش تخمین نیوتنی $\Rightarrow \beta = 22.5°$

۶-۲۲۳.

پایستاری مومنتوم زاویه‌ای : $I_O\omega_O = (I_O + m r^2)\omega$



$\dot\theta = \omega = \dfrac{I_O\omega_O}{I_O + m r^2}$

$\Sigma F_r = ma_r = m(\ddot r - r\dot\theta^2) \;:\; 0 = m(\ddot r - r\dot\theta^2)$

$\ddot r = \dot r\dfrac{d\dot r}{dr} = r\left(\dfrac{I_O\omega_O}{I_O + m r^2}\right)^2 \Rightarrow \int_0^{\dot r}\dot r\,d\dot r = I_O{}^2\omega_O{}^2\int_0^r \dfrac{r\,dr}{(I_O + m r^2)^2}$

انتگرال‌گیری : $\dot r = \left(\dfrac{I_O\omega_O r^2}{I_O + m r^2}\right)^{\frac12} = \omega_O r\sqrt{\dfrac{I_O}{I_O + m r^2}}$

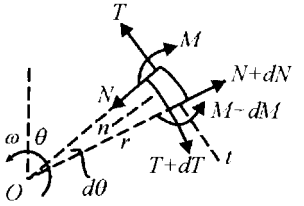
۶-۲۲۴. ◀

$\Sigma F_n = m\,\bar a_n$

$2T\sin\dfrac{d\theta}{2} + dT\sin\dfrac{d\theta}{2} + N\cos\dfrac{d\theta}{2} - (N + dN)\cos\dfrac{d\theta}{2} =$

$= \rho\,r\,d\theta\,(r\omega^2)$

$\Rightarrow T - \rho\,r^2\omega^2 = \dfrac{dN}{d\theta} \quad (1)$



$\Sigma F_t = m\,\bar a_t = 0$

$$-T\cos\frac{d\theta}{2}+(T+dT)\cos\frac{d\theta}{2}+N\sin\frac{d\theta}{2}+(N+dN)\sin\frac{d\theta}{2}=0 \Rightarrow N=-\frac{dT}{d\theta} \quad (2)$$

ترکیب (1) و (2) $\Rightarrow \frac{d^2N}{d\theta^2}+N=0$

$\Rightarrow N=A\sin\theta+B\sin\theta$

(برای $\theta=0$) $\Rightarrow N=A\sin\theta$ طبق تقارن $N=0$

از رابطه (1) $T=\rho r^2\omega^2+A\cos\theta$

(وقتی $\theta=\pi$) $\Rightarrow A=\rho r^2\omega^2$

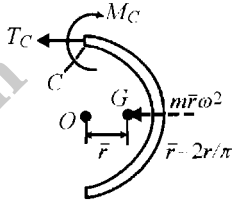
$\Rightarrow N=\rho r^2\omega^2\sin\theta$

$T=\rho r^2\omega^2(1+\cos\theta)$

$\Sigma M_C=m\bar{a}d \;:\; M_C=m\frac{2r}{\pi}\omega^2 r$

$=\rho\pi r\left(\frac{2r^2}{\pi}\omega^2\right)$

$\Rightarrow M_C=2\rho r^2\omega^2$



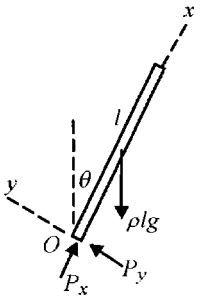
۲۲۵-۶ ◀

(جرم طول واحد $=\rho$) برای کل دودکش (میله) : $\Sigma M_O=I_O\alpha$

$\rho l g\frac{l}{2}\sin\theta=\frac{1}{3}\rho l(l^2)\alpha \Rightarrow \alpha=\frac{3}{2}\frac{g}{l}\sin\theta$

$\Sigma F_y=m\bar{a}_y$

$P_y-\rho l g\sin\theta=\rho l\left(-\frac{l}{2}\frac{3}{2}\frac{g}{l}\sin\theta\right) \Rightarrow P_y=\frac{1}{4}\rho l g\sin\theta$



برای قسمتی به طول x : $M=$ گشتاور خمشی $V=$ نیروی برشی

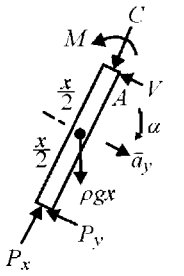
$\Sigma M_A=\bar{I}\alpha+m\bar{a}d$

$\bar{a}d=\bar{a}_y\frac{x}{2}=\frac{x}{2}\alpha\frac{x}{2}=\frac{x^2}{4}\alpha$

$\bar{I}\alpha=\frac{1}{12}\rho x(x^2)\alpha=\frac{1}{12}\rho x^3\alpha$

$-M-\rho g x\frac{x}{2}\sin\theta+\frac{1}{4}\rho l g x\sin\theta=\frac{\rho x^3}{12}\alpha-\rho x\frac{x^2}{4}\alpha$

$=-\frac{1}{4}\rho x^3\frac{g}{l}\sin\theta$



$$\Rightarrow M = \frac{\rho g \sin\theta}{4l}(x^3 - 2lx^2 + l^2x)$$

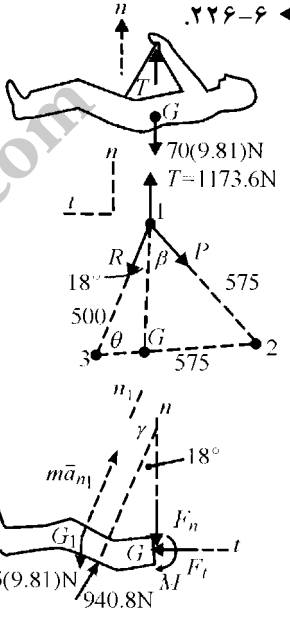
برای M_{max} : $\frac{dM}{dx} = \frac{\rho g \sin\theta}{4l}(3x^2 - 4lx + l^2) = 0 \Rightarrow (3x - l)(x - l) = 0 \Rightarrow x = \frac{l}{3}\ (M_{max})$

$x = l \quad (M = 0)$

$x = \frac{l}{3}$ در مرکز ضربه از سردودکش قرار دارد که: $q = \frac{k_O^2}{\bar{r}} = \frac{1}{3}\frac{l^2}{\frac{l}{2}} = \frac{2}{3}l$

◀ ۶-۲۲۶.

دوران محور ثابت



$\Sigma F_n = m\bar{a}_n : T - 70(9.81) = 70\frac{4^2}{2.3} \Rightarrow T = 1173.6\,\text{N}$

$\theta = \cos^{-1}\left(\frac{250}{575}\right) = 64.2°$

$\beta = \theta - 18° = 46.2°$

$\Sigma F_n = 0 : 1173.6 - R\cos 18° - P\cos 46.2° = 0$

$\Sigma F_t = 0 : R\sin 18° - P\sin 46.2° = 0$

$\Rightarrow P = 402.8\,\text{N}\ ,\ R = 940.8\,\text{N}$

$\gamma = \sin^{-1}\frac{325}{2300} = 8.12°$

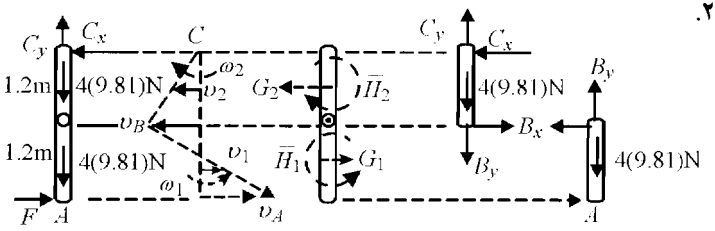
$\Sigma F_t = m\bar{a}_t : 940.8\sin 18° - F_t = 35\frac{4^2}{2.3}\sin 8.12°$

$F_t = 256.3\,\text{N}$

$+\circlearrowright \Sigma M_O = I_O\alpha = 0 : -940.8\ \sin 18°(2.3 - 0.455) - 35(9.81)(0.325) + 256.3(2.3) + M = 0$

$\Rightarrow M = 58.4\ \text{N.m}$

◀ ۶-۲۲۷.



$$\bar{I} = \frac{1}{12} m l^2 = \frac{1}{12}(4)(1.2)^2 = 0.48\,\text{kg.m}^2$$

$$\int F\,dt = 14\,\text{N.s}$$

$$\omega_2 = \frac{v_2}{0.6}$$

$$\omega_1 = \frac{(v_1 + v_B)}{0.6} = \frac{(v_1 + 2v_2)}{0.6}$$

$$m = 4\,\text{kg}$$

برای سیستم $\int \Sigma M_C\,dt = \Sigma \Delta H_C$: $14(2.4) = 4v_1(1.8) + 0.48\omega_1 - 4v_2(0.6) - 0.48\omega_2$ (1)

AB : $\int \Sigma M_C\,dt = \Delta H_C$: $14(2.4) - \int 1.2B_x\,dt = 4v_1(1.8) + 0.48\omega_1$ (2)

$\int \Sigma F_x\,dt = \Delta G_x$: $14 - \int B_x\,dt = 4v_1$ (3)

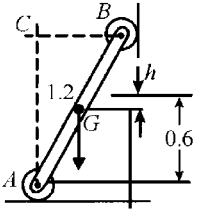
(2) و (3) : $2v_1 + v_2 = 10.5$

ω_2 ، ω_1 و (1) : $5v_1 - v_2 = 21$

$\Rightarrow$ $v_1 = 4.5\,\text{m/s}$

$v_2 = 1.5\,\text{m/s} \Rightarrow \omega_2 = 2.50\,\text{rad/s}$

۲۲۸-۶ *



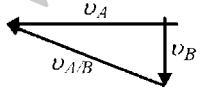
$$U = \Delta T \;:\; T = \frac{1}{2} I_C \omega^2 = \frac{1}{2}\frac{1}{3} m (1.2)^2 \omega^2$$

$$U = mgh = mg(0.6 - 0.6\cos\theta) = 0.6mg(1 - \cos\theta)$$

$$\Rightarrow 0.6mg(1 - \cos\theta) = \frac{1.44m}{6}\omega^2 \Rightarrow \omega^2 = 2.5g(1 - \cos\theta)$$

$$\omega = \sqrt{2.5(9.81)(1 - \cos\theta)} = 4.95\sqrt{1 - \cos\theta}\ \text{rad/s}$$

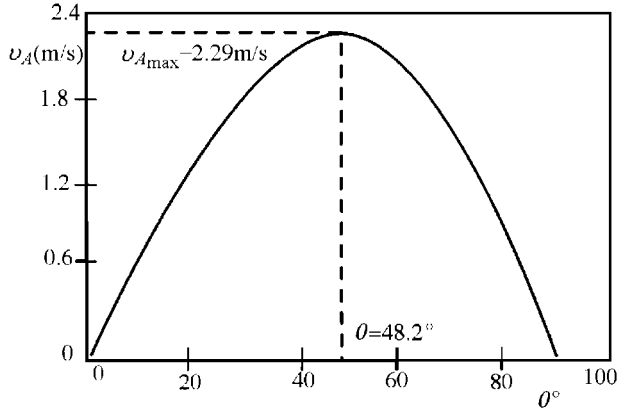
$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$



$$v_A = v_{A/B}\cos\theta = L\omega\cos\theta$$

$$= 1.2(4.95)\sqrt{1 - \cos\theta}\cos\theta$$

$$= 5.94\cos\theta\sqrt{1 - \cos\theta}\ \text{m/s}$$



۶-۲۲۹ *

$$\Sigma M_O = I_O\alpha \;:\; 78.5(0.220\cos\theta) = 8(0.235^2)\,\alpha \;\Rightarrow\; \alpha = 39.1\cos\theta$$

$$\int_0^{\omega}\omega\,d\omega = \int_0^{\theta}\alpha\,d\theta \;:\; \frac{\omega^2}{2} = 39.1\sin\theta \;\Rightarrow\; \omega^2 = 78.2\sin\theta$$

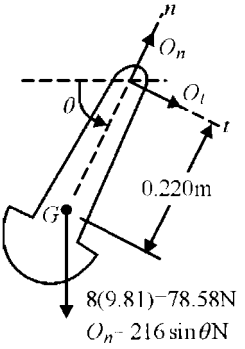
$$\Sigma F_t = m\,\bar{a}_t \;:\; O_t + 78.5\cos\theta = 8\times 0.220\,\alpha$$

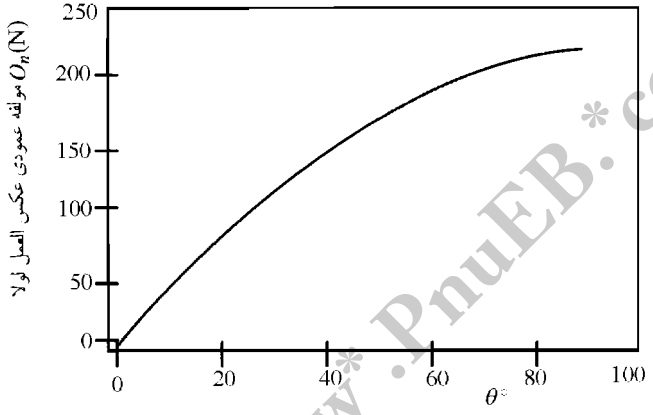
$$O_t = 8(0.220)39.1\cos\theta - 78.5\cos\theta$$

$$O_t = -9.70\cos\theta \;\Rightarrow$$

$$(O_t)_{\max} = 9.70\,\text{N} \;@\; \theta = 0 \;(-t \text{ در راستای})$$

$$\Sigma F_n = m\,\bar{a}_n \;:\; O_n - 78.5\sin\theta = 8\times 0.220\,\omega^2$$





۶-۲۳۰ *

$$U' = \Delta T + \Delta V_e + \Delta V_g \;,\; U' = 0$$

$$\Delta T = \frac{1}{2}m\,v^2 - 0 = \frac{1}{2}5\,v^2$$

$$\Delta V_e = \frac{2}{2}k\,(x_2^{\,2} - x_1^{\,2})$$

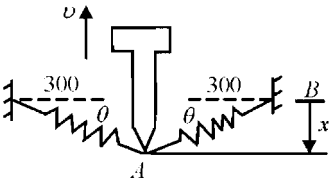
$$= 1050\left[(\sqrt{x^2 + 0.3^2} - 0.3)^2 - (0.375 - 0.300)^2\right]$$

$$= 1050\left[x^2 - 0.6\sqrt{x^2 + 0.3^2} + 0.174375\right] \qquad (\overline{AB} = 375\,\text{mm} \Leftarrow x = 225\,\text{mm} \text{ وقتی})$$

$$\Delta V_g = 5(9.81)(0.225 - x) = 49.05(0.225 - x)$$

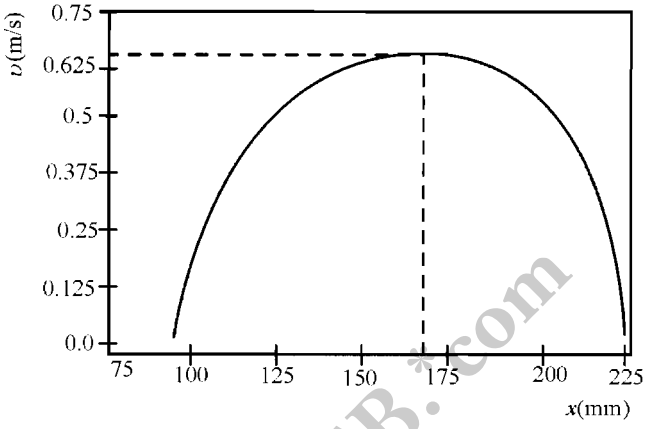
$$\Rightarrow\; 2.5\,v^2 + 1050\,x^2 - 630\sqrt{x^2 + 0.09} + 183.09375 + 11.03625 - 49.05\,x = 0$$

$$\Rightarrow\; v^2 = 252\sqrt{x^2 + 0.09} - 420\,x^2 + 19.62\,x - 77.652\,(\text{m/s})^2$$



حالا نمودار سرعت برحسب x را رسم می‌کنیم (به پیوستگی توجه کنید)

$v_{max} = 0.665\ \text{m/s} \quad @ \quad x = 173.6\ \text{mm}$

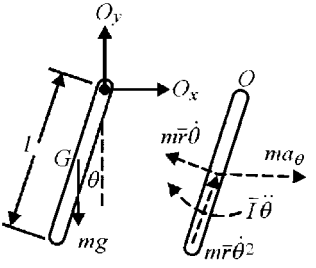


* ۶-۲۳۱. $k\theta - \frac{5}{2} m g l \sin\theta - \frac{5}{2} m a l \cos\theta = 0$: طبق جواب مسئله ۶-۱۶

حل عددی : $\theta = 12.17°$ ⇒ $75\theta - 7.36\sin\theta - 14.72\cos\theta = 0$: جای‌گذاری اعداد

* ۶-۲۳۲. $\Sigma M_O = \bar{I}\alpha + m\bar{a}d$:

$$\curvearrowleft_+ \; -mg\frac{l}{2}\sin\theta = \frac{1}{12}ml^2\ddot{\theta} + m\frac{l}{2}\ddot{\theta}\left(\frac{l}{2}\right) - ma_O\frac{l}{2}\cos\theta$$



$$\ddot{\theta} = \frac{3}{l}\left(\frac{a_O}{2}\cos\theta - \frac{g}{2}\sin\theta\right)$$

$$\int_0^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = \int_0^{\theta} \ddot{\theta}\,d\theta$$

$$\frac{\dot{\theta}^2}{2} = \frac{3}{l}\int_0^{\theta}\left(\frac{a_O}{2}\cos\theta - \frac{g}{2}\sin\theta\right)d\theta$$

$$\dot{\theta}^2 = \frac{b}{l}\left(\frac{a_O}{2}\sin\theta - \frac{g}{2}[1-\cos\theta]\right) = 1.5\left(\sin\theta - \frac{9.81}{2}[1-\cos\theta]\right)$$

$$\dot{\theta} = 0 \;\Rightarrow\; R = \sin\theta - \frac{9.81}{2}(1-\cos\theta) = 0$$

حل عددی : $\theta_{max} = 23.0°$

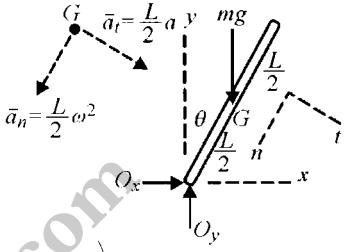
$\dot{\theta}$ حداکثر است وقتی $\ddot{\theta} = 0$ بنابراین

$$\frac{a_O}{2}\cos\theta - \frac{g}{2}\sin\theta = 0 \Rightarrow \theta = \tan^{-1}\frac{a_O}{g} = 11.52°$$

$$(\dot{\theta}^2)_{max} = 0.1513 \Rightarrow \dot{\theta}_{max} = 0.389\,\text{rad/s}$$

۲۳۳-۶ *

$\Sigma M_O = I_O\alpha$:



$$mg\frac{L}{2}\sin\theta = \frac{1}{3}mL^2\alpha \Rightarrow \alpha = \frac{3}{2}\frac{g}{L}\sin\theta$$

$$\omega\,d\omega = \alpha\,d\theta$$

$$\int_0^\omega \omega\,d\omega = \frac{3}{2}\frac{g}{L}\int_0^\theta \sin\theta\,d\theta \Rightarrow \omega^2 = \frac{3g}{L}(1 - \cos\theta)$$

$$\bar{a}_x = \bar{a}_t\cos\theta - \bar{a}_n\sin\theta = \frac{L}{2}\left(\frac{3g}{2L}\sin\theta\right)\cos\theta - \frac{L}{2}\left(\frac{3g}{L}[1 - \cos\theta]\right)\sin\theta$$

$$= \frac{3g}{4}\sin\theta(3\cos\theta - 2)$$

$$\bar{a}_y = -\bar{a}_t\sin\theta - \bar{a}_n\cos\theta = -\frac{L}{2}\left(\frac{3g}{2L}\sin\theta\right)\sin\theta - \frac{L}{2}\left(\frac{3g}{L}[1 - \cos\theta]\right)\cos\theta$$

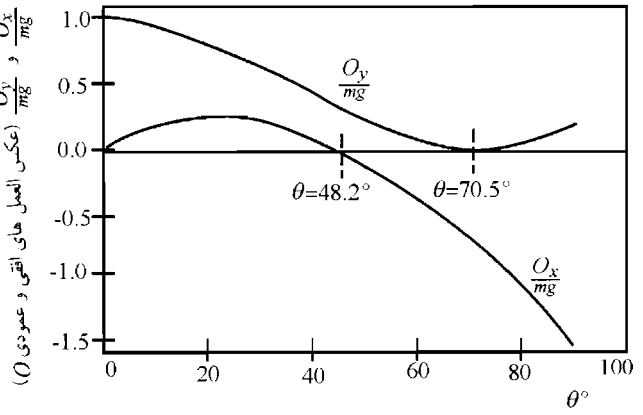
$$= \frac{-3g}{4}(1 + 2\cos\theta - 3\cos^2\theta)$$

$$= \frac{3g}{4}(3\cos\theta + 1)(\cos\theta - 1)$$

$$\Sigma F_x = m\bar{a}_x : \quad O_x = \frac{3mg}{4}\sin\theta(3\cos\theta - 2)$$

$$\Sigma F_y = m\bar{a}_y : \quad O_y - mg = \frac{3mg}{4}(3\cos\theta + 1)(\cos\theta - 1)$$

$$O_y = \frac{mg}{4}(1 - 6\cos\theta + 9\cos^2\theta) = \frac{mg}{4}(3\cos\theta - 1)^2$$

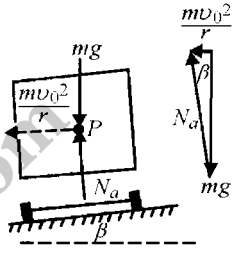


افزایش O_y از $\theta = 70.5°$ تا $\theta = 90°$ بیانگر این واقعیت است که در $\overline{a}_y = \overline{a}_n \cos\theta + \overline{a}\sin\theta$ – عبارت $\overline{a}_n \cos\theta$ سریعتر از افزایش $\overline{a}_t \sin\theta$ کم می‌شود که نشان دهندهٔ اثر ضرایب $\cos\theta$ و $\sin\theta$ برای زوایای θ نزدیک $90°$ می‌باشد.

* **۶-۲۳۴.** (مسافر به صورت ذرهٔ P در شکل‌ها نشان داده شده است)

$$F = 0.3\,m\frac{v^2}{r}$$

(الف) $$\tan\beta = \frac{\frac{m v_0^{\,2}}{r}}{mg} = \frac{v_0^{\,2}}{gr} = \frac{\left(\frac{160}{3.6}\right)^2}{9.81(1900)} \Rightarrow \beta = 6.05°$$



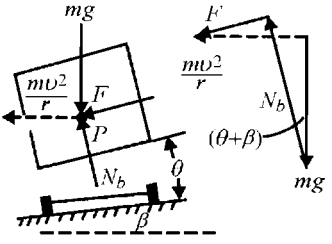
(ب) طبق دیاگرام نیروها :

$$m\,g\sin(\theta+\beta) + \frac{0.3\,m\,v^2}{r} = \frac{m\,v^2}{r}\cos(\theta+\beta)$$

$$9.81\sin(\theta+\beta) + \frac{\left(\frac{260}{3.6}\right)^2}{1900}(0.3 - \cos(\theta+\beta)) = 0$$

$$9.81\sin(\theta+\beta) + 2.75[0.3 - \cos(\theta+\beta)] = 0$$

حل عددی $\Rightarrow \theta = 4.95°$



* **۶-۲۳۵.**

$$2\beta + \left(\frac{\pi}{2} - \theta\right) = \pi \Rightarrow \beta = \frac{\pi}{4} + \frac{\theta}{2}$$

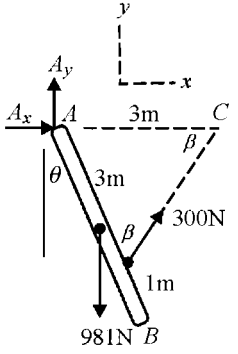
$$\sin\beta = \sin\frac{\pi}{4}\cos\frac{\theta}{2} + \cos\frac{\pi}{4}\sin\frac{\theta}{2}$$

$$= \frac{1}{\sqrt{2}}\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)$$

$$I_A = \frac{1}{3}m\,l^2 = \frac{1}{3}(100)(4)^2 = 533\ \text{kg.m}^2$$

$$\curvearrowright + \Sigma M_A = I_A\alpha : 300(3\sin\beta) - 9.81(2\sin\theta) = 533\,\alpha$$

$$\frac{900}{\sqrt{2}}\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right) - 1962\sin\theta = 533\,\alpha \quad (1)$$



$$\int_0^{\omega}\omega\,d\omega = \int_0^{\theta}\alpha\,d\theta \;:\; \omega^2 = \frac{2}{533}\int_0^{\theta}\left[\frac{900}{\sqrt{2}}\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right) - 1962\sin\theta\right]d\theta$$

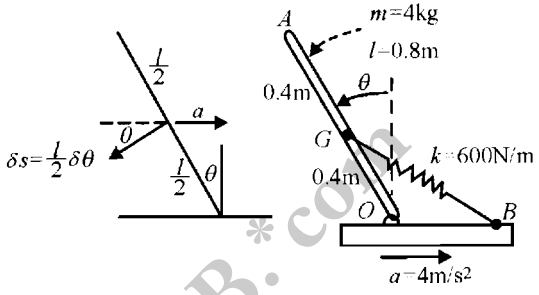
$$\omega^2 = \frac{2}{533}\left[\frac{1800}{\sqrt{2}}\left(1 - \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right) - 1962(1 - \cos\theta)\right] \quad (2)$$

(الف) ω_{max} برای : $\alpha = 0$ ((1) در رابطه) $\Rightarrow \theta = 22.4°$

$\Rightarrow$ (2) از رابطه $\omega_{max} = 0.680\,\text{rad/s}$

(ب) $\omega = 0$ ((2) در رابطه) $\Rightarrow \theta_{max} = 45.9°$

۲۳۶-۶ *



$G\hat{O}B = \theta + \frac{\pi}{2} = \frac{2\theta + \pi}{2}$

یک جابجایی مجازی $\delta\theta$ از حالت تعادل در نظر بگیرید. نیروها در O و B عمل نمی‌کنند.

$$\delta U' = 0 = \delta T + \delta V_g + \delta V_e$$

$$\delta T = \delta\left(\frac{1}{2} m \bar{v}^2\right) = m\bar{v}\delta\bar{v} = m\bar{a}.\delta s$$

$$= ma\left(-\frac{l}{2}\delta\theta\right)\cos\theta = -ma\frac{l}{2}\cos\theta\,\delta\theta = -4(4)(0.4)\cos\theta\,\delta\theta = -6.4\cos\theta\,\delta\theta$$

$$\delta V_g = \delta(mgh) = \delta\left(mg\frac{l}{2}\cos\theta\right) = -mg\frac{l}{2}\sin\theta\,\delta\theta =$$

$$= -4(9.81)(0.4)\sin\theta\,\delta\theta = -15.70\sin\theta\,\delta\theta$$

$\overline{GB} = 2\left(\frac{l}{2}\right)\sin\frac{2\theta+\pi}{4}$ $\Rightarrow$ کشیدگی فنر $x = l\sin\frac{2\theta+\pi}{4} - l\frac{\sqrt{2}}{2}$

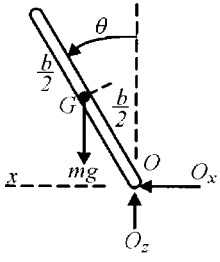
$$\delta V_e = \delta\left(\frac{1}{2}kx^2\right) = kx\delta x = kl\left(\sin\frac{2\theta+\pi}{4} - \frac{\sqrt{2}}{2}\right).\frac{1}{2}l\cos\frac{2\theta+\pi}{4}\delta\theta$$

$$= \frac{kl^2}{2}\left(\sin\frac{2\theta+\pi}{4} - \frac{\sqrt{2}}{2}\right)\cos\frac{2\theta+\pi}{4}\delta\theta = 192\left(\sin\frac{2\theta+\pi}{4} - \frac{\sqrt{2}}{2}\right)\cos\frac{2\theta+\pi}{4}\delta\theta$$

$$\Rightarrow 0 = -6.4\cos\theta\,\delta\theta - 15.70\sin\theta\,\delta\theta + 192\left(\sin\frac{2\theta+\pi}{4} - \frac{\sqrt{2}}{2}\right)\cos\frac{2\theta+\pi}{4}\delta\theta$$

حل عددی : $\theta = 15.62°$ ($\theta = 46.4°$ یک موقعیت ناپایدار است)

۶-۲۳۷ *



$$\Sigma \curvearrowleft_+ M_O = I_O \ddot{\theta} \ :$$

$$mg\frac{b}{2}\sin\theta = \frac{1}{3}m b^2 \ddot{\theta} \Rightarrow \ddot{\theta} = \frac{3}{2}\frac{g}{b}\sin\theta$$

$$\dot{\theta}\,d\dot{\theta} = \ddot{\theta}\,d\theta \ : \ \int_{\theta_0}^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = \frac{3g}{2b}\int_{\theta_0}^{\theta} \sin\theta\,d\theta$$

$$\frac{\dot{\theta}^2}{2} - \frac{\dot{\theta}_0^{\,2}}{2} = \frac{3g}{2b}(\cos\theta_0 - \cos\theta) \Rightarrow \frac{d\theta}{dt} = \left[\dot{\theta}_0^{\,2} + \frac{3g}{b}(\cos\theta_0 - \cos\theta)\right]^{\frac{1}{2}}$$

$$\int_0^t dt = \int_{\theta_0}^{\dot{\theta}} \frac{d\theta}{\left[\dot{\theta}_0^{\,2} + \frac{3g}{b}(\cos\theta_0 - \cos\theta)\right]^{\frac{1}{2}}}$$

$\theta_0 = 10°\ (= 0.1745\,\text{rad})$, $b = 18\,\text{m}$, $g = 9.81\,\text{m/s}^2$, $\dot{\theta}_0 = \frac{(v_A)_0}{b} = \frac{1.35}{18} = 0.075\,\text{rad/s}$

حل عددی : $t = 2.83\,\text{s}$

از $\theta_0 = 10°$ تا $\theta = 90°$: $\Delta T + \Delta V_g = 0$

$$\Delta T = \frac{1}{2}I_O\left[\frac{v_A}{b}\right]^2 - \frac{1}{2}I_O\left[\frac{(v_A)_0}{b}\right]^2 = \frac{1}{6}m\,[v_A^{\,2} - (v_A)_0^{\,2}]$$

$$\Delta V_g = -mgh = -mg\frac{b}{2}\cos 10°$$

$$\Rightarrow \frac{1}{6}m\,[v_A^{\,2} - 1.35^2] - m(9.81)\frac{18}{2}\cos 10° = 0 \Rightarrow v_A = 22.9\,\text{m/s}$$

پیوست ب

ممان اینرسی جرمی

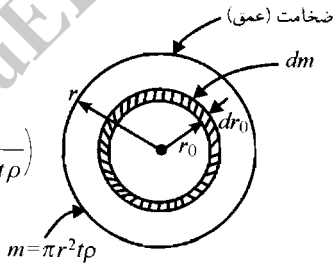
ب-۱.

$$dm = \rho\, dv = \rho\, 2\pi\, r_0\, dr_0\, t$$

$$dI = dm\, r_0^2 = 2\pi\rho\, t\, r_0^3\, dr_0$$

$$I = \int dI = \int_0^r 2\pi\rho\, t\, r_0^3\, dr_0^3 = 2\pi\rho\, t\, \frac{r^4}{4}\left(\frac{m}{\pi r^2 t\rho}\right)$$

$$= \frac{1}{2} m r^2$$



ب-۲.

$$I_{xx} = 2\, m\, L^2 \quad , \quad I_{zz} = 2\, m\, L^2$$

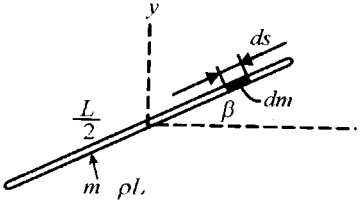
$$I_{yy} = 2\, m\, (L^2 + L^2) = 4\, m\, L^2$$

ب-۳.

$$ds = \frac{dx}{\cos\beta} \quad , \quad dm = \rho\, ds = \frac{\rho}{\cos\beta}\, dx$$

$$I_{yy} = \int x^2\, dm = \int_{-\frac{1}{2}\cos\beta}^{\frac{1}{2}\cos\beta} x^2\, \frac{\rho}{\cos\beta}\, dx$$

$$= \frac{\rho}{\cos\beta}\, \frac{x^3}{3}\, \Big|_{-\frac{L}{2}\cos\beta}^{\frac{L}{2}\cos\beta} = \frac{1}{12}\rho L^3 \cos^2\beta \left(\frac{m}{\rho L}\right) = \frac{1}{12} m L^2 \cos^2\beta$$



$$I_{xx} = \int y^2\,dm = \int_{-\frac{L}{2}\sin\beta}^{\frac{L}{2}\sin\beta} y^2\,\frac{\rho}{\sin\beta}\,dy = \frac{1}{12}\rho L^3\sin^2\beta\left(\frac{m}{\rho L}\right) = \frac{1}{12}m L^2\sin^2\beta$$

برای اجسام تخت $I_{zz} = I_{xx} + I_{yy} = \frac{1}{12}m L^2$

ب-۴.

$$\text{درصد خطا}\quad e = \frac{\frac{1}{12}m l^2 - \left(\frac{1}{12}m l^2 + \frac{1}{4}m r^2\right)}{\frac{1}{12}m l^2 + \frac{1}{4}m r^2}(100) = \frac{-100}{1 + \frac{\frac{1}{3}}{\left(\frac{r}{l}\right)^2}}$$

مقادیر عددی :

$\frac{r}{l}$	e
0.01	−0.030%
0.1	−2.91%
0.5	−42.9%

ب-۵.

$$\text{خطا} = \frac{\left(\frac{1}{2}m r^2 + m d^2\right) - m d^2}{\frac{1}{2}m r^2 + m d^2} = \frac{1}{1 + 2\left(\frac{d}{r}\right)^2}$$

(الف) $d = 10r$: $e = \frac{100}{1 + 200} = 0.498\%$

(ب) $d = 2r$: $e = \frac{100}{1 + 8} = 11.11\%$

ب-۶.

جرم کرهٔ کامل $= 2m \Rightarrow I_{zz} = \frac{2}{5}(2m)r^2$

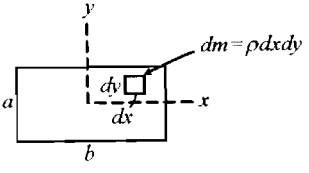
برای نیمکره $I_{zz} = I_{xx} = \frac{2}{5}m r^2$

ب-۷.

$$m = \rho A = \rho a b \qquad \left(\rho = \frac{\text{جرم}}{\text{سطح}}\right)$$

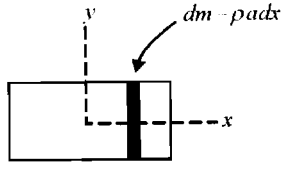
(الف) $I_{xx} = \int(y^2 + z^2)dm = \int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} y^2\rho\,dy\,dx$

$$= \rho\left(\frac{1}{12}a^3\right)\int_{-\frac{b}{2}}^{\frac{b}{2}} dx = \frac{1}{12}\rho a^3 b\left(\frac{m}{\rho a b}\right) = \frac{1}{12}m a^2$$



(ب) $dI_{xx} = \frac{1}{12}dm\,(a^2) = \frac{1}{12}\rho a^3\,dx$

$$I_{xx} = \int dI_{xx} = \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{1}{12}\rho a^3\,dx = \frac{1}{12}\rho a^3 b\left(\frac{m}{\rho a b}\right) = \frac{1}{12}m a^2$$



ملاحظه می‌کنید که روش (الف) در میانهٔ روش (ب) آغاز می‌شود.

با بررسی و مقایسه : $I_{yy} = \frac{1}{12} m b^2$

$I_{zz} = I_{xx} + I_{yy} = \frac{1}{2} m (a^2 + b^2)$

ب–۸.

طبق جدول د–۳ : $I_{xx} = \rho t \left(\frac{b h^3}{12}\right) = \frac{1}{6} m h^2$

$\Rightarrow I_{yy} = \frac{1}{6} m \left(b \frac{\sqrt{3}}{2}\right)^2 = \frac{1}{8} m b^2$

$I_{y_0 y_0} = I_{yy} - m\left(\frac{h}{3}\right)^2 = \frac{1}{8} mb^2 - m \frac{b^2}{12} = \frac{1}{24} mb^2$

دوباره طبق مسئله ب–۶ : $I_{x_0 x_0} = 2\left(\frac{1}{6} \frac{m}{2} \left(\frac{b}{2}\right)^2\right) = \frac{1}{24} m b^2$

$I_{zz} = I_{x_0 x_0} + I_{y_0 y_0} = \frac{1}{12} m b^2$

ب–۹.

$I_{xx} = \frac{1}{2} m_2 r_2^2 - \frac{1}{2} m_1 r_1^2$

$= \frac{1}{2} (\rho \pi r_2^2 l) r_2^2 - \frac{1}{2} (\rho \pi r_1^2 l) r_1^2$

$= \frac{1}{2} \rho \pi l (r_2^4 - r_1^4) = \underbrace{\frac{1}{2} \rho \pi l (r_2^2 - r_1^2)}_{m} (r_2^2 + r_1^2)$

$= \frac{1}{2} m (r_2^2 + r_1^2)$

ب–۱۰.

$I = I_{2_0} - I_{1_0} = \frac{1}{2} m_2 r_2^2 - \left[\frac{1}{2} m_1 r_1^2 + m_1 d^2\right]$

$= \rho \pi t \left[\frac{r_2^4}{2} - \frac{r_1^4}{2} - r_1^2 d^2\right]$

$m = \rho \pi t [r_2^2 - r_1^2]$

$k_O^2 = \frac{I}{m} = \frac{1}{2} (r_2^2 + r_1^2) - \frac{r_1^2 d^2}{(r_2^2 - r_1^2)} = \frac{1}{2} (300^2 + 150^2) - \frac{(150)^2 (100)^2}{(300^2 - 150^2)} = 52917 \text{ mm}^2$

$\Rightarrow k_O = 230 \text{ mm}$

ب–۱۱.

$I_{y'y'} = \int x^2 dm = \int_0^L x^2 \rho A \, dx = \rho A \int_0^L x^2 dx = \rho A \frac{L^3}{3} = \frac{1}{3} m L^2$

$I_{yy} = \overline{I}_{yy} = I_{y'y'} - m \overline{x}^2 = \frac{1}{3} m L^2 - m \left(\frac{L}{2}\right)^2 = \frac{1}{12} m L^2$

ب-۱۲.

دیسک : $I_{zz} = \frac{1}{2} m r^2$ ، $I_{xx} = \frac{1}{4} m r^2$

هر دو میله : $I_{zz} = 0$ ، $I_{xx} = 2\left(\frac{1}{3}\frac{m}{2}L^2\right) = \frac{1}{3} m L^2$

$$I_{zz} = I_{xx} = \frac{1}{2} m r^2 + 0 = \frac{1}{4} m r^2 + \frac{1}{3} m L^2 \Rightarrow \frac{3}{4} r^2 = L^2 \Rightarrow L = \frac{r\sqrt{3}}{2}$$

ب-۱۳.

جرم قاب $= \rho\left(\pi \frac{L}{2}\right)$

جرم دسته $= \rho L$

$$I_{yy} = \frac{1}{12}\rho L . L^2 + \rho L \left(\frac{3L}{8}\right)^2 + \frac{1}{2}\left(\rho \pi \frac{L}{2}\right)\left(\frac{L}{4}\right)^2 + \rho \pi \frac{L}{2}\left(\frac{9}{8}L\right)^2$$
$$= \left[\frac{43}{192} + \frac{83}{128}\pi\right]\rho L^3$$

ب-۱۴.

طوقه خارجی : $I = \frac{1}{2} m_2 r_2^2 - \frac{1}{2} m_1 r_1^2 = \frac{1}{2}\rho \pi r_2^4 b - \frac{1}{2}\rho \pi r_1^4 b$

$$= \frac{1}{2}\rho \pi (r_2^4 - r_1^4) b$$
$$= \frac{1}{2}(7830)\,\pi\,(\overline{0.2}^4 - \overline{0.15}^4)(0.075) = 1.009\,\text{kg.m}^2$$

توپی $I = \frac{1}{2}\rho \pi (r_2^4 - r_1^4) b$

$$= \frac{1}{2}(7830)\,\pi\,(\overline{0.05}^4 - \overline{0.025}^4)(0.12) = 0.00865\,\text{kg.m}^2$$

پره‌ها $I = 8\left[\frac{m l^2}{12} + m d^2\right] = 8m\left[\frac{l^2}{12} + d^2\right]$

$$= 8(7830)(0.1)(200 \times 10^{-6})\left[\frac{\overline{0.1}^2}{12} + \overline{0.1}^2\right] = 0.01357\,\text{kg.m}^2$$

کل $I = 1.009 + 0.00865 + 0.01357 = 1.031\,\text{kg.m}^2$

$$n = \frac{1.009}{1.031}(100) = 97.8\%$$

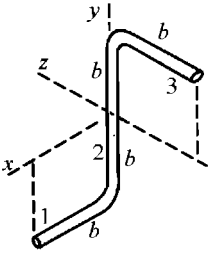
ب-۱۵.

قسمت ۱ : $m_1 = \frac{m}{4}$ ، $I_{xx} = \frac{m}{4} b^2$

$$I_{yy} = \frac{1}{3}\frac{m}{4} b^2 = \frac{1}{12} m b^2$$
$$I_{zz} = \frac{1}{12}\frac{m}{4} b^2 + \frac{m}{4}\left[\left(\frac{b}{2}\right)^2 + b^2\right] = \frac{1}{3} m b^2$$

قسمت 2 : $m_2 = \frac{1}{2} m$ ، $I_{xx} = \frac{1}{12}\frac{m}{2}(2b)^2 = \frac{1}{6} m b^2$

$$I_{yy} = 0 \quad , \quad I_{zz} = \frac{1}{6} m b^2$$



قسمت 3 : $m_3 = \frac{m}{4}$, $I_{xx} = \frac{1}{12}\frac{m}{4}b^2 + \frac{m}{4}\left[\left(\frac{b}{2}\right)^2 + b^2\right] = \frac{1}{3}mb^2$

$$I_{yy} = \frac{1}{3}\frac{m}{4}b^2 = \frac{1}{12}mb^2 \quad , \quad I_{zz} = \frac{m}{4}b^2$$

کل : $I_{xx} = mb^2\left(\frac{1}{4} + \frac{1}{6} + \frac{1}{3}\right) = \frac{3}{4}mb^2$

$$I_{yy} = mb^2\left(\frac{1}{12} + 0 + \frac{1}{12}\right) = \frac{1}{6}mb^2$$

$$I_{zz} = mb^2\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{4}\right) = \frac{3}{4}mb^2$$

ب-۱۶.

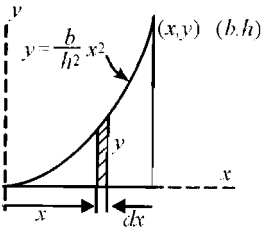
$$I_O = \frac{1}{3}ml^2 + 7mx^2 = m\left(7x^2 + \frac{1}{3}l^2\right)$$

برای $x = \frac{3}{4}l$: $I_O = m\left(7\left(\frac{3}{4}l\right)^2 + \frac{1}{3}l^2\right) = \frac{205}{48}ml^2$

برای $x = l$: $I_O = m\left(7l^2 + \frac{1}{3}l^2\right) = \frac{32}{3}ml^2$

$$R = \frac{205/48}{22/3} = 0.582$$

ب-۱۷. ضخامت ورق $= t$



$$dm = \rho\, dV = \rho\, y\, dx\, t = \rho\, t\, \frac{h}{b^2}x^2\, dx$$

$$dI_{xx} = \frac{1}{3}dm\, y^2 = \frac{1}{3}\rho\, t\, \frac{h^3}{b^6}x^6\, dx$$

$$dI_{yy} = x^2\, dm = \rho\, t\, \frac{h}{b^2}x^4\, dx$$

$$m = \int dm = \int_0^b \rho\, t\, \frac{h}{b^2}x^2\, dx = \rho\, t\, \frac{h}{b^2}\frac{b^3}{3} = \frac{1}{3}\rho\, t\, h\, b$$

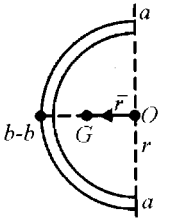
$$I_{xx} = \int dI_{xx} = \int_0^b \frac{1}{3}\rho\, t\, \frac{h^3}{b^6}x^6\, dx$$

$$= \frac{1}{3}\rho\, t\, \frac{h^3}{b^6}\frac{b^7}{7} = \frac{1}{21}\rho\, t\, h^3\, b\left(\frac{m}{\frac{1}{3}\rho\, t\, h\, b}\right) = \frac{1}{7}mh^2$$

$$I_{yy} = \int dI_{yy} = \int_0^b \rho\, t\, \frac{h}{b^2}x^4\, dx = \rho t\, \frac{h}{b^2}\frac{b^5}{5} = \frac{1}{5}\rho\, t\, h\, b^3\left(\frac{m}{\frac{1}{3}\rho\, t\, h\, b}\right) = \frac{3}{5}mb^2$$

$$I_{zz} = I_{xx} + I_{yy} = m\left(\frac{3b^2}{5} + \frac{h^2}{7}\right)$$

ب–۱۸.



برای حلقه کامل به جرم $2m$: $I_{()} = (2m)r^2$ ، $I_{aa} = \frac{1}{2}(2m)r^2$

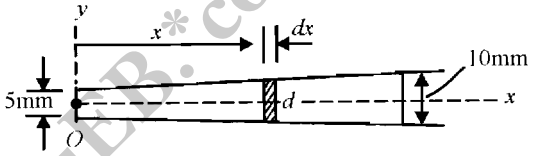
برای نیم حلقه : $I_{aa} = \frac{1}{2}mr^2$

$$I_{bb} = \bar{I} + (r-\bar{r})^2 m = I_{()} - m\bar{r}^2 + (r-\bar{r})^2 m$$

$$= I_{()} + m(r^2 - 2r\bar{r}) = mr^2 + m(r^2 - 2r\bar{r}) = 2mr^2\left(1 - \frac{\bar{r}}{r}\right)$$

$$\Rightarrow I_{bb} = 2mr^2\left(1 - \frac{2}{\pi}\right)$$ $\left(\text{که در آن } \bar{r} = \frac{2r}{\pi}\right)$

ب–۱۹.



قطر $d = 5 + kx$

$$10 = 5 + k(100) \Rightarrow k = 0.05$$

$$\Rightarrow d = 5 + 0.05x$$

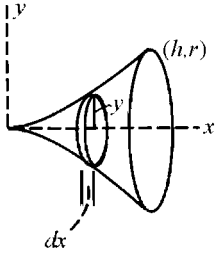
$$dm = \rho\, dv = \rho\frac{\pi d^2}{4}dx$$

$$dI_{()} = x^2\, dm = \frac{\rho\pi}{4}(5 + 0.05x)^2 x^2\, dx$$

$$I_{()} = \frac{\rho\pi}{4}\int_0^{100}(25x^2 + 0.5x^3 + 0.0025x^4)dx = \frac{\rho\pi}{4}(25.8(10^6))\ \text{kg.mm}^2$$

$$\rho = 7830(10^{-9})\ \text{kg/mm}^3 \Rightarrow I_{()} = 158.9\,\text{kg.mm}^2$$

ب–۲۰.



$y = kx^2$: $r = kh^2 \Rightarrow k = \frac{r}{h^2} \Rightarrow y = \frac{r}{h^2}x^2$

$$dm = \rho\, dv = \rho\pi y^2\, dx$$

$$dI_{xx} = \frac{1}{2}dm\, y^2 = \frac{1}{2}\rho\pi y^4\, dx = \frac{1}{2}\rho\pi\frac{r^4}{h^8}x^8\, dx$$

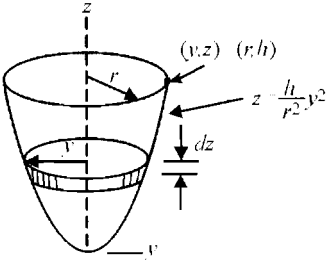
$$I_{xx} = \int dI_{xx} = \frac{1}{2}\rho\pi\frac{r^4}{h^8}\int_0^h x^8\, dx = \frac{1}{18}\rho\pi r^4 h$$

جرم $m = \rho v = \int_0^h \rho\pi y^2\, dx = \int_0^h \rho\pi\frac{r^2}{h^4}x^4\, dx = \frac{1}{5}\rho\pi r^2 h$

$$I_{xx} = \frac{1}{18}\rho\pi r^4 h\left(\frac{m}{\frac{1}{5}\rho\pi r^2 h}\right) = \frac{5}{18}mr^2$$

ب-۲۱.

$$z = ky^2 \;:\; h = kr^2 \Rightarrow k = \frac{h}{r^2} \Rightarrow z = \frac{h}{r^2}y^2$$



$$dm = \rho\pi y^2 dz$$

$$dI_{zz} = \frac{1}{2}\rho\pi y^4 dz$$

$$I_{zz} = \int dI = \int_0^h \frac{1}{2}\rho\pi \frac{r^2}{h} z^2 dz$$

$$I_{zz} = \frac{1}{2}\rho\pi \frac{r^4}{h^2}\frac{h^3}{3} = \frac{1}{6}\rho\pi r^4 h$$

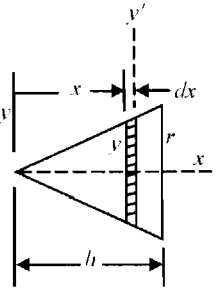
$$m = \rho V = \rho\int_0^h \pi y^2 dz = \rho\pi \int_0^h \frac{r^2}{h} z\, dz = \rho\pi \frac{r^2}{h}\frac{h^2}{2} = \frac{1}{2}\rho\pi r^2 h$$

$$\Rightarrow I_{zz} = \frac{1}{6}\rho\pi r^4 h \qquad \frac{m}{\frac{1}{2}\rho\pi r^2 h} = \frac{1}{3}mr^2 \Rightarrow k_z = \sqrt{\frac{I_{zz}}{m}} = \frac{r}{\sqrt{3}}$$

ب-۲۲.

$$y = \frac{r}{h}x : \quad dI_{xx} = \frac{1}{2}dm(y^2) = \frac{1}{2}(\pi y^3 \rho\, dx)y^2$$

$$= \frac{\pi}{2}\rho\frac{r^4}{h^4}x^4 dx$$



$$\Rightarrow I_{xx} = \frac{\pi}{2}\rho\frac{r^4}{h^4}\int_0^h x^4 dx = \frac{\pi}{10}\rho\frac{r^4}{h^4}h^5 = \frac{\pi}{10}\rho h r^4$$

از طرفی $m = \frac{1}{3}\rho\pi r^2 h \Rightarrow I_{xx} = \frac{3}{10}mr^2$

$$dI_{yy} = dI_{y'y'} + x^2 dm = \frac{1}{4}dm\, y^2 + x^2 dm = \left(\frac{y^2}{4} + x^2\right)dm$$

$$= \frac{1}{4}\frac{r^2}{h^2}+1\,x^2\rho\pi y^2 dx = \left(\frac{r^2}{4h^2}+1\right)\rho\pi\frac{r^2}{h^2}x^4 dx$$

$$\Rightarrow I_{yy} = \frac{\rho\pi r^2}{h^2}\left(\frac{r^2}{4h^2}+1\right)\int_0^h x^4 dx = \frac{\rho\pi r^2 h^3}{5}\left(\frac{r^2}{4h^2}+1\right) \Rightarrow I_{yy} = \frac{3}{5}m\left(\frac{r^2}{4}+h^2\right)$$

ب-۲۳. (جدول د-۴)

$$I_{aa} = \frac{3}{10}mr^2 \;,\; I_{cc} = \frac{3}{20}mr^2 + \frac{3}{5}mh^2$$

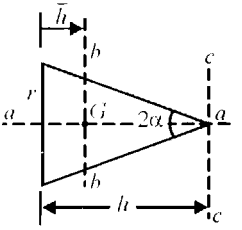
که در آن $\bar{h} = \frac{h}{4}$

$$I_{bb} = I_{cc} - m\bar{h}^2$$

$$= I_{cc} - m\frac{9}{16}h^2$$

$$= \frac{3}{20}mr^2 + \frac{3}{5}mh^2 - \frac{9}{16}mh^2$$

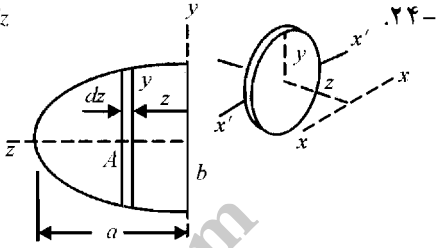
$$= \frac{3}{20}mr^2 + \frac{3}{80}mh^2$$



$$I_{aa} = I_{bb} \Rightarrow \frac{3}{10} m r^2 = \frac{3}{20} m r^2 + \frac{3}{80} m h^2 \Rightarrow \frac{3}{20} r^2 = \frac{3}{80} h^2$$

$$\Rightarrow \left(\frac{r}{h}\right)^2 = \frac{1}{4} \Rightarrow r = \frac{1}{2} h \quad , \quad \tan\alpha = \frac{r}{h} = \frac{1}{2} \Rightarrow \alpha = 26.6°$$

ب-۲۴.



$$dI_{x'x'} = \frac{1}{4} dm\, y^2 = \frac{1}{4} \rho \pi y^2 dz\, y^2 = \frac{1}{4} \rho \pi y^4 dz$$

$$\frac{y^2}{b^2} + \frac{z^2}{a^2} = 1 \Rightarrow y^2 = b^2\left(1 - \frac{z^2}{a^2}\right)$$

$$dI_{xx} = dI_{x'x'} + z^2 dm$$

$$= \frac{1}{4} \rho \pi y^4 dz + (\rho \pi y^2 dz)\, z^2 = \rho \pi \left(\frac{1}{4} b^4 \left[1 - \frac{z^2}{a^2}\right]^2 + b^2 z^2 \left[1 - \frac{z^2}{a^2}\right]\right) dz$$

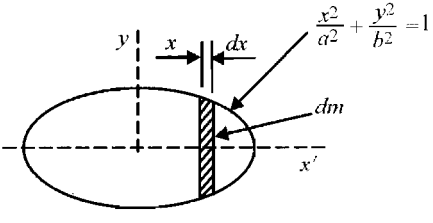
$$\Rightarrow I_{xx} = \int_0^a \rho \pi b^2 \left[\frac{b^2}{4} + \left(1 - \frac{b^2}{4a^2}\right) z^2 + \left(\frac{b^2}{4a^2} - 1\right) \frac{z^4}{a^2}\right] dz$$

$$= \rho \pi b^2 \left[\frac{ab^2}{4} + \frac{a^3}{3}\left(1 - \frac{b^3}{2a^2}\right) + \frac{a^3}{5}\left(\frac{b^2}{4a^2} - 1\right)\right] = \frac{2}{15} \rho \pi a b^2 (a^2 + b^2)$$

از طرفی $m = \int \rho\, dV = \rho \int \pi y^2 dz = \rho \pi \int_0^a b^2 \left(1 - \frac{z^2}{a^2}\right) dz = \frac{3}{4} \rho \pi b^2 a$

$$\Rightarrow I_{xx} = \frac{1}{5} m (a^2 + b^2)$$

ب-۲۵.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$$

$$dm = \rho\, dA = \rho\, 2y\, dx$$

$$dI_{xx} = \frac{1}{12} dm\, (2y)^2 = \frac{1}{12} (\rho\, 2y\, dx)\, (2y)^2$$

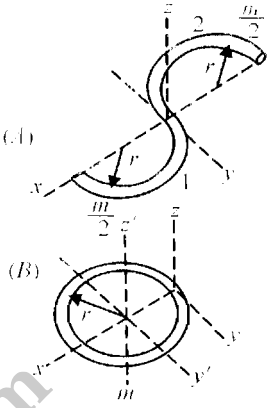
$$= \frac{2}{3} \rho y^3 dx = \frac{2}{3} \rho b^3 \left(1 - \frac{x^2}{a^2}\right)^{\frac{3}{2}} dx$$

$$I_{xx} = \int dI_{xx} = \frac{2}{3} \rho b^3 \int_{-a}^{a} \left(1 - \frac{x^2}{a^2}\right)^{\frac{3}{2}} dx = \frac{2}{3} \rho \frac{b^3}{a^3} \int_{-a}^{a} (a^2 - x^2)^{\frac{3}{2}} dx$$

$$= \frac{2}{3} \rho \frac{b^3}{a^3} \left[\frac{x}{8} (-2x^2 + 5a^2) \sqrt{a^2 - x^2} + \frac{3a^4}{8} \sin^{-1} x\sqrt{\frac{1}{a^2}}\right]_{-a}^{a}$$

$$= \frac{1}{4} \rho b^3 a \left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] \left[\frac{m}{\rho \pi a b}\right] = \frac{1}{4} m b^2$$

ب-۲۶.



$$I_{xx_A} = I_{xx_B}$$

$$I_{xx_B} + I_{x'x'_B} = I_{zz_B} = m r^2$$

$$\Rightarrow I_{xx} = \frac{1}{2} m r^2$$

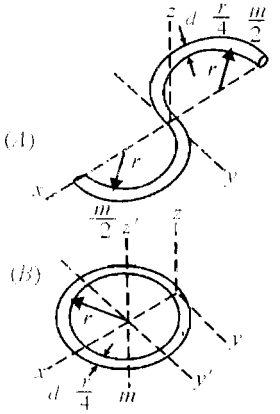
$$I_{yy_A} = I_{yy_B} = I_{y'y'} + m r^2$$

$$\Rightarrow I_{yy} = \frac{1}{2} m r^2 + m r^2 = \frac{3}{2} m r^2$$

$$I_{zz_A} = I_{zz_B} = I_{z'z'} + m r^2$$

$$= m r^2 + m r^2 = 2 m r^2$$

ب-۲۷.



از جدول د-۴ : $I_{xx_A} = I_{xx_B} = \frac{1}{2} m r^2 + \frac{5}{8} m \left(\frac{r}{8}\right)^2 = \frac{261}{512} m r^2$

$$\Rightarrow I_{xx} = 0.510\, m r^2$$

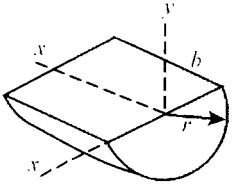
$$I_{yy_A} = I_{yy_B} = I_{y'y'} + m r^2 = \frac{261}{512} m r^2 + m r^2 = \frac{773}{512} m r^2$$

$$\Rightarrow I_{yy} = 1.510\, m r^2$$

$$I_{zz_A} = I_{zz_B} = m r^2 + \frac{3}{4} m \left(\frac{r}{8}\right)^2 + m r^2 = \frac{515}{256} m r^2$$

$$\Rightarrow I_{zz} = 2.01\, m r^2$$

ب-۲۸.



از جدول د-۴ : $I_{yy} = \frac{1}{4} m r^2 + \frac{1}{3} m b^2$

از جدول د-۱ برای فولاد : $\rho = 7830\ \text{kg/m}^3$

استوانه مثبت

$r = 80\ \text{mm}$, $b = 90\ \text{mm}$

$$m = \rho V = 7830 \times \frac{1}{2} \pi (0.080)^2 (0.090) = 7.08\ \text{kg}$$

$$I_{yy} = 7.08 \left(\frac{0.080^2}{4} + \frac{0.090^2}{3}\right) = 0.0305\ \text{kg.m}^2$$

استوانه منفی

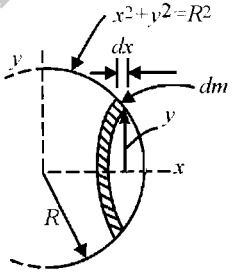
$r = 40\,\text{mm} \quad , \quad b = 90\,\text{mm}$

$$m = \rho V = 7830 \times \frac{1}{2}\pi (0.040)^2 (0.090) = 1.771\,\text{kg}$$

$$I_{yy} = 1.771\left(\frac{0.040^2}{4} + \frac{0.090^2}{3}\right) = 0.00549\,\text{kg.m}^2$$

خالص $I_{yy} = 0.0305 - 0.00549 = 0.0250\,\text{kg.m}^2$

ب-۲۹.



$$x^2 + y^2 = R^2 \quad \Rightarrow \quad y^2 = R^2 - x^2$$

$$dm = \rho\, dV = \rho \pi y^2\, dx$$

$$dI_{xx} = \frac{1}{2} dm\, y^2 = \frac{1}{2}\rho \pi y^4\, dx$$

$$\Rightarrow \quad I_{xx} = \int dI_{xx} = \int_{\frac{R}{2}}^{R} \frac{1}{2}\rho \pi (R^2 - x^2)^2\, dx = \frac{53\rho \pi R^5}{960}$$

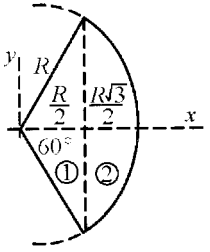
جرم جسم $m = \rho V = \rho \int dV = \rho \int \pi y^2\, dx = \rho \int_{\frac{R}{2}}^{R} \pi (R^2 - x^2)\, dx = \frac{5}{24}\rho \pi R^3$

$$\Rightarrow \quad I_{xx} = \frac{53\rho \pi R^5}{960}\left(\frac{m}{\frac{5}{24}\rho \pi R^3}\right) = \frac{53}{200} m R^2$$

ب-۳۰. طبق پاسخ مسئله ب-۲۹، جرم و ممان اینرسی قسمت ② برابر است با:

$$m_2 = \frac{5}{24}\rho \pi R^3$$

$$I_{xx_2} = \frac{53}{960}\rho \pi R^5$$



برای مخروط ① : $m_1 = \rho \frac{1}{3}\frac{R^2 3}{4}\frac{R}{2} = \frac{1}{8}\rho \pi R^3$

$$I_{xx_1} = \frac{3}{10}\frac{1}{8}\rho \pi R^3 \frac{R^2 3}{4} = \frac{9}{320}\rho \pi R^5$$

$$m = m_1 + m_2 = \rho \pi R^3\left(\frac{1}{8} + \frac{5}{24}\right) = \frac{1}{3}\rho \pi R^3$$

$$I_{xx} = I_{xx_1} + I_{xx_2} = \rho \pi R^5\left(\frac{9}{320} + \frac{53}{960}\right) = \frac{1}{12}\rho \pi R^5\left(\frac{m}{\frac{1}{3}\rho \pi R^3}\right) = \frac{1}{4} m R^2$$

ب-۳۱. توزیع جرم اساساً معادل یک پوسته استوانه‌ای است.

از جدول د-۴ : $I_{zz} = m r^2$

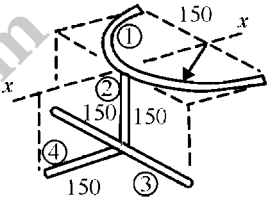
$$I_{xx} = I_{yy} = \frac{1}{2} m r^2 + \frac{1}{12} m L^2$$

$$\Rightarrow I_{zz} = 2\left(\frac{75}{2}\right)^2 = 2812.5\ \text{kg.mm}^2$$

$$I_{xx} = I_{yy} = \frac{1}{2}(2812.5) + \frac{1}{12}(2)(250)^2 = 11823\ \text{kg.mm}^2$$

ب-۳۲. جرم واحد طول $\rho = 0.993\ \text{kg/m}$ $\quad r = a = 150\ \text{mm}$

$$m_1 = \pi r \rho = \pi a \rho \quad , \quad m_2 = a\rho \ , \ m_3 = 2a\rho \quad , \quad m_4 = a\rho$$



① $I_{xx} = \frac{1}{2} m_1 r^2 = \frac{1}{2}\pi a \rho r^2 = \frac{1}{2}\pi\rho a^3$

② $I_{xx} = \frac{1}{3} m_2 a^2 = \frac{1}{3}\rho a^3$

③ $I_{xx} = \frac{1}{12} m_3 (2a)^2 + m_3 a^2 = \frac{4}{3}(2a\rho) a^2 = \frac{8}{3}\rho a^3$

④ $I_{xx} = m_4 a^2 = \rho a^3$

کل $I_{xx} = \rho a^3 \left(\frac{\pi}{2} + \frac{1}{3} + \frac{8}{3} + 1\right) = 5.57\rho a^3 = 5.57(0.993)(0.15)^3 = 0.01867\ \text{kg.m}^2$

ب-۳۳.

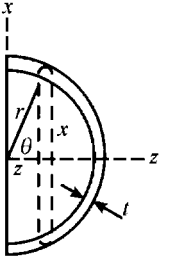
$$dm = \rho\, dV = \rho t\, 2\pi x r d\theta = 2\pi\rho t r^2 \sin\theta\, d\theta$$

$$m = 2\pi\rho t r^2 \int_0^{\frac{\pi}{2}} \sin\theta\, d\theta = 2\pi\rho t r^2$$

$$I_{zz} = \int x^2\, dm = 2\pi\rho t r^2 \int_0^{\frac{\pi}{2}} r^2 \sin^3\theta\, d\theta$$

$$= 2\pi\rho t r^4 \left[-\frac{\cos\theta}{3}(2 + \sin^2\theta)\right]_0^{\frac{\pi}{2}} = \frac{4}{3}\pi\rho t r^4 = \frac{2}{3} m r^2$$

$$I_{xx} = I_{zz} = \frac{2}{3} m r^2$$



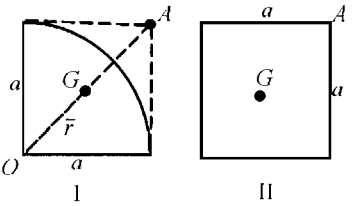
ب-۳۴. جرم واحد طول $= \rho$

$$I_A = I_{A_{II}} - I_{A_I}$$

برای II :

$$I_A = I_G + m\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{1}{6} m a^2 + \frac{1}{2} m a^2 = \frac{2}{3} m a^2$$

$$= \frac{2}{3}(\rho a^2) a^2 = 0.667\rho a^4$$



برای I :

$$I_O = \frac{1}{2} m a^2 \quad , \quad I_G = I_O - m \bar{r}^2 = \frac{1}{2} m a^2 - m\left(\frac{4a}{3\pi}\sqrt{2}\right)^2 = 0.1397\, m a^2$$

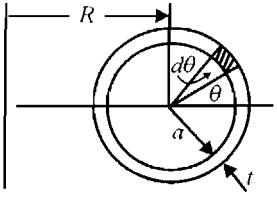
$$I_A = I_G + m\left(a\sqrt{2} - \frac{4a}{3\pi}\sqrt{2}\right)^2 = 0.802\, m a^2 = 0.802\left(\frac{1}{4}\pi a^2 \rho\right) a^2 = 0.630 \rho a^4$$

کل $I_A = 0.667 \rho a^4 - 0.630 \rho a^4 = 0.0365 \rho a^4$

از طرفی $m = \rho\left(a^2 - \frac{1}{4}\pi a^2\right) = 0.215 \rho a^2$

$$\Rightarrow I_A = 0.0365 \rho a^4 \left(\frac{m}{0.215 \rho a^2}\right) = 0.1701\, m a^2$$

ب-۳۵. فرض کنید ρ جرم واحد سطح باشد. برای المان حلقه به مساحت $a\, d\theta\, (t)$ و محیط $2\pi (R + a\cos\theta)$:



$$dm = \rho (a\, d\theta)\, 2\pi (R + a\cos\theta)$$

$$dI = (R + a\cos\theta)^2\, dm = 2\pi\rho (R + a\cos\theta)^3\, d\theta$$

$$\Rightarrow m = 2\pi\rho a \int_0^{2\pi} (R + a\cos\theta)\, d\theta = 4\pi^2 \rho a R$$

$$\Rightarrow I = 2\pi\rho a \int_0^{2\pi} (R^3 + 3R^2 a\cos\theta + 3R a^2 \cos^2\theta + a^3 \cos^3\theta)\, d\theta$$

$$= 2\pi\rho a [\, ① + ② + ③ + ④ \,]$$

$$① = \int_0^{2\pi} R^3\, d\theta = 2\pi R^3$$

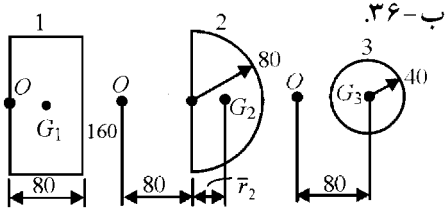
$$② = 3R^2 a \int_0^{2\pi} \cos\theta\, d\theta = 0$$

$$③ = 3R a^2 \int_0^{2\pi} \cos^2\theta\, d\theta = 3R a^2 \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{2\pi} = 3\pi R a^2$$

$$④ = a^3 \int_0^{2\pi} \cos^3\theta\, d\theta = \frac{a^3}{3}\left[\sin\theta (\cos^2\theta + 2)\right]_0^{2\pi} = 0$$

$$\Rightarrow I = 2\pi^2 \rho a R\, [2R^2 + 3a^2] \left(\frac{m}{4\pi^2 \rho a R}\right) = \frac{1}{2} m (2R^2 + 3a^2)$$

ب-۳۶. قطعه از ۳ بخش 1+ و 2+ و 3- تشکیل شده است. فرض کنید ρ = جرم واحد سطح وجه باشد.



① قسمت :

$$m_1 = 0.16\times 0.080\rho = 0.0128\rho \text{ kg}$$

$$I_{OO} = \frac{1}{12}m_1(0.160)^2 + \frac{1}{3}m_1(0.080)^2 = 54.6\times 10^{-6}\rho \text{ kg.m}^2 \quad \text{جدول د-۴ :}$$

② قسمت :

$$m_2 = \frac{1}{2}\pi(0.080)^2\rho = 0.01005\rho \text{ kg}$$

$$I_A = \frac{1}{2}m_2 r^2 = \frac{1}{2}0.01005\rho(0.080)^2 = 32.2\times 10^{-6}\rho \text{ kg.m}^2$$

$$\bar{r} = \frac{4r}{3\pi} = \frac{4(0.080)}{3\pi} = 0.0340 \text{ m}$$

$$I_{G_2} = I_A - m_2\bar{r}_2^{\,2} = 32.2\times 10^{-6}\rho - 0.01005\rho(0.0340)^2 = 20.6\times 10^{-6}\rho \text{ kg.m}^2$$

$$I_{OO} = I_{G_2} + m_2(0.080 + 0.0340)^2 = 20.6\times 10^{-6}\rho + 0.01005\rho(0.1140)^2$$

$$= 151.1\times 10^{-6}\rho \text{ kg.m}^2$$

③ قسمت :

$$m_3 = \pi(0.040)^2\rho = 0.00503\rho \text{ kg}$$

$$I_{G_3} = \frac{1}{2}m_3 r^2 = \frac{1}{2}\times 0.00503\rho(0.040)^2 = 4.02\times 10^{-6}\rho \text{ kg.m}^2$$

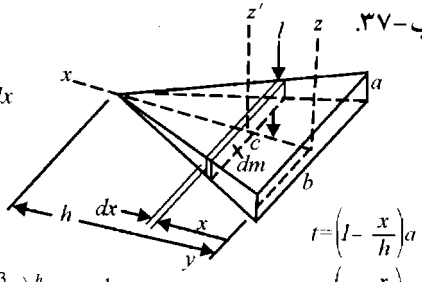
$$I_{OO} = I_{G_3} + m_3 d^2 = 4.02\times 10^{-6}\rho + 0.00503\rho(0.080)^2 = 36.2\times 10^{-6}\rho \text{ kg.m}^2$$

کل $m = m_1 + m_2 - m_3 = (0.0128 + 0.01005 - 0.00503)\rho = 0.01783\rho$ kg

کل $I_{OO} = (54.6 + 151.1 - 36.2)\times 10^{-6}\rho = 169.5\times 10^{-6}\rho$ kg.m^2

$$I_{OO} = m k_O^2 \quad \Rightarrow \quad k_O = \sqrt{\frac{169.5\times 10^{-6}\rho}{0.01783\rho}} = 0.0975 \text{ m} \quad \text{یا} \quad k_O = 97.5 \text{ mm}$$

ب-۳۷.



برای θ کوچک : $I_{yy} = \int x^2\,dm$

$$dm = \rho\,dV = \rho\,t\,c\,dx = \rho\left(1-\frac{x}{h}\right)a\left(1-\frac{x}{h}\right)b\,dx$$

$$= \rho\,a\,b\left(1-\frac{x}{h}\right)^2 dx$$

$$m = \rho\,a\,b\int_0^h\left(1-\frac{x}{h}\right)^2 dx = \rho\,a\,b\left(x - \frac{x^2}{h} + \frac{x^3}{3h^2}\right)_0^h = \frac{1}{3}\rho\,a\,b\,h$$

$$I_{yy} = \rho\,a\,b\int_0^h\left(1-\frac{x}{h}\right)^2 x^2\,dx = \rho\,a\,b\left(\frac{x^3}{3} - \frac{2x^4}{4h} + \frac{x^5}{5h^2}\right) = \frac{1}{30}\rho\,a\,b\,h^3 \quad \Rightarrow \quad I_{yy} = \frac{1}{10}m\,h^2$$

ب-۳۸. طبق شکل پاسخ مسئله قبل و عبارت نوشته شده برای dm یعنی $\rho a b \left(1 - \frac{x}{h}\right)^2 dx$، ممان اینرسی dm حول محور z برابر است با $dI_{zz} = dI_{z'z'} + x^2 dm$ (طبق تئوری انتقال محورها). همچنین از نتایج مسئله قبل یا از جدول د-۴ خواهیم داشت:

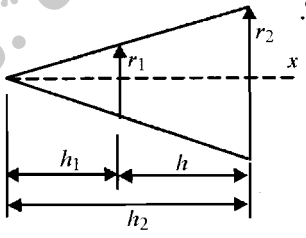
$$dI_{z'z'} = \frac{1}{12} dm\, c^2 = \frac{1}{12} dm \left(1 - \frac{x}{h}\right)^2 b^2$$

$$\Rightarrow dI_{zz} = \left[\frac{1}{12} b^2 \left(1 - \frac{x}{h}\right)^2 + x^2\right] dm = \rho a b \left[\frac{1}{12} b^2 \left(1 - \frac{x}{h}\right)^4 + x^2 \left(1 - \frac{x}{h}\right)^2\right] dx$$

$$\Rightarrow I_{zz} = \rho a b \int_0^h \left[\frac{1}{12} b^2 \left(1 - \frac{x}{h}\right)^4 + x^2 \left(1 - \frac{x}{h}\right)^2\right] dx = \rho a b \left[\frac{b^2 h}{60} + \frac{h^3}{30}\right] = \frac{\rho a b h}{30} \left(\frac{b^2}{2} + h^2\right)$$

طبق پاسخ مسئله قبل، $m = \frac{1}{3} \rho a b h$ بنابراین $I_{zz} = \frac{1}{10} m \left(\frac{b^2}{2} + h^2\right)$

ب-۳۹. قسمت حذف شدهٔ بالا : ① ، کل مخروط : ②



$$\frac{h_1}{r_1} = \frac{h_1 + h}{r_2} \Rightarrow h_1 = \frac{h r_1}{r_2 - r_1}$$

$$h_2 = h_1 + h = \frac{h r_1}{r_2 - r_1} + h = \frac{h r_2}{r_2 - r_1}$$

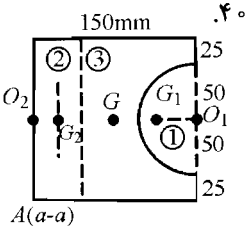
جدول د-۴ : $I_2 = \frac{3}{10} m_2 r_2^2 = \frac{3}{10} \left(\rho \frac{1}{3} \pi r_2^2 \frac{h r_2}{r_2 - r_1}\right) r_2^2 = \frac{1}{10} \rho \pi \frac{h r_2^5}{r_2 - r_1}$

$$I_1 = \frac{3}{10} m_1 r_1^2 = \frac{3}{10} \left(\rho \frac{1}{3} \pi r_1^2 \frac{h r_1}{r_2 - r_1}\right) r_1^2 = \frac{1}{10} \rho \pi \frac{h r_1^5}{r_2 - r_1}$$

جرم مخروط ناقص $m = \rho \frac{1}{3} \pi \left[r_2^2 \frac{h r_2}{r_2 - r_1} - r_1^2 \frac{h r_1}{r_2 - r_1}\right] = \frac{1}{3} \rho \pi h \frac{r_2^3 - r_1^3}{r_2 - r_1}$

$$\Rightarrow I = I_2 - I_1 = \frac{1}{10} \rho \pi h \frac{r_2^5 - r_1^5}{r_2 - r_1} \left(\frac{m}{\frac{1}{3} \rho \pi h \frac{r_2^3 - r_1^3}{r_2 - r_1}}\right) = \frac{3}{10} m \frac{r_2^5 - r_1^5}{r_2^3 - r_1^3}$$

◄ **ب-۴۰.**



$$\overline{G_1 O_1} = \overline{G_2 O_2} = \frac{4(0.05)}{3} = 0.0212\,\text{m}\,,$$

$$\overline{G_1 A}^2 = (0.15 - 0.0212)^2 + (0.075)^2 = 0.0222\,m^2$$

① شیار : $I_{aa} = \bar{I} + m(\overline{G_1 A}^2)$

$$= I_{O_1 O_1} - m(\overline{G_1 O_1}^2) + m(\overline{G_1 A}^2)$$

$$= m\left(\frac{1}{2}r^2 - \overline{G_1O_1}^2 + \overline{G_1A}^2\right)$$

$$= 11370\,\frac{\pi(0.05)^2(0.15)}{2}\left[\frac{0.05^2}{2} - 0.0212^2 + 0.0222\right] = 0.1541\ \text{kg.m}^2 \quad (\text{منفی})$$

② شیار : $I_{aa} = \frac{1}{2}$ (استوانه کامل) ← طبق تقارن

از جدول د-۴ : $I_{aa} = \frac{1}{2}\,\frac{2m}{12}\,(3r^2 + 4l^2) = \frac{11370}{2}\,\frac{\pi(0.05)^2(0.15)}{12}\,[3(0.05)^2 + 4(0.15)^2]$

$= 0.0544\ \text{kg.m}^2$ (منفی)

③ مکعب : $I_{aa} = \frac{1}{12}\,m(a^2 + a^2) + m\left(\frac{a^2}{4} + \frac{a^2}{4}\right) = \frac{2}{3}\,m\,a^2$

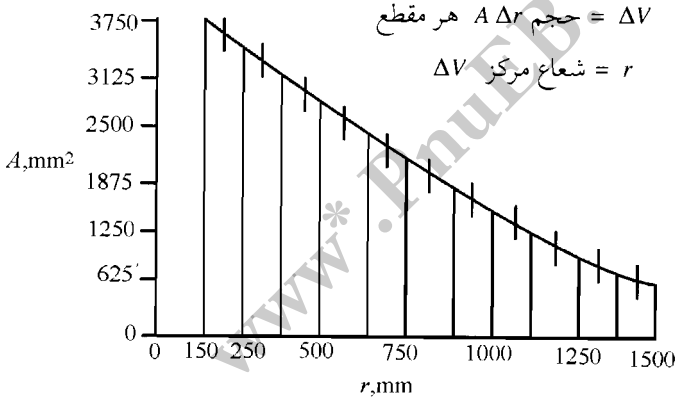
$= \frac{2}{3}(11370)(0.15)^3(0.15)^2 = 0.576\,\text{kg.m}^2$ (مثبت)

کل $I_{aa} = 0.576 - 0.0544 - 0.1541 = 0.367\,\text{kg.m}^2$

◀ ب-۴۱. که در آن : ρ = جرم واحد حجم $I = \Sigma m r^2 = \rho \Sigma \Delta V r^2$

هر مقطع ΔV = حجم $A\,\Delta r$

r = شعاع مرکز ΔV



بازه : ۶ قسمت	Δr	r_{Av}	A	$\Delta V = A\,\Delta r$	$\Delta V(r_{Av})^2$
150-250	100	200	3585	358500	$1.434(10^{10})$
250-500	250	375	3055	763750	$10.74(10^{10})$
500-750	250	625	2370	592500	$23.145(10^{10})$
750-1000	250	875	1772	443000	$33.917(10^{10})$
1000-1250	250	1125	1257	314250	$39.77(10^{10})$
1250-1500	250	1375	822	205500	$38.85(10^{10})$

$$\Sigma \Delta V (r_{Av})^2 = 147.856(10^{10})\ \text{mm}^5$$

$$I = \rho \Sigma \Delta V r^2 = 2850\,\frac{(147.856 \times 10^{10})}{10^{15}} = 4.22\,\text{kg.m}^2 \quad (n = 6)$$

بازه : ۱۱ قسمت	Δr	r_{Av}	A	$\Delta V = A\,\Delta r$	$\Delta V(r_{Av})^2$
150-250	100	200	3585	358500	$1.434(10^{10})$
250-375	125	312.5	3219	402375	$3.93(10^{10})$
375-500	125	437.5	2863	357875	$6.85(10^{10})$
500-625	125	562.5	2513	314125	$9.94(10^{10})$
625-750	125	687.5	2207	275875	$13.04(10^{10})$
750-875	125	812.5	1907	238375	$15.736(10^{10})$
875-1000	125	937.5	1632	204000	$17.93(10^{10})$
1000-1125	125	1062.5	1369	171125	$19.318(10^{10})$
1125-1250	125	1187.5	1125	140625	$19.83(10^{10})$
1250-1375	125	1312.5	900	112500	$19.38(10^{10})$
1375-1500	125	1437.5	700	87500	$18.08(10^{10})$

$$\Sigma\,\Delta V\,(r_{Av})^2 = 145.468(10^{10})\ \text{mm}^5$$

$$I = 2850\,\frac{(145.468\times 10^{10})}{10^{15}} = 4.15\,\text{kg.m}^2 \quad (n = 11)$$

تغییر قابل ملاحظه‌ای با تغییر گام در I موجود نخواهد آمد زیرا توزیع جرم نسبت به محور با تغییر گام تغییر قابل توجهی نخواهد کرد.

◄ ب-۴۲.

$dm = \rho\,(2\pi\,y\,a\,d\theta)$ ρ = واحد سطح

$dI_{zz} = y^2\,dm = 2\pi\rho\,a\,y^2\,d\theta$

از طرفی $y = r + a(1 - \cos\theta) = (r + a) - a\cos\theta$

$\Rightarrow y^3 = (r + a)^3 - 3a(r + a)^2\cos\theta + 3a^2(r + a)\cos^2\theta - a^3\cos^3\theta$

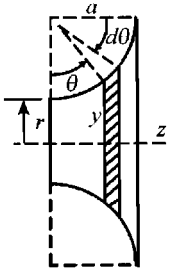
$I_{zz} = 2\pi\rho\,a\,(\,① - ② + ③ - ④\,)$

$① = (r + a)^3\int_0^{\frac{\pi}{2}} d\theta = (r + a)^3\,\frac{\pi}{2}$

$② = 3a(r + a)^2\int_0^{\frac{\pi}{2}} \cos\theta\,d\theta = 3a(r + a)^2$

$③ = 3a^2(r + a)\int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta = 3a^2(r + a)\,\frac{\pi}{4}$

$④ = a^3\int_0^{\frac{\pi}{2}} \cos^3\theta\,d\theta = a^3\,\frac{2}{3}$



$$\Rightarrow I_{zz} = 2\pi\rho a\left[(r+a)^3\frac{\pi}{2} - 3a(r+a)^2 + 3a^2(r+a)\frac{\pi}{4} - \frac{2}{3}a^3\right]$$

$$r = \frac{a}{3} \text{ برای } : I_{zz} = 2\pi\rho a^4\left[\frac{59}{27}\pi - 6\right] = 2\pi\rho a^4(0.865)$$

$$m = 2\pi\rho a\int_0^{\frac{\pi}{2}}[(r+a) - a\cos\theta]\,d\theta = 2\pi\rho a^2\left(\frac{2\pi}{3} - 1\right) = 2\pi\rho a^2(1.0944)$$

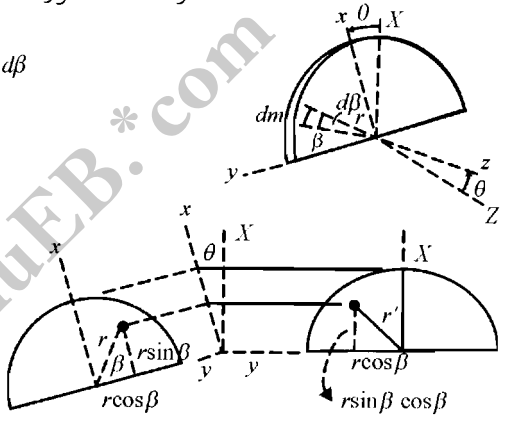
$$k_z^2 = \frac{I_{zz}}{m} = \frac{0.865}{1.0944}a^2 \Rightarrow k_z = 0.890\,a$$

◀ ب-۴۳. ρ = جرم واحد سطح

$dm = \rho\,dA = \rho\,r\,dr\,d\beta$

r' = فاصله از dm تا محور Z

$r'^2 = r^2(\cos^2\theta\sin^2\beta + \cos^2\beta)$



$dI_{zz} = r'^2\,dm = \rho\,r^3(\cos^2\theta\sin^2\theta + \cos^2\beta)\,dr\,d\beta$

$$I_{zz} = \rho\frac{R^4}{4}\int_0^{\pi}(\cos^2\theta\sin^2\beta + \cos^2\beta)\,d\beta$$

$$= \rho\frac{R^4}{4}\left[\frac{\pi}{2}(1+\cos^2\theta)\right].\frac{m}{\frac{\rho\pi R^2}{2}}$$

$$= \frac{1}{4}mR^2(1+\cos^2\theta)$$

* ب-۴۴. ρ = جرم واحد سطح

$m = \rho(2r)^2$ برای هر ورق

برای ورق‌ها : $I_{xx} = 2\left\{\frac{1}{12}m(2r)^2 + m(2r)^2\right\} = \frac{104}{3}\rho r^4$

$$I_{yy} = 2\left\{\frac{1}{12}m(2r)^2\right\} = \frac{2}{3}mr^2 = \frac{8}{3}\rho r^4$$

$$I_{zz} = 2\left\{\frac{1}{6}m(2r)^2\right\} + m(2r)^2 = \frac{28}{3}mr^2 = \frac{112}{3}\rho r^4$$

برای پوسته استوانه‌ای (طبق جدول د-۴) :

$$I_{xx} = I_{yy} = \frac{1}{2}mr^2 + \frac{1}{12}mL^2 = \frac{m}{2}\left(r^2 + \frac{L^2}{6}\right) = \pi rL\rho\left(r^2 + \frac{L^2}{6}\right)$$

$$I_{zz} = mr^2 = 2\pi r^3 L\rho$$

برای مدل کامل : $I_{xx} = \frac{104}{3}\rho r^4 + \pi rL\rho\left(r^2 + \frac{L^2}{6}\right)$

$$I_{yy} = \frac{8}{3}\rho r^4 + \pi r L \rho \left(r^2 + \frac{L^2}{6}\right)$$

$$I_{zz} = \frac{112}{3}\rho r^4 + 2\pi r^3 L \rho$$

از آنجا که $I_{xx} < I_{yy}$ ، I_{zz} باید کوچکتر از I_{yy} باشد بنابراین:

$$\left(\frac{112}{3}\rho r^4 + 2\pi r^3 L\rho\right) < \left(\frac{8}{3}\rho r^4 + \pi r L \rho \left[r^2 + \frac{L^2}{6}\right]\right) \Rightarrow \frac{\pi}{6}\left(\frac{L}{r}\right)^3 - \pi\frac{L}{r} - \frac{104}{3} > 0$$

حل معادلهٔ درجه سوم فوق نتیجه می‌دهد $\frac{L}{r} = 4.54$ در نتیجه نامساوی صادق است برای $L > 4.54\,r$

ب-۴۵.

$$I_{xy} = 0$$

$$I_{xz} = m(-l)(2l) = -2ml^2$$

$$I_{yz} = m(l)(-l) + m(-l)(l) = -2ml^2$$

ب-۴۶.

$$I_{xy} = m(l)(-l) + m(-l)(l) = -2ml^2$$

$$I_{xz} = m(2l)(-l) + m(-2l)(l) = -4ml^2$$

$$I_{yz} = 0$$

ب-۴۷.

$$I_{xy} = \bar{I}_{xy} + m\,d_x d_y = 0 + 0.6(0.045)(0.150)$$

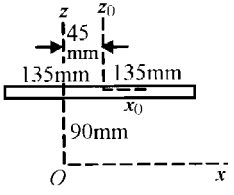
$$= 4.05(10^{-3})\ \text{kg.m}^2$$

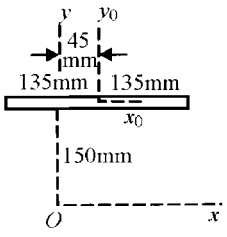
$$I_{yz} = m\,d_y d_z = 0.6(0.15)(0.09)$$

$$= 8.10(10^{-3})\ \text{kg.m}^2$$

$$I_{xz} = \bar{I}_{xz} + m\,d_x d_z = 0 + 0.6(0.045)(0.090)$$

$$= 2.43(10^{-3})\ \text{kg.m}^2$$



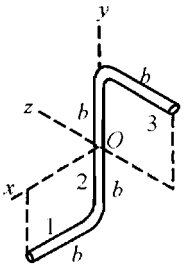


ب-۴۸.

$$I_{xy} = -\left(\frac{b}{4}\right)\left(\frac{b}{4}\right)\left(\rho\pi\left(\frac{b}{8}\right)^2\right) - \left(-\frac{b}{4}\right)\left(-\frac{b}{4}\right)\left(\rho\pi\left(\frac{b}{8}\right)^2\right) = -\frac{\rho\pi b^4}{512}$$

$$I_{xz} = I_{yz} = 0$$

ب-۴۹.



قسمت 1 : $m_1 = \frac{m}{4}$

$$I_{xy} = \frac{m}{4}\left(\frac{b}{2}\right)(-b) = -\frac{1}{8}mb^2$$

$$I_{yz} = 0 \quad , \quad I_{xz} = 0$$

قسمت 2 : $m_2 = \frac{m}{2}$

$$I_{xy} = 0 \quad , \quad I_{yz} = 0 \quad , \quad I_{xz} = 0$$

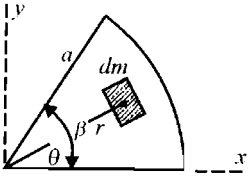
قسمت 3 : $m_3 = \frac{m}{4}$

$$I_{xy} = 0 \quad , \quad I_{xz} = 0$$

$$I_{yz} = \frac{m}{4}(b)\left(-\frac{b}{2}\right) = -\frac{1}{8}mb^2$$

ترکیب : $I_{xy} = -\frac{1}{8}mb^2 \quad , \quad I_{xz} = 0 \quad , \quad I_{yz} = -\frac{1}{8}mb^2$

ب-۵۰.



$dm = \rho t r dr d\theta \quad , \quad I_{xz} = I_{yz} = 0$

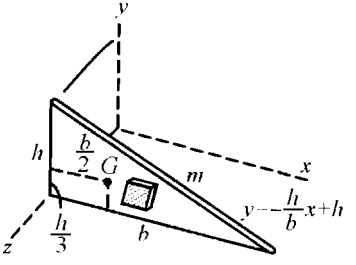
$$I_{xy} = \int xy\,dm = \int_0^{\beta}\int_0^{a}(r\cos\theta)(r\sin\theta)\rho t r dr d\theta$$

$$= \rho t\int_0^{\beta}\int_0^{a} r^3\cos\theta\sin\theta\,dr\,d\theta$$

$$= \rho t\int_0^{\beta}\frac{a^4}{4}\cos\theta\sin\theta\,d\theta$$

$$= \frac{\rho t a^4}{4}\frac{\sin^2\beta}{2} = \frac{\rho t a^4\sin^2\beta}{8}\left(\frac{m}{\frac{\rho t\beta a^2}{2}}\right) = \frac{m a^2\sin^2\beta}{4\beta}$$

ب-۵۱. مبدأ محورهای $x_0 - y_0 - z_0$ نقطهٔ G می‌باشد:



$$I_{x_0\tilde{z}_0} = 0 \quad , \quad I_{y_0\tilde{z}_0} = 0$$

$$I_{xz} = I_{x_0\tilde{z}_0} + m\,d_x d_z = 0 + m\left(\frac{b}{3}\right)(a) = \frac{1}{3}mab$$

$$I_{yz} = I_{y_0\tilde{z}_0} + m\,d_y d_z = 0 + m\left(\frac{b}{3}\right)(a) = \frac{1}{3}mah$$

انتگرال‌گیری مستقیم : $I_{xy} = \int xy\,dm = \int_0^{b}\int_0^{-\frac{b}{h}x+h} xy\rho t\,dy\,dx$

از طرفی $m = \frac{\rho t h b}{2} \Rightarrow I_{xy} = \frac{1}{12}mbh$

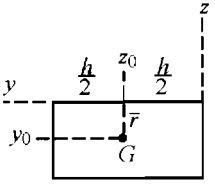
ب-۵۲.

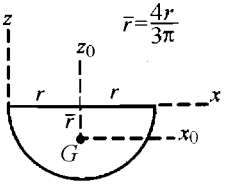
طبق تقارن : $\bar{I}_{xy} = \bar{I}_{yz} = \bar{I}_{zx} = 0$

$$\Rightarrow I_{xy} = 0 + \frac{mrh}{2} = \frac{mrh}{2}$$

$$I_{yz} = 0 + m\,\frac{h}{2}\left(-\frac{4r}{3\pi}\right) = -\frac{2}{3\pi}\,m\,r\,h$$

$$I_{xz} = 0 + mr\left(-\frac{4r}{3\pi}\right) = -\frac{4}{3\pi}\,m\,r^2$$





ب-۵۳.

از معادلهٔ ب-۱۰ :

$$I_{ZZ} = I_M = I_{xx}l^2 + I_{yy}m^2 + I_{zz}n^2 - 2I_{xy}lm - 2I_{xz}ln - 2I_{yz}mn$$

استفاده از بردار یکه : $\boldsymbol{\lambda} = l\,\mathbf{i} + m\,\mathbf{j} + n\,\mathbf{k}$

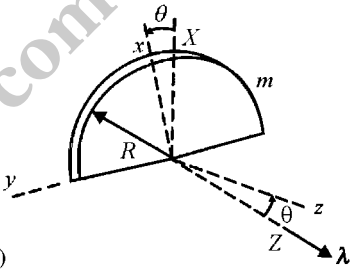
$$l = -\sin\theta\ ,\ m = 0\ ,\ n = \cos\theta$$

$$I_{xx} = \frac{1}{4}m\,R^2\ ,\quad I_{yy} = \frac{1}{4}m\,R^2\ ,\quad I_{zz} = \frac{1}{2}m\,R^2$$

$$I_{xy} = I_{xz} = I_{yz} = 0$$

$$\Rightarrow I_{ZZ} = \frac{1}{4}m\,R^2(\sin^2\theta) + \frac{1}{4}m\,R^2(0) + \frac{1}{2}m\,R^2(\cos^2\theta)$$

$$= \frac{1}{4}m\,R^2(\sin^2\theta + 2\cos^2\theta) = \frac{1}{4}m\,R^2(1 + \cos^2\theta)$$



ب-۵۴.

$$I_{xx} = I_{yy} = I_{zz} = 2\left(\frac{2}{5}m\,r^2 + m\,b^2\right) + \frac{2}{5}m\,r^2 = m\left(\frac{6}{5}r^2 + 2b^2\right) = I$$

$$I_{xy} = I_{yz} = I_{xz} = 0$$

بنابراین برای تمام محورهای OM گذرنده از O طبق معادله ب-۱۰ خواهیم داشت:

$$I_M = I(l^2 + m^2 + n^2) = I \qquad (n\ ,\ m\ ,\ l \text{ مستقل از})$$

ب-۵۵.

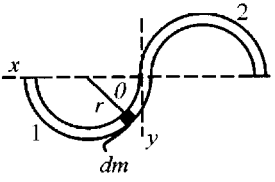
$$I_{xy_1} = I_{xy_2}$$

$$m_1 = \pi\,r\rho\ ,\quad m = 2\pi\,r\rho$$

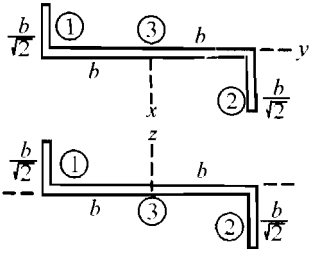
$$I_{xy_1} = \int xy\,dm = \int_0^{\pi} r(1 - \cos\theta)(r\sin\theta)\,\rho\,r\,d\theta$$

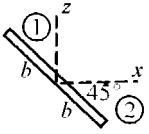
$$= \rho\,r^3\int_0^{\pi}(\sin\theta - \sin\theta\cos\theta)\,d\theta = \rho\,r^3\left[-\cos\theta - \frac{1}{2}\sin^2\theta\right]_0^{\pi} = 2\rho\,r^3$$

$$I_{xy} = 2\,I_{xy_1} = 4\rho\,r^3 = \frac{2m\,r^2}{\pi}\ ,\quad I_{xz}\ ,\ I_{yz} = 0$$



ب-۵۶.





① قسمت : $I_{xy} = \frac{m}{4}(-b)\left(-\frac{b}{2\sqrt{2}}\right) = \frac{m\,b^2}{8\sqrt{2}}$

② قسمت : $I_{xy} = \frac{m}{4}(b)\left(\frac{b}{2\sqrt{2}}\right) = \frac{m\,b^2}{8\sqrt{2}}$

③ قسمت : $I_{xy} = 0$

برای قسمت‌های ① و ② با جرم کلی $\frac{m}{2}$، از طریق انتگرال‌گیری می‌توان نشان داد که

$I_{xz} = -\frac{1}{12}m\,b^2$

③ قسمت : $I_{xz} = 0$

① قسمت : $I_{yz} = \frac{m}{4}(-b)\left(\frac{b}{2\sqrt{2}}\right) = -\frac{m\,b^2}{8\sqrt{2}}$

② قسمت : $I_{yz} = \frac{m}{4}(b)\left(-\frac{b}{2\sqrt{2}}\right) = -\frac{m\,b^2}{8\sqrt{2}}$

③ قسمت : $I_{yz} = 0$

کل مجموعه
$$\begin{cases} I_{xy} = \dfrac{m\,b^2}{4\sqrt{2}} \\ I_{xz} = -\dfrac{1}{12}m\,b^2 \\ I_{yz} = -\dfrac{m\,b^2}{4\sqrt{2}} \end{cases}$$

ب-۵۷. مبدأ محورهای x-y-z را در نقطه G در نظر بگیرید. طبق تقارن، کسینوسهای هادی AA عبارتند از:

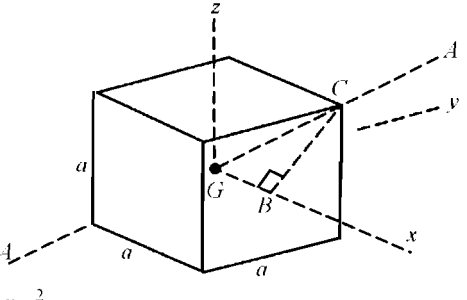
$l = m = n = \cos B\hat{G}C$

$\overline{GB} = \frac{a}{2}$, $BC = \frac{a\sqrt{2}}{2}$, $\overline{GC} = \frac{a}{2}\sqrt{3}$

$\Rightarrow l = m = n = \frac{1}{\sqrt{3}}$

$I_{xx} = I_{yy} = I_{zz} = \frac{1}{6}m\,a^2$, $I_{xy} = I_{xz} = I_{yz} = 0$

از معادله ب-۱۰ : $I_{AA} = 3\left(\frac{1}{6}m\,a^2\right)\left(\frac{1}{\sqrt{3}}\right)^2 = \frac{m\,a^2}{6}$



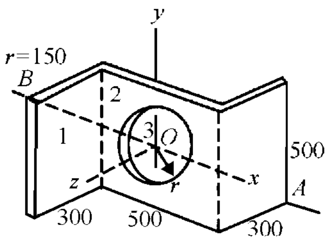
ب-۵۸.

$$\overline{OB} = \sqrt{(250)^2 + (250)^2 + (300)^2} = 464\ \text{mm}$$

OB کسینوسهای هادی $l = -\dfrac{250}{464} = -0.539$

$$m = \frac{250}{464} = 0.539$$

$$n = \frac{300}{464} = 0.647$$



جرم‌ها : $m_1 = m_4 = 7830(0.3)(0.5)(0.015) = 17.62\,\text{kg}$

$$m_3 = -7830\,(\pi)(0.150)^2(0.015) = -8.30\,\text{kg}$$

$$m_2 = 7830\,(0.5)^2(0.015) = 29.36\,\text{kg}$$

	①	②	③	④	مجموع (kg.m^2)
I_{xx}	0.896	0.612	−0.047	0.896	2.356
I_{yy}	1.630	0.612	−0.047	1.630	3.825
I_{zz}	1.468	1.223	−0.093	1.468	4.067
I_{xz}	−0.661	0	0	−0.661	−1.321

جای‌گذاری در معادله ب-۱۰ :

$$I_{AB} = 2.356(-0.539)^2 + 3.825(0.539)^2 + 4.067(0.647)^2 - 2(-1.321)(-0.539)(0.647)$$

($I_{xy} = I_{yz} = 0$ که در آن)

$$I_{AB} = 2.58\,\text{kg.m}^2$$

* ب-۵۹.

$$I_{xx} = m(\sqrt{2}\,l)^2 + m(\sqrt{2}\,l)^2 + m(2\,l)^2 = 8\,m\,l^2$$

$$I_{yy} = ml^2 + ml^2 + m(\sqrt{5}\,l)^2 = 7\,m\,l^2$$

$$I_{zz} = ml^2 + ml^2 + ml^2 = 3\,ml^2$$

$I_O = \dfrac{I}{m\,l^2}$ با ب-۱۱ :

طبق مسئله ب-۴۵ $\begin{cases} I_{xy} = 0 \\ I_{xz} = I_{yz} = -2\,m\,l^2 \end{cases}$

$$ml^2 \begin{vmatrix} (8 - I_O) & 0 & +2 \\ 0 & (7 - I_O) & +2 \\ +2 & +2 & (3 - I_O) \end{vmatrix} = 0$$

حل عددی معادله درجه ۳ : $I_1 = 9\,m\,l^2$, $I_2 = 7.37\,m\,l^2$, $I_3 = 1.628\,m\,l^2$

برای I_1، جواب معادلات ب-۱۲ به همراه $l_1^2 + m_1^2 + n_1^2 = 1$ منتهی می‌شود به:

$l_1 = 0.816$
$m_1 = 0.408$
$n_1 = 0.408$

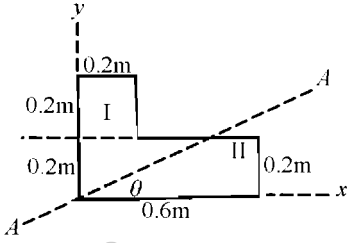
* ب-۶۰. معادله ب-۱۰ : $I_{AA} = I_{xx}l^2 + I_{yy}m^2 + I_{zz}n^2 - 2I_{xy}lm - 2I_{xz}ln - 2I_{yz}mn$

قسمت I : $m = 160(0.2)^2 = 6.40\,\text{kg}$

$$I_{xx} = \bar{I}_{xx} + md^2 = \frac{1}{12}(6.40)(0.2)^2 + 6.40(0.3)^2 = 0.597\ \text{kg.m}^2$$

$$I_{yy} = \frac{1}{3}(6.40)(0.2)^2 = 0.0853\,\text{kg.m}^2$$

$$I_{xy} = 6.40(0.1)(0.3) = 0.1920\,\text{kg.m}^2$$



قسمت II : $m = 160(0.6)(0.2) = 19.20\,\text{kg}$

$$I_{xx} = \frac{1}{3}(19.20)(0.2)^2 = 0.256\,\text{kg.m}^2$$

$$I_{yy} = \frac{1}{3}(19.20)(0.6)^2 = 2.30\,\text{kg.m}^2$$

$$I_{zz} = 19.20(0.3)(0.1) = 0.576\,\text{kg.m}^2$$

کل قطعه : $I_{xx} = 0.597 + 0.256 = 0.853\,\text{kg.m}^2$

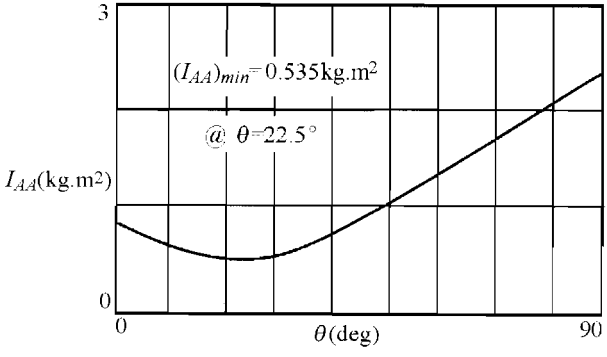
$$I_{yy} = 0.0853 + 2.30 = 2.39\,\text{kg.m}^2$$

$$I_{xy} = 0.1920 + 0.576 = 0.768\,\text{kg.m}^2$$

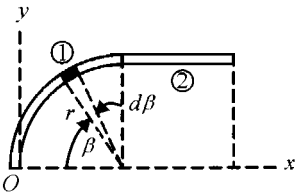
$I_{xz} = I_{yz} = 0$

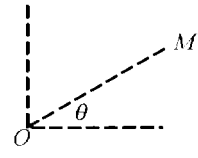
$l = \cos\theta$, $m = \sin\theta$, $n = \cos 90° = 0$

از معادله ب-۱۰ : $I_{AA} = 0.853\cos^2\theta + 2.39\sin^2\theta - 1.536\sin\theta\cos\theta$



* ب-۶۱.





$$① I_x = \frac{1}{4}\frac{1}{2}(4\,m\,r^2) = \frac{1}{4}\rho\pi r^3$$

$$I_y = \int_0^{\frac{\pi}{2}} [r(1-\cos\beta)]^2 \rho\, r\, d\beta$$

$$= \rho r^3\left(\frac{3\pi}{4} - 2\right)$$

$$I_{xy} = \int_0^{\frac{\pi}{2}} r(1-\cos\beta)\, r\sin\beta\, \rho\, r\, d\beta$$

$$= \rho r^3\left(-\cos\beta - \frac{1}{2}\sin^2\beta\right)_0^{\frac{\pi}{2}} = \frac{1}{2}\rho r^3$$

$$② I_x = \rho r^3$$

$$I_y = \rho r\left(\frac{r^2}{12} + \left[\frac{3r}{2}\right]^2\right) = \frac{7}{3}\rho r^3$$

$$I_{xy} = \rho r\left(\frac{3r}{2}\right)r = \frac{3}{2}\rho r^3$$

مجموع : $I_x = \rho r^3\left(\frac{\pi}{4} + 1\right) = 1.785\rho r^3$

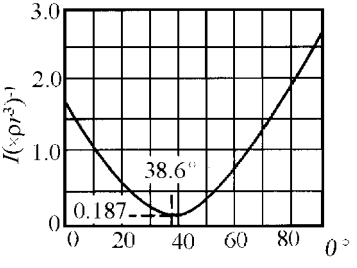
$$I_y = \rho r^3\left(\frac{3\pi}{4} - 2 + \frac{7}{3}\right) = 2.690\rho r^3$$

$$I_{xy} = \rho r^3\left(\frac{1}{2} + \frac{3}{2}\right) = 2\rho r^3$$

معادله الف-۹ کتاب استاتیک : $I_{OM} = I = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$

برنامه کامپیوتری بنویسید که رابطه قبل را برای $0 < \theta < 90$ حل نمایید.

چک با معادلات الف-۱۰ و الف-۱۱ کتاب استاتیک :



$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{4}{2.690 - 1.785}$$

$$2\alpha = 77.26^\circ \Rightarrow \alpha = 38.6^\circ$$

$$I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$= (2.237 - 2.050)\rho r^3 = 0.1870\rho r^3$$

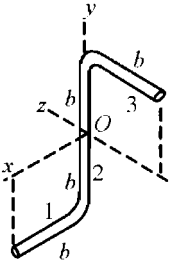
* ب-۶۲.

		①	②	③	مجموع
نتایج مسئله ب-۱۵	I_{xx}	$\frac{1}{4}mb^2$	$\frac{1}{6}mb^2$	$\frac{1}{3}mb^2$	$\frac{3}{4}mb^2$
	I_{yy}	$\frac{1}{12}mb^2$	0	$\frac{1}{12}mb^2$	$\frac{1}{6}mb^2$
	I_{zz}	$\frac{1}{3}mb^2$	$\frac{1}{6}mb^2$	$\frac{1}{4}mb^2$	$\frac{3}{4}mb^2$
نتایج مسئله ب-۴۹	I_{xy}	$-\frac{1}{8}mb^2$	0	0	$-\frac{1}{8}mb^2$
	I_{xz}	0	0	0	0
	I_{yz}	0	0	$-\frac{1}{8}mb^2$	$-\frac{1}{8}mb^2$

$I = I_O\, mb^2$

جای‌گذاری در معادله ب-۱۱

$$\begin{vmatrix} \frac{3}{4}-I_O & \frac{1}{8} & 0 \\ \frac{1}{8} & \frac{1}{6}-I_O & \frac{1}{8} \\ 0 & \frac{1}{8} & \frac{3}{4}-I_O \end{vmatrix} = 0$$



بسط : $I_O{}^3 - \frac{5}{3}I_O{}^2 + \frac{25}{32}I_O - \frac{9}{128} = 0$

معادله فوق را با برنامه کامپیوتری یا فرمول جبری حل کنید (در این مورد، بسط دترمینان منتهی به یک ضریب مشترک $\left(\frac{3}{4}-I_O\right)$ می‌شود. بنابراین معادله درجه سه به صورت زیر حل می‌شود:

$$\left(\frac{3}{4}-I_O\right)\left(\left[\frac{1}{6}-I_O\right]\left[\frac{3}{4}-I_O\right]-\frac{1}{32}\right)$$

یا

$$\left(\frac{3}{4}-I_O\right)\left(I_O{}^2-\frac{11}{12}I_O+\frac{3}{32}\right)=0$$

$$\Rightarrow I_{O_1} = 0.750 \qquad I_1 = 0.750\, mb^2$$

$$I_{O_2} = 0.799 \quad \text{یا} \quad I_2 = 0.799\, mb^2$$

$$I_{O_3} = 0.1173 \qquad I_3 = 0.1173\, mb^2$$

برای $I_3 = 0.1173\, mb^2$ (ممان اینرسی مینیمم) کسینوس‌های هادی که در معادله ب-۱۲ صدق کنند و نیز $l^2 + m^2 + n^2 = 1$ خواهیم داشت:

$$\left.\begin{aligned} 0.633\,l + 0.125\,m + 0 &= 0 \\ 0.125\,l + 0.0494\,m + 0.125n &= 0 \\ 0 + 0.125\,m + 0.633\,n &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} l &= 0.1903 \\ m &= -0.963 \\ n &= 0.1903 \end{aligned}$$

* ب-۶۳.

$$I_{xx} = \frac{1}{3}\rho b^4 + \frac{1}{12}(2b^2)(b^2 + 4b^2) + \rho(2b^2)\left(\frac{b^2}{4} + b^2\right) = \frac{11}{3}\rho b^4$$

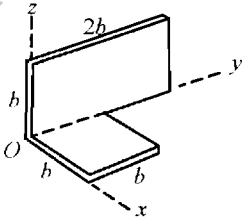
$$I_{yy} = \frac{1}{3}\rho b^4 + \frac{1}{3}\rho(2b^2)b^2 = \rho b^4$$

$$I_{zz} = \frac{1}{12}\rho b^2(b^2 + b^2) + \rho b^2\left(\frac{b^2}{4} + \frac{b^2}{4}\right) + \frac{1}{3}\rho(2b^2)(2b)^2 = \frac{10}{3}\rho b^4$$

$$I_{xy} = \rho b^2\left(\frac{b}{2}\,\frac{b}{2}\right) = \frac{1}{4}\rho b^4$$

$$I_{xz} = 0$$

$$I_{yz} = \rho b^4$$



جای‌گذاری در معادله ب-۱۱ و $I = I_O \rho b^4$:

$$\begin{vmatrix} \frac{11}{3} - I_O & -\frac{1}{4} & 0 \\ -\frac{1}{4} & 1 - I_O & -1 \\ 0 & -1 & \frac{10}{3} - I_O \end{vmatrix} = 0$$

بسط : $I_O^3 - 8I_O^2 + 18.160 I_O - 8.347 = 0$

با حل کامپیوتری یا فرمول جبری خواهیم داشت:

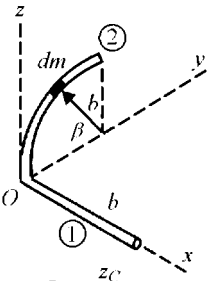
$$I_1 = 3.78\rho b^4 \quad , \quad I_2 = 0.612\rho b^4 \quad , \quad I_3 = 3.61\rho b^4$$

* ب-۶۴.

① قسمت : $I_{xx} = 0 \quad , \quad I_{yy} = I_{zz} = \frac{1}{3}\rho b^3$

$$I_{xy} = I_{xz} = I_{yz} = 0$$

② قسمت : I_{xx} , I_{yy} , I_{zz} طبق تقارن نصف ممان‌های نیم‌حلقه هستند



$$I_O = I_G + m(b - \overline{r})^2$$

$$= I_C - m\overline{r}^2 + m(b - \overline{r})^2$$

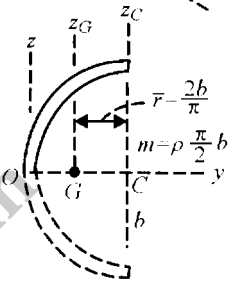
$$= m b^2 + m(b^2 - 2b\overline{r})$$

$$= \rho \frac{\pi}{2} b^3 \left[1 + 1 - \frac{4}{\pi}\right] = \rho b^3 (\pi - 2)$$

$$I_{yy} = \frac{1}{2} m b^2 = \frac{1}{4} \rho \pi b^3$$

$$I_{zz} + I_{yy} = I_O$$

$$\Rightarrow I_{zz} = \rho b^3 \left(\frac{3\pi}{4} - 2\right)$$



$$I_{xy} = 0 \quad , \quad I_{xz} = 0 \quad , \quad I_{yz} = \int y z \, dm$$

$$I_{yz} = \int_0^{\frac{\pi}{2}} (b - b\cos\beta)(b\sin\beta) \rho b \, d\beta = \frac{1}{2} \rho b^3$$

برای میله کامل :

$$I_{xx} = \rho b^3 (\pi - 2) = 1.1416 \rho b^3 \quad , \quad I_{xy} = 0$$

$$I_{yy} = \rho b^3 \left(\frac{\pi}{4} + \frac{1}{3}\right) = 1.1187 \rho b^3 \quad , \quad I_{xz} = 0$$

$$I_{zz} = \rho b^3 \left(\frac{3\pi}{4} - \frac{5}{3}\right) = 0.6895 \rho b^3 \quad , \quad I_{yz} = \frac{1}{2} \rho b^3$$

جای‌گذاری در معادله ب-۱۱ $\left(\frac{I}{\rho b^3} = I'\right)$:

$$\begin{vmatrix} (1.1416 - I') & 0 & -0 \\ -0 & (1.1187 - I') & -0.5 \\ -0 & -0.5 & (0.6895 - I') \end{vmatrix} = 0$$

بسط : $I'^3 - 2.950 I'^2 + 2.586 I' - 0.5952 = 0$

حل عددی یا جبری :

$$I_1 = 1.448 \rho b^3$$

$$I_2 = 0.360 \rho b^3$$

$$I_3 = 1.142 \rho b^3$$

طبق معادله ب-۱۲، کسینوس‌های هادی برای محور I_2 عبارتند از:

$$(1.1416 - 0.360) l - (0) m - (0) n = 0 \quad (1)$$

$$-(0) l + (1.1187 - 0.360) m - 0.5 n = 0 \quad (2)$$

$$-(0) l - 0.5 m + (0.6895 - 0.360) n = 0 \quad (3)$$

$$\Rightarrow l = 0 \quad , \quad m = 0.5503 \quad , \quad n = 0.8350$$