توابع مثلثاتی

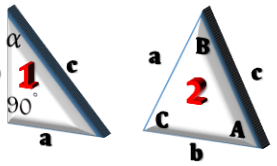
مقدمات

1

$c^2 = a^2 + b^2$

$\sin\alpha = \frac{a}{c} \qquad \cos\alpha = \frac{b}{c}$

$\tan\alpha = \frac{a}{b} \qquad \cot\alpha = \frac{b}{a}$



2

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

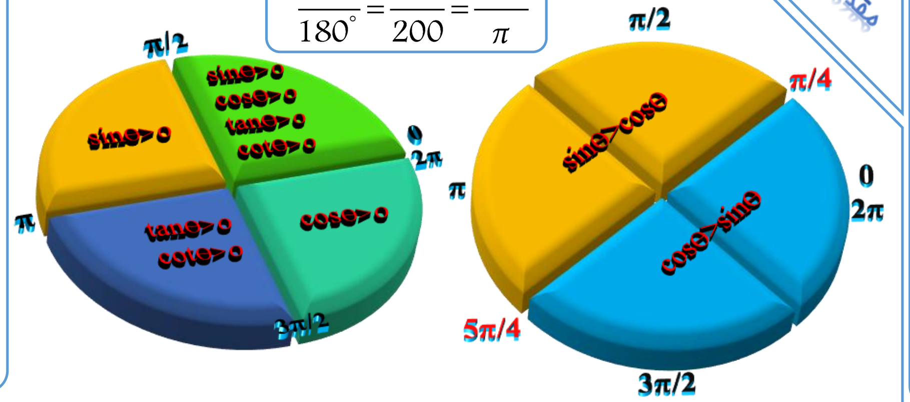
$c^2 = a^2 + b^2 - 2ab\cos C$

$a = b\cos C + c\cos B$

$b = a\cos C + c\cos A$

$c = a\cos B + b\cos A$

$\frac{D^\circ}{180^\circ} = \frac{G^{gr}}{200} = \frac{R^{Rad}}{\pi}$



جدول زوایا

نسبت \ زاویه	0	30 $\frac{\pi}{6}$	45 $\frac{\pi}{4}$	60 $\frac{\pi}{3}$	90 $\frac{\pi}{2}$	180 π	270 $\frac{3\pi}{2}$	360 2π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	+1	0	−1	0
$\cos\theta$	+1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	−1	0	+1
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	+1	$\sqrt{3}$	∞	0	∞	0
$\cot\theta$	∞	$\sqrt{3}$	+1	$\frac{\sqrt{3}}{3}$	0	∞	0	∞

روابط مقدماتی

$\sin^2 x + \cos^2 x = 1$

$-\sqrt{a^2+b^2} \le a\sin x \pm b\cos x \le +\sqrt{a^2+b^2}$

$\tan x = \frac{\sin x}{\cos x}$ * $\cot x = \frac{\cos x}{\sin x}$ * $\tan x\cot x = 1 \to \begin{cases}\tan x = \frac{1}{\cot x}\\ \cot x = \frac{1}{\tan x}\end{cases}$

$\sec x = \frac{1}{\cos x}$ *** $\csc x = \frac{1}{\sin x}$

$1+\tan^2 x = \frac{1}{\cos^2 x} = \sec^2 x$ *** $1+\cot^2 x = \frac{1}{\sin^2 x} = \csc^2 x$

روابط کلیدی و مهم

$\cos^2 x = \frac{1+\cos 2x}{2}$ *** $\sin^2 x = \frac{1-\cos 2x}{2}$ *** $\tan^2 x = \frac{1-\cos 2x}{1+\cos 2x} = \frac{\tan x}{\cot x}$ *** $\cot^2 x = \frac{1+\cos 2x}{1-\cos 2x} = \frac{\cot x}{\tan x}$

$(\sin x \pm \cos x)^2 = 1 \pm \sin 2x$ * $\sin x + \cos x = \sqrt{2}\sin\left(x+\frac{\pi}{4}\right) = +\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)$ * $\sin x - \cos x = \sqrt{2}\sin\left(x-\frac{\pi}{4}\right) = -\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)$

$\tan x + \cot x = \frac{2}{\sin 2x}$ *** $\tan x - \cot x = -2\cot 2x$ *** $\frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4} - x\right)$ *** $\frac{1+\tan x}{1-\tan x} = \tan\left(\frac{\pi}{4} + x\right)$

$\cos^4 x + \sin^4 x = 1 - \frac{1}{2}\sin^2 2x \quad \because \begin{cases} 1 = (\cos^2 x + \sin^2 x)^2 = \cos^4 x + \sin^4 x + 2\cos^2 x\sin^2 x \ldots \\ 2\cos^2 x\sin^2 x = 2\left(\frac{1}{4}\sin^2 2x\right) = \frac{1}{2}\sin^2 2x \end{cases}$ *** $\cos^4 x - \sin^4 x = \cos 2x$

$\tan^2 x - \sin^2 x = \tan^2 x\sin^2 x \qquad \cot^2 x - \cos^2 x = \cot^2 x\cos^2 x$

$\sin(\alpha-\beta)\sin(\alpha+\beta) = \cos^2\beta - \cos^2\alpha = \sin^2\alpha - \sin^2\beta \qquad \cos(\alpha-\beta)\cos(\alpha+\beta) = \cos^2\alpha - \sin^2\beta$

$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x} \quad \because \quad \sin 2x = \frac{2\tan x}{1+\tan^2 x} = 2\tan x\cos^2 x = 2\tan x\left(\frac{1+\cos 2x}{2}\right) \Rightarrow \tan x = \frac{\sin 2x}{1+\cos 2x}$

if $\alpha+\beta = \frac{\pi}{2} \Rightarrow \sin\alpha = \cos\beta$ * $\tan\alpha = \cot\beta$ *** if $\alpha+\beta = \pi \Rightarrow \sin\alpha = \sin\beta$ * $\cos\alpha = -\cos\beta$ * $\tan\alpha = -\tan\beta$ * $\cot\alpha = -\cot\beta$

if $\alpha+\beta = \frac{\pi}{4} \Rightarrow \tan\alpha + \tan\beta = 1 - \tan\alpha\tan\beta$ *** if $\alpha+\beta = \frac{3\pi}{4} \Rightarrow \tan\alpha + \tan\beta = \tan\alpha\tan\beta - 1$

معادلات مثلثاتی

$\sin x = \sin\alpha \Rightarrow \begin{cases} x = 2k\pi + \alpha \\ x = 2k\pi + (\pi - \alpha)\end{cases} \quad \therefore \quad \sin x = 0 \Rightarrow x = k\pi$

$\cos x = \cos\alpha \Rightarrow x = 2k\pi \pm \alpha \quad \therefore \quad \cos x = 0 \Rightarrow x = k\pi \pm \frac{\pi}{2}$

$\tan x = \tan\alpha \Rightarrow x = k\pi + \alpha \quad \therefore \quad \tan x = 0 \Rightarrow x = k\pi$

$\cot x = \cot\alpha \Rightarrow x = k\pi + \alpha \quad \therefore \quad \cot x = 0 \Rightarrow x = k\pi \pm \frac{\pi}{2}$

$\tan x = \cot x \Rightarrow x = k\pi \pm \frac{\pi}{4}$

$\left.\begin{aligned} \sin^2 x &= \sin^2\alpha \\ \cos^2 x &= \cos^2\alpha \\ \tan^2 x &= \tan^2\alpha \\ \cot^2 x &= \cot^2\alpha \end{aligned}\right\} \Rightarrow x = k\pi \pm \alpha$

تبدیل کمان

1. برای بدست آوردن نسبت های مثلثاتی کمان هایی که بصورت $k\pi + \alpha$ هستند مضارب زوج π را نادیده و مضارب فرد آن را همان π در نظر میگیریم و سپس ناحیه کمان را مشخص و علامت نسبت مذکور را در آن ناحیه بر اساس دایره فوق تعیین و پشت آن قرار داده و سپس π را حذف می کنیم.

2. این کار (تعیین علامت) عیناً برای کمان هایی که بصورت $\frac{k\pi}{2} + \alpha$ هستند انجام می شود اما در مضارب فرد $\frac{\pi}{2}$ جای $\sin$ و $\cos$ و همچنین $\tan$ و $\cot$ را با هم عوض می کنیم.

مثال :

$\sin\left(\frac{3\pi}{2} + \alpha\right) = -\cos\alpha$

$\because \quad \frac{3\pi}{2} + \alpha \to$ ربع چهارم در $\sin\alpha < 0$

$\tan(\pi + \alpha) = +\tan\alpha$

$\because \quad \pi + \alpha \to$ ربع سوم در $\tan\alpha > 0$

کمان(α-)

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\tan(-x) = -\tan x$

$\cot(-x) = -\cot x$

Trigonometry

نسبت مجموع

$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$

$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha\pm\beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\cot(\alpha\pm\beta) = \frac{\cot\alpha\cot\beta \mp 1}{\cot\beta \pm \cot\alpha}$

نسبت های 3x

$\sin 3x = 3\sin x - 4\sin^3 x$

$\cos 3x = 4\cos^3 x - 3\cos x$

$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

$\cot 3x = \frac{3\cot x - \cot^3 x}{1 - 3\cot^2 x}$

نسبت های 2x

$\sin 2x = 2\sin x\cos x \to \begin{cases} = \frac{2\tan x}{1+\tan^2 x} \\ = 2\tan x\cos^2 x \end{cases}$

$\cos 2x = \cos^2 x - \sin^2 x \to \begin{cases} = 2\cos^2 x - 1 = 1 - 2\sin^2 x \\ = \frac{1-\tan^2 x}{1+\tan^2 x} = \frac{\cot^2 x - 1}{\cot^2 x + 1} \end{cases}$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2}{\cot x - \tan x}$

$\cot 2x = \frac{\cot^2 x - 1}{2\cot x} = \frac{\cot x - \tan x}{2}$

جمع به ضرب

$\sin p \pm \sin q = 2\sin\left(\frac{p\pm q}{2}\right)\cos\left(\frac{p\mp q}{2}\right)$

$\cos p + \cos q = 2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$

$\cos p - \cos q = -2\sin\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$

$\tan p \pm \tan q = \frac{\sin(p\pm q)}{\sin p\sin q}$

$\cot p \pm \cot q = \frac{\sin(q\pm p)}{\sin p\sin q}$

ضرب به جمع

$\sin\alpha\cos\beta = \frac{1}{2}\left(\sin(\alpha+\beta) + \sin(\alpha-\beta)\right)$

$\cos\alpha\cos\beta = \frac{1}{2}\left(\cos(\alpha-\beta) + \cos(\alpha+\beta)\right)$

$\sin\alpha\sin\beta = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)$

$\tan\alpha\tan\beta = \frac{\tan\alpha + \tan\beta}{\cot\alpha + \cot\beta}$ *** $\cot\alpha\cot\beta = \frac{\cot\alpha + \cot\beta}{\tan\alpha + \tan\beta}$

توابع معکوس مثلثاتی

$\text{Arctan}\,x + \text{Arctan}\,y = \text{Arctan}\left(\frac{x+y}{1-xy}\right)$

$xy < 1$

$\text{Arctan}\,x - \text{Arctan}\,y = \text{Arctan}\left(\frac{x-y}{1+xy}\right)$

$\text{Arcsin}\,x + \text{Arccos}\,x = \frac{\pi}{2}$ *** $-\frac{\pi}{2} \le \text{Arcsin}\,x \le +\frac{\pi}{2}$ *** $0 \le \text{Arccos}\,x \le \pi$

$\sin(\text{Arcsin}\,x) = \cos(\text{Arccos}\,x) = x$ *** $\text{Arcsin}(\sin x) = \text{Arccos}(\cos x) = x \quad -1 \le x \le +1$

$\sin(\text{Arccos}\,x) = \cos(\text{Arcsin}\,x) = \sqrt{1-x^2}$

$\text{Arcsin}\,x = \text{Arccos}\sqrt{1-x^2}$ *** $\text{Arccos}\,x = \text{Arcsin}\sqrt{1-x^2}$

$\text{Arctan}\,x = \begin{cases} \text{Arccot}\frac{1}{x} & x>0 \\ \pi - \text{Arccot}\frac{1}{x} & x<0 \end{cases}$

$\text{Arccot}\,x = \begin{cases} \text{Arctan}\frac{1}{x} & x>0 \\ \pi + \text{Arctan}\frac{1}{x} & x<0 \end{cases}$

$\text{Arctan}\,x + \text{Arccot}\,x = \frac{\pi}{2}$ *** $-\frac{\pi}{2} \le \text{Arctan}\,x \le +\frac{\pi}{2}$ *** $0 \le \text{Arccot}\,x \le \pi$

$\tan(\text{Arctan}\,x) = \cot(\text{Arccot}\,x) = x$ *** $\tan(\text{Arccot}\,x) = \cot(\text{Arctan}\,x) = \frac{1}{x}$

if $x + y + z = xyz \Rightarrow \text{Arctan}\,x + \text{Arctan}\,y + \text{Arctan}\,z = 0\ ,\ \pi$

$\text{Arctan}\,x + \text{Arctan}\frac{1}{x} = \begin{cases} +\frac{\pi}{2} & x>0 \\ -\frac{\pi}{2} & x<0 \end{cases}$

$\text{Arccot}\,x + \text{Arccot}\frac{1}{x} = \begin{cases} \frac{\pi}{2} & x>0 \\ \frac{3\pi}{2} & x<0 \end{cases}$

$\text{Arcsin}(-x) = -\text{Arcsin}(x)$

$\text{Arctan}(-x) = -\text{Arctan}(x)$

****

$\text{Arccos}(-x) = \pi - \text{Arccos}(x)$

$\text{Arccot}(-x) = \pi - \text{Arccot}(x)$

$y = \sin x \Rightarrow x = \text{Arcsin}\,y \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \ ,\ y \in [-1,1]$

$y = \cos x \Rightarrow x = \text{Arccos}\,y \quad x \in [0,\pi] \ ,\ y \in [-1,1]$

$y = \tan x \Rightarrow x = \text{Arctan}\,y \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \ ,\ y \in R$

$y = \cot x \Rightarrow x = \text{Arccot}\,y \quad x \in [0,\pi] \ ,\ y \in R$